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On Artin Cokernel of the Group ($Q_{2m} \times D_3$) where m= 2^h p and p is prime number

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Abstract:

The main purpose of This paper is to find Artin's character table $\boldsymbol{Ar(Q_{2m} \times D_3)}$ *when m is even number such that m =2p,and p is a prime number ;where* Q_{2m} *is denoted to Quaternion group of order 4m, time is said to have only one dimension and space to have three dimension ,the mathematical quaternion partakes of both these elements; in technical language it may be said to be "time plus space", or "space plus time" and in this sense it has , or at least involves a reference to four dimensions ,and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols.,1882,1885,1889)) ,and* $\boldsymbol{D_3}$ *is Dihedral group of order 6.In 1962, C.W.Curits & I.Reiner studied Representation Theory of finite groups ,*

In 1976, I.M.Isaacs studied Characters Theory of finite groups, In 1982, M.S.Kirdar studied The Factor Group of the Z-Valued class function modulo the group of the Generalized Characters , In1994, H.H.Abass studies The Factor Group of class function over the group of Generalized Characters of D_n *and found* $\equiv^*(D_n)$*, In 1995,*

N.R.Mahmood studies The Cyclic Decomposition of the factor Group cf(Q_{2m},Z)/ $\bar{R}$(Q_{2m}), In 2002, K-Sekiguchi studies Extensions and the Irreducibilies of the Induced Characters of Cyclic P-Group, In 2006, A.S.Abid studies Characters Table of Dihedral Group for Odd number.

Key words: words: even number, prime number, Quaternion group, and Dihedral group.

1. INTRODUCTION:

Representation Theory is a branch of mathematics that studies abstract algebra structures by representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication ,In which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication. Moreover, representation and character theory provide applications ,no only in other branches of mathematics but also in physics and chemistry.

For a finite group G ,The factor group **$\bar{R}$(G)/T(G)** is called the Artin cokernel of **G** denoted **AC(G)**, **$\bar{R}$(G)** denoted the a belian group generated by Z-valued characters of G under the operation of pointwise addition**, T(G)** is asubgroup of **$\bar{R}$(G)** which is generated by Artin›s characters.

2. PRELIMINARS: [1]:(3,1)

<u>The Generalized Quaternion Group Q_{2m}:</u>

For each positive integer m≥2 ,The generalized Quaternion Group **Q_{2m}** of order 4m with two generators x and y satisfies **Q_{2m}** =$\{x^h\ y^k, 0 \leq h \leq 2m\text{-}1, k=0,1\}$

Which has the following properties $\{x^{2m}=y^4=I, yx^my^{-1}=x^{-m}\}$.

Let **G** be a finite group, all the characters of group **G** induced from a principal character of cyclic subgroup of **G** are called Artin characters of **G**.

Artin characters of the finite group can be displayed in a table called Artin characters table of **G** which is denoted by **Ar(G);** The first row is Γ-conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G(cl_a)|$ and other rows contains the values of Artin characters .

Theorem:[2]:(3,3)

The general form of Artin characters table of $\mathbf{Cp^s}$
When p is a prime number and s is a positive integer number is given by :-
$\mathbf{Ar(Cp^s)=}$

Γ-classes	*[1]*	$[x^{ps-1}]$	$[x^{ps-2}]$	…	*[x]*
$\lvert cl_a \rvert$	1	1	1	…	1
$\lvert cp^s(cl_a) \rvert$	p^s	ps	p^s	…	p^s
Φ_1	p^s	0	0	…	0
Φ_2	P^{s-1}	P^{s-1}	0	…	0
⋮	⋮	⋮	⋮	…	⋮
Φ_s	p	p	p	…	0
Φ_{s+1}	1	1	1	…	1

Table(1)

3. THE MAIN RESULTS:

Theorem: (4,1)

The Artin's character table of the group $(\mathbf{Q_{2.2^hp} \times D_3})$ where $m=2^hp$ such that h is any positive integer and p is prime number, it is given as follows:

$\mathbf{Ar(Q_{2.2^hp} \times D_3)=}$

	Γ-classes of $Q_{2.2^h p}$ × { 1 }															
Γ-classes	[l,1]	$[x^{2^{\frac{h}{p}}},1]$	$[x^{2^{\frac{h}{p}-1}},1]$	$[x^{2^{\frac{h}{p}-2}},1]$	⋯	$[x^p,1]$	$[x^{2^{\frac{h}{2}}},1]$	$[x^{2^{\frac{h}{2}-1}},1]$	$[x^{2^{\frac{h}{2}-2}},1]$	⋯	$[x^2,1]$	[x,1]	[y,1]	[xy,1]		
$\|CL_a\|$	1	1	2	2	⋯	2	2	2	2	⋯	2	2	2m	2m		
$\|CQ_{2.2^h p}\times D_3(CL_a)\|$	24m	24m	12m	12m	⋯	12m	12m	12m	12m	⋯	12m	12m	24	24		
Φ(l,1)																
Φ(2,1)																
Φ(3,1)	6Ar($Q_{2.2^h p}$)															
⋮																
Φ(l,1)																
Φ(l+1,1)																
Φ(l+2,1)																

	Γ-classes of $Q_{2.2^h p}$ × { r }															
Γ-classes	[l,r]	$[x^{2^{\frac{h}{p}}},r]$	$[x^{2^{\frac{h}{p}-1}},r]$	$[x^{2^{\frac{h}{p}-2}},r]$	⋯	$[x^p,r]$	$[x^{2^{\frac{h}{2}}},r]$	$[x^{2^{\frac{h}{2}-1}},r]$	$[x^{2^{\frac{h}{2}-2}},r]$	⋯	$[x^2,r]$	[x,r]	[y,r]	[xy,r]		
$\|CL_a\|$	2	2	4	4	⋯	4	4	4	4	⋯	4	4	4m	4m		
$\|CQ_{2.2^h p}\times D_3(CL_a)\|$	12m	12m	6m	6m	⋯	6m	6m	6m	6m	⋯	6m	6m	24	24		
Φ(l,1)																
Φ(2,1)																
Φ(3,1)																
⋮	0															
Φ(l,1)																
Φ(l+1,1)																
Φ(l+2,1)																

	Γ-classes of $Q_{2.2^h p}$ × { s}															
Γ-classes	[l,s]	$[x^{2^{\frac{h}{p}}},s]$	$[x^{2^{\frac{h}{p}-1}},s]$	$[x^{2^{\frac{h}{p}-2}},s]$	⋯	$[x^p,s]$	$[x^{2^{\frac{h}{2}}},s]$	$[x^{2^{\frac{h}{2}-1}},s]$	$[x^{2^{\frac{h}{2}-2}},s]$	⋯	$[x^2,s]$	[x,s]	[y,s]	[xy,s]		
$\|CL_a\|$	3	3	6	6	⋯	6	6	6	6	⋯	6	6	6m	6m		
$\|CQ_{2.2^h p}\times D_3(CL_a)\|$	8m	8m	4m	4m	⋯	4m	4m	4m	4m	⋯	4m	4m	8	8		
Φ(l,1)																
Φ(2,1)																
Φ(3,1)																
⋮	0															
Φ(l,1)																
Φ(l+1,1)																
Φ(l+2,1)																

	Γ-classes of $Q_{2.2^h p}$ × { 1 }															
Γ-classes	[l,1]	$[x^{2^{\frac{h}{p}}},1]$	$[x^{2^{\frac{h}{p}-1}},1]$	$[x^{2^{\frac{h}{p}-2}},1]$	⋯	$[x^p,1]$	$[x^{2^{\frac{h}{2}}},1]$	$[x^{2^{\frac{h}{2}-1}},1]$	$[x^{2^{\frac{h}{2}-2}},1]$	⋯	$[x^2,1]$	[x,1]	[y,1]	[xy,1]		
$\|CL_a\|$	1	1	2	2	⋯	2	2	2	2	⋯	2	2	2m	2m		
$\|CQ_{2.2^h p}\times D_3(CL_a)\|$	24m	24m	12m	12m	⋯	12m	12m	12m	12m	⋯	12m	12m	24	24		
Φ(l,2)																
Φ(2,2)																
Φ(3,2)	2Ar($Q_{2.2^h p}$)															
⋮																
Φ(l,2)																
Φ(l+1,2)																
Φ(l+2,2)																

	Γ-classes of $Q_{2.2^h_p}\times\{r\}$														
Γ-classes	[I,r]	$[x^{2_p^h},r]$	$[x^{2_p^{h-1}},r]$	$[x^{2_p^{h-2}},r]$	⋯	$[x^p,r]$	$[x^{2_2^h},r]$	$[x^{2_2^{h-1}},r]$	$[x^{2_2^{h-2}},r]$	⋯	$[x^2,r]$	[x,r]	[y,r]	[xy,r]
$\lvert CL_a\rvert$	2	2	4	4	⋯	4	4	4	4	⋯	4	4	4m	4m
$\lvert C_{Q_{2.2^h_p}\times D_3}(CL_a)\rvert$	12m	12m	6m	6m	⋯	6m	6m	6m	6m	⋯	6m	6m	24	24
Φ(l,2)														
Φ(2,2)														
Φ(3,2)														
⋮	$2Ar(Q_{2.2^h_p})$													
Φ(l,2)														
Φ(l+1,2)														
Φ(l+2,2)														

	Γ-classes of $Q_{2.2^h_p}\times\{s\}$														
Γ-classes	[I,s]	$[x^{2_p^h},s]$	$[x^{2_p^{h-1}},s]$	$[x^{2_p^{h-2}},s]$	⋯	$[x^p,s]$	$[x^{2_2^h},s]$	$[x^{2_2^{h-1}},s]$	$[x^{2_2^{h-2}},s]$	⋯	$[x^2,s]$	[x,s]	[y,s]	[xy,s]
$\lvert CL_a\rvert$	3	3	6	6	⋯	6	6	6	6	⋯	6	6	6m	6m
$\lvert C_{Q_{2.2^h_p}\times D_3}(CL_a)\rvert$	8m	8m	4m	4m	⋯	4m	4m	4m	4m	⋯	4m	4m	8	8
Φ(l,2)														
Φ(2,2)														
Φ(3,2)														
⋮	0													
Φ(l,2)														
Φ(l+1,2)														
Φ(l+2,2)														

	Γ-classes of $Q_{2.2^h_p}\times\{1\}$														
Γ-classes	[I,1]	$[x^{2_p^h},1]$	$[x^{2_p^{h-1}},1]$	$[x^{2_p^{h-2}},1]$	⋯	$[x^p,1]$	$[x^{2_2^h},1]$	$[x^{2_2^{h-1}},1]$	$[x^{2_2^{h-2}},1]$	⋯	$[x^2,1]$	[x,1]	[y,1]	[xy,1]
$\lvert CL_a\rvert$	1	1	2	2	⋯	2	2	2	2	⋯	2	2	2m	2m
$\lvert C_{Q_{2.2^h_p}\times D_3}(CL_a)\rvert$	24m	24m	12m	12m	⋯	12m	12m	12m	12m	⋯	12m	12m	24	24
Φ(l,3)														
Φ(2,3)														
Φ(3,3)	$3Ar(Q_{2.2^h_p})$													
⋮														
Φ(l,3)														
Φ(l+1,3)														
Φ(l+2,3)														

	Γ-classes of $Q_{2.2^h_p}\times\{r\}$														
Γ-classes	[I,r]	$[x^{2_p^h},r]$	$[x^{2_p^{h-1}},r]$	$[x^{2_p^{h-2}},r]$	⋯	$[x^p,r]$	$[x^{2_2^h},r]$	$[x^{2_2^{h-1}},r]$	$[x^{2_2^{h-2}},r]$	⋯	$[x^2,r]$	[x,r]	[y,r]	[xy,r]
$\lvert CL_a\rvert$	2	2	4	4	⋯	4	4	4	4	⋯	4	4	4m	4m
$\lvert C_{Q_{2.2^h_p}\times D_3}(CL_a)\rvert$	12m	12m	6m	6m	⋯	6m	6m	6m	6m	⋯	6m	6m	24	24
Φ(l,3)														
Φ(2,3)														
Φ(3,3)														
⋮														
Φ(l,3)														
Φ(l+1,3)	0													
Φ(l+2,3)														

	Γ-classes of $Q_{2.2^h p}\times\{s\}$													
Γ-classes	$[I,s]$	$[x^{2^h_p},s]$	$[x^{2^{h-1}_p},s]$	$[x^{2^{h-2}_p},s]$	$\cdots$	$[x^p,s]$	$[x^{2^h_2},s]$	$[x^{2^{h-1}_2},s]$	$[x^{2^{h-2}_2},s]$	$\cdots$	$[x^2,s]$	$[x,s]$	$[y,s]$	$[xy,s]$
$\lvert CL_\alpha\rvert$	3	3	6	6	$\cdots$	6	6	6	6	$\cdots$	6	6	6m	6m
$\lvert C_{Q_{2.2^h p}\times D_3}(CL_\alpha)\rvert$	8m	8m	4m	4m	$\cdots$	4m	4m	4m	4m	$\cdots$	4m	4m	8	8
$\Phi(1,3)$														
$\Phi(2,3)$														
$\Phi(3,3)$														
$\vdots$							$Ar(Q_{2.2^h p})$							
$\Phi(l,3)$														
$\Phi(l+1,3)$														
$\Phi(l+2,3)$														

Table 2

<u>Proof:-</u>

Let g_{ij}=(qi,dj) ; qi $\in Q_{2.2^h p}$,dj $\in D_3$

Case (I):-

*Consider the group **G=($Q_{2.2^h p}$×D_3)** and if H is a cyclic subgroup of **($Q_{2.2^h p}$×{I})** then H=<(q,1)> and Φ the principle character of H and Φ_j Artin's characters of **$Q_{2.2^h p}$** ,1≤j≤i+2 ,The cyclic subgroup of **$Q_{2.2^h p}$** which are {<I>},{$<x^{2^h_p}>$},*
{$<x^{2^{h-1}_p}>$},{$<x^{2^{h-2}_p}>$},⋯ ,{$<x^p>$},{$<x^{2^h_2}>$},{$<x^{2^{h-1}_2}>$},{$<x^{2^{h-2}_2}>$},
⋯, {$<x^2>$},{<x>},{<y>},{<xy>}
*and cyclic subgroup of **D_3** which are {<I>},{<r>},{<s>}, by using theorem:*

$$\emptyset_j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & \text{if } hi \in H\cap cl(g) \\ 0 & \text{if } H\cap cl(g)=\emptyset \end{cases}$$

H=<q,1>:-

1:-H_{11} =<(I,1)> if g=(I,1) then $\emptyset_{11}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset$ (g)=$\frac{24.2^h p}{|CH(g)|}$ .1=$\frac{6|CQ2m(1)|}{|C<x>(1)|}$ .1=6.$\emptyset_j(1)$
since H∩cl(I,1)=(I,1)

2:-H_{21} =<($x^{2^h_p}$,1)>; (a)if g=(I,1)then$\emptyset_{21}$ (g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$ =$\frac{24.2^{hp}}{|CH(g)|}$ .1=$\frac{6|CQ2m(1)}{|C<x>(x2^h p)|}$ .1= 6$\emptyset_j(I)$

(b)if $g=(x^{2^h_p},1)$ *then* $\emptyset_{11}(g)=\frac{|CG(g)|}{|CH(g)|}.\emptyset(g)=\frac{24.2^hp}{|CH(g)|}.1=\frac{24.2^hp}{|C<x>(x^{2^h}p)|}$ $.1=\frac{6|CQ2m(x^{2h}p)|}{|C<x>(x^{2^{hp}})|}.1=6\emptyset_j(x^{2^h_p})$

since $H\cap cl(x^{2^h_p})=(I,1),(x^{2^h_p},1)$ *otherwise=0.*

3:- if $g\neq(x^{2^h_p},1)$ *then* $\emptyset_{j,1}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2^hp}{|C<x>(x^q)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^q)|}.2=6\emptyset_j(x^q)$

Since $H\cap cl(g)=\{g,g^{-1}\}$ *and* $\emptyset(g)=\emptyset(g^{-1})=1$; *since* $g=(q,I)$, $q\in Q_{2.2^hp}$, $q\neq x^{2^h_p}$

If $g\notin H$ *then* $\Phi_{j,1}(g)=0=6.0=0\ \emptyset_j(q)$ *since* $H\cap cl(g)=\emptyset$.

4:- $H_{41}=<(y,1)>$ *;(a) if* $g=(I,1)$ *then* $\emptyset_{41}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^hp}{4}.1=6\ \emptyset_{i+1}(I)$.

(b) if $g=(y^2,1)=(x^{2^h_p},1)$ *then* $\emptyset_{41}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h_p}{4}.1=6\ \emptyset_{i+1}(x^{2^h_p})$.

(c) if $g=(y,1)$ *or* $(y^3,1)$ *then* $\emptyset_{41}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{24}{4}(1+1)=6.2=12$.

Since $H\cap cl(g)=\{(I,1),(y^2,1),(y,1)\}$ *and* $\emptyset(g)=\emptyset(g^{-1})$*otherwise =0.*

5:- $H_{51}=<(xy,1)>$ *; (a) if* $g=(I,1)$ *then* $\emptyset_{51}(g)=\frac{|CG(G)|}{|cH(g)|}\emptyset(g)=\frac{24.2^h_p}{4}.1=6\ \emptyset_{i+2}(I)$.

(b) if $g=((xy)^2,1)=(x^{2^h_p},1)$ *then* $\emptyset_{51(g)}=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h_p}{4}.1=6\ \emptyset_{i+2}(x^{2^h_p})$.

(c) if $g=(xy,1)$ *or*$((xy)^3,1)$ *then* $\emptyset_{51}(g)=|\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})).\frac{24}{4}(1+1)=6.2=12$.

Since $H\cap cl(xy)=\{(I,1),(xy^2,1),(xy,1)\}$ *and* $\emptyset(g)=\emptyset(g^{-1})=1$

Case (II):-

Consider the group $\boldsymbol{G=(Q_{2.2^h p}\times D_3)}$ *and if H is a cyclic subgroup of* $\boldsymbol{(Q_{2.2^h p}\times\{r\})}$ *then H=<(q,r)> and* $\emptyset$ *the principle character of H and* $\emptyset_j$ *Artin's character of* $\boldsymbol{Q_{2.2^h p}}$ *, 1≤j≤i+2 ,by using theorem:-*

$$\Phi j(g)=\begin{cases}\frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & if\ hi\in H\cap cl(g)\\ 0 & if\ H\cap cl(g)=\emptyset\end{cases}$$

H=<(q,r)>:-

1:- H_{12}*=<(I,r)> ; (a) if g=(I,1) then* $\emptyset_{12}(g)$ $=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{242^h p.}{|C<x>(1)|}.1=\frac{6|CQ2m(1)|}{3|C<x>(1)|}.1=2.\emptyset_j(I).$

(b) If g=(I,r)or=(I,r²) then

$\emptyset_{12}(g)=$

$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2^h p}{|C<x>(1)|}(1+1)=\frac{3|CQ2m(1)|}{3|C<x>(1)|}.2=2.\emptyset_j(q)$

Since $H\cap cl(g)=\{(I,1),(I,r),(I,r^2)\}$ *and* $\emptyset(g)=\emptyset(g^{-1})=1$ *othrewise=0.*

2:- $H_{22}=<(x^{2^h_p},r)>$ *(a) if g=(I,1) then* $\emptyset_{22}(g)=$ $\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h p}{|C<x>\left(x^{2^h_p}\right)|}(1)=\frac{6|CQ2m|}{3|C<x>\left(x^{2^h_p}\right)|}(1)=2\emptyset_j(I)$

(b) if $g=(x^{2^h_p},1)$ *then* $\emptyset_{22}(g)$ $=$ $\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h p}{|C<x>\left(x^{2^h_p}\right)|}(1)=\frac{6|CQ2m(x^{2^h_p})|}{3|c<x>\left(x^{2^h_p}\right)|}(1)=2\emptyset_j(x^{2^h_p}).$

(c) if g=(I,r) then$\emptyset_{22}(g)$ $=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2^h p}{3|C<x>(q)|}(1+1)=\frac{3|CQ2m|}{3|C<x>(q)|}.2=2\emptyset_j(q).$

(d) if $g=(x^{2^h_p},r)$ *then*$\emptyset_{22}(g)$ $=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+$ $\emptyset(g^{-1}))=\frac{12.2^h p}{3|C<x>(q)|}(1+1)=\frac{3|CQ2m|}{3|C<x>(q)|}.2=2\emptyset_j(q).$

Since $H\cap cl(g)=\left\{(I,1),\left(x^{2^h_p},1\right),(I,r),(x^{2^h_p},r)\right\}$*.and* $\emptyset(g)=$ $\emptyset(g^{-1})=1$*.othrewise=0.*

3:- if g $\neq(x^{2^h_p},1),(x^{2^h_p},r)$ *then* $\emptyset_{j,2}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=$ $\frac{6.(2^h p)}{3|C<x>(x^q)|}(1+1+1+1)=\frac{6.4(2^h p)}{3|C<x>(x^q)|}=\frac{6|CQ2m|}{3|C<x>(x^q)|}=2.\emptyset_j(q)$

since $H \cap cl(g)=\{g,g^{-1}\}$ *,and* $g=\{(q,r),(q,r^2)\}$ *,q* $\in Q_{2.2^h p}$ *,q* $\neq x^{2^h_p}$, $\emptyset(g) = \emptyset(g^{-1}) = 1$

if $g \notin H$ *then* $\Phi_{j,2}(g)=0 = 6.0 =0 . \Phi j(q)$ *since* $H \cap cl(g)=\emptyset$.

4:- $H_{42}=<(y,r)>$ *(a) if* $g=(I,1)$ *then* $\emptyset_{42}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h_p}{12}.1=2.(2^h_p) =2.\emptyset_{i+1}(I)$.

(b) if $g=(y^2,1)$ *or* $(x^{2^h_p},1)$ *then* $\emptyset_{42}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h_p}{12}.1=2.(2^h p)$ $=2.\emptyset_{i+1}(x^{2^h_p})$.

(c) if $g=(y,1)$ *or* $(y^3,1)$ *then* $\emptyset_{42}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))= \frac{24}{12}(1+1)=4$.

(d) if $g=(I,r)$ *then* $\emptyset_{42}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))= \frac{12.2^h_p}{12}(1+1)=2.(2^h_p)=2. \emptyset_{i+1}(q)$.

(e) if $g=(y^2,r)$ *or* $(x^{2^h_p},r)$ *then* $\emptyset_{42}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2^h_p}{12}(1 + 1)=2.(2^h_p)= 2. \emptyset_{i+1}(q)$.

(f) if $g=((y,r)$ *or* (y^3,r) *then* $\emptyset_{42}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12}{12}(1+1+1+1)=4$.

Since $H \cap cl(g)=\{(I,1),(y^2,1),(y,1),(I,r),(y^2,r),(y,r)\}$ *And* $\emptyset(g)=\emptyset((g^{-1}) = 1$.*othrewise=0.*

5:-$H_{52}=<(xy,r)>$ *(a) if* $g=(I,1)$ *then*$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h_p}{12}.1=2.(2^h_p) =2.\emptyset_{i+2}(I)$.

(b) if $g=((xy)^2,1)=(x^{2^h_p},1)$ *then*$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h_p}{12}.1=2.(2^h_p)$ $=2.\emptyset_{i+2}(x^{2^h_p})$.

(c) if $g=(xy,1)$ *or* $((xy)^3,1)$ *then* $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{24}{12}(1 + 1)=4$.

(d) if $g=(I,r)$ *then* $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) =\frac{12.2^h_p}{12}(1 + 1)=2.(2^h_p)= 2. \emptyset_{i+2}(q)$.

(e) if $g=((xy)^2,r)=(x^{2^h_p},r)$ *then* $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$ $=\frac{12.2^h_p}{12}(1+1)=2.(2^h_p)=2.\ \emptyset_{i+2}(q)$.

(f) if $g=(xy,r)$ *or* $((xy)^3,r)$ *then* $\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12}{12}(1+1+1+1)=4$.

Since
$H\cap cl(g)=$
$\{(I,1),((xy)^2,1)(xy,1),(I,r),((xy)^2,r),(xy,r)\} And\ \emptyset(g)=\emptyset((g^{-1})=1. othrewise =0$

Case (III):-

Consider the group $G=(Q_{2.2^h p}\times D_3)$ *and if H is a cyclic subgroup of* $(Q_{2.2^h p}\times\{s\})$ *then* $H=\langle(q,s)\rangle$ *and* $\emptyset$ *the principle character of H and* $\emptyset j$ *Artin's character of* $Q_{2.2^h p}$, $1\le j\le i+2$ *,by using theorem:-*

$$\emptyset_j(g)=\begin{cases}\frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & if\ hi\in H\cap cl(g)\\ 0 & if\ H\cap cl(g)=\emptyset\end{cases}$$

$H=\langle(q,s)\rangle$

1:-$H_{13}=\langle(I,s)\rangle$ *(a) if* $g=(I,1)$ *then* $\emptyset_{13}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h p}{2|C\langle x\rangle(I)|}.1=\frac{6|CQ2m|}{2|C\langle x\rangle(I)|}.1=3.\emptyset_j(I)$.

(b) if $g=(I,s)$ *then* $\emptyset_{13}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^h p}{2|C\langle x\rangle(I)|}.1=\frac{2|CQ2m|}{2|C\langle x\rangle(I)|}.1=\emptyset_j(q)$.

Since $H\cap cl(g)=\{(I,1),(I,s)\}$ $othrewise=0$

:-$H_{23}=\langle(x^{2^h_p},s)\rangle$ *(a) if* $g=(I,1)$ *then* $\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h p}{2|C\langle x\rangle(x^{2^h}p)|}.1=\frac{6|CQ2m|}{2|C\langle x\rangle(x^{2^h}p)|}.1=3.\emptyset_j(I)$.

(b) if $g=(x^{2^h_p},1)$ *then* $\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^h p}{2|C\langle x\rangle(x^{2^h}p)|}.1=\frac{6|CQ2m|}{2|C\langle x\rangle(x^{2^h}p)|}.1==3.\emptyset_j(x^{2^h}p)$.

(c) if $g=(I,s)$ *then* $\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^h p}{2|C\langle x\rangle(x^{2^h}p)|}.1=\frac{2|CQ2m|}{2|C\langle x\rangle(x^{2^h}p)|}.1=\emptyset_j(q)$.

(d) if g=($x^{2^h}p$,s) then $\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^h_p}{2|C<x>\left(x^{2^h}p\right)|}.1=\frac{2|CQ2m|}{2|C<x>\left(x^{2^h}p\right)|}.1=\emptyset_j(q)$.

Since $H\cap cl(g)=\left\{(I,1),\left(x^{2^hp},1\right),(I,s),(x^{2^h}p,s)\right\}$ *And* $\emptyset(g)=\emptyset((g^{-1})=1$ *othrewise* $=0$.

3:- if $g\neq(x^{2^h_p},1),(x^{2^h_p},s)$ *and* $g\in H$ *, if* $g\neq(x^{2^h_p},1)$ *,* $g\in(Q_{2.2^h p}\times\{1\})$ *then* $\emptyset_{j,3}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2^hp}{2|C<x>(x^q)|}(1+1)=\frac{3|CQ2m(q)|}{2|c<x>(x^q)|}.2=3\emptyset_j(x^q)$

since $g=(q,1)$,

$q\in Q_{2.2^h p}, q\neq x^{2^h_p}$,

if $g\neq(x^{2^h_p},s)$ *and* $g\in(Q_{2.2^h p}\times\{s\})$ *then* $\emptyset_{j,3}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2^hp}{2|C<x>(x^q)|}(1+1)=\frac{|CQ2m|}{2|C<x>(x^q)|}.2=\emptyset_j(q)$. *since* $H\cap cl(g)=\{g,g^{-1}\}$

,

And $\emptyset(g)=\emptyset((g^{-1})=1$ *if* $g\notin H$ *then* $\Phi_{j,3}(g)=0=\emptyset_j(q)$ *since* $H\cap cl(g)=\emptyset$.

4:- $H_{43}=<(y,s)>$ *(a) if g=(I,1) then* $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^hp}{8}.1=3.\emptyset_{i+1}(I)$.

(b) if $g=(y^2,1)=(x^{2^h_p},1)$ *then* $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^hp}{8}.1=3.\emptyset_{i+1}(x^{2^h_p})$.

(c) if g=(y,1) or $(y^3,1)$ *then* $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right)=\frac{24}{8}(1+1)=6$.

(d) if g=(I,s) then $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^hp}{8}.1=\emptyset_{i+1}(q)$.

(e) if $g=(y^2,s)=(x^{2^h_p},s)$ *then* $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^h_p}{8}.1=\emptyset_{i+1}(q)$.

(f) if g=(y,s) or (y^3,s) *then* $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right)=\frac{8}{8}(1+1)=2$.

Since $H\cap cl(g)=\{(I,1),(y^2,1),(y,1),(I,s),(y^2,s),(y,s)\}$ *And* $\emptyset(g)=\emptyset((g^{-1})=1$ *othrewise* $=0$.

5:-H_{53}=<(xy,s)> (a) if g=(I,1) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^hp}{8}.1=3.\emptyset_{i+2}(I)$.

(b) if g=((xy)^2,1) =($x^{2^h_p}$,1) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2^hp}{8}.1=3.\emptyset_{i+2}(x^{2^hp})$.

(c) if g=(xy,1) or ((xy)^3,1) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\big(\emptyset(g)+\emptyset(g^{-1})\big)=\frac{24}{8}(1+1)=6$.

(d) if g=(I,s) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^hp}{8}.1=\emptyset_{i+2}(q)$.

(e) if g=((xy)^2,s) =(x^{2^hp},s) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2^hp}{8}.1=\emptyset_{i+2}(q)$.

(f) if g=(xy,s) or ((xy)^3,s) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\big(\emptyset(g)+\emptyset(g^{-1})\big)=\frac{8}{8}(1+1)=2$.

Since $H\cap cl(g)=\{(I,1),((xy)^2,1),(xy,1),(I,s),((xy)^2,s),(xy,s)\}$ And $\emptyset(g)=\emptyset((g^{-1})=1$ othrewise $=0$.

Note: $(xy)^2=y^2$ since $(xy)^2=xyxy=xyxy.y^2y^2=xyxy^3y^2=x(yxy^3)y^2=xx^{-1}y^2=y^2$.

Example:-(4,2)

Let **m=2^hp** *,such that* **p=3** *,and* **h=2** *,* **m=12** *,* **Q_{2m}=Q_{24}** *; To find Artin's character of the group* **(Q_{24} × D_3)** *the cyclic subgroup of* **Q_{24}** *which are* {<1>},{<x^{12}>},{<x^6>},{<x^3>},{<x^8>},{<x^4>},{<x^2>},{<x>},{<y>}, {<xy>} *and cyclic subgroup of* **D_3** *which are* {<1>},{<r>},{<s>}

∴*The cyclic subgroup of* **(Q_{24} × D_3)** *which are*
{<I,1>},{<x^{12},1>},{<x^6,1>},{<x^3,1>},{<x^8,1>},{<x^4,1>},{<x^2,1>},
{<x,1>},{<y,1>},{<xy,1>} ,
{<I,r >},{<x^{12},r >},{<x^6,r >},{<x^3,r >},{<x^8,r >},{<x^4,r >},
{<x^2,r >},{<x,r >},{<y,r >},{<xy,r >} ,
{<I,s >},{<x^{12},s >},{<x^6,s >},{<x^3,s >},{<x^8,s >},{<x^4,s >},
{<x^2,s >},{<x,s >},{<y,s >},{<xy,s >} ,*by using theorem:-*

$$\Phi j(g)=\begin{cases}\frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & \text{if } hi\in H\cap cl(g)\\ 0 & \text{if } H\cap cl(g)=\emptyset\end{cases}$$

	Γ-classes of Q_{24} × {1 }	Γ-classes of Q_{24} × {r }	Γ-classes of Q_{24} × {s}
	[I,1][x^{12},1][x^6,1][x^3,1][x^8,1][x^4,1][x^2,1][x,1][y,1][xy,1]	[I,r][x^{12},r][x^6,r][x^3,r][x^8,r][x^4,r][x^2,r][x,r][y,r][xy,r]	[I,s][x^{12},s][x^6,s][x^3,s][x^8,s][x^4,s][x^2,s][x,s][y,s][xy,s]
Φ(1,1)	288 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(2,1)	144 144 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(3,1)	72 72 72 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(4,1)	36 36 36 36 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(5,1)	96 0 0 0 96 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(6,1)	48 48 0 0 48 48 0 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(7,1)	24 24 24 0 24 24 24 0 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(8,1)	12 12 12 12 12 12 12 12 0 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(9,1)	72 72 0 0 0 0 0 0 12 0	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(10,1)	72 72 0 0 0 0 0 0 0 12	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(1,2)	96 0 0 0 0 0 0 0 0 0	96 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(2,2)	48 48 0 0 0 0 0 0 0 0	48 48 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(3,2)	24 24 24 0 0 0 0 0 0 0	24 24 24 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(4,2)	12 12 12 12 0 0 0 0 0 0	12 12 12 12 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(5,2)	32 0 0 0 32 0 0 0 0 0	32 0 0 0 32 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(6,2)	16 16 0 0 16 16 0 0 0 0	16 16 0 0 16 16 0 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(7,2)	8 8 8 0 8 8 8 0 0 0	8 8 8 0 8 8 8 0 0 0	0 0 0 0 0 0 0 0 0 0
Φ(8,2)	4 4 4 4 4 4 4 4 0 0	4 4 4 4 4 4 4 4 0 0	0 0 0 0 0 0 0 0 0 0
Φ(9,1)	24 24 0 0 0 0 0 0 4 0	24 24 0 0 0 0 0 0 4 0	0 0 0 0 0 0 0 0 0 0
Φ(10,1)	24 24 0 0 0 0 0 0 0 4	24 24 0 0 0 0 0 0 0 4	0 0 0 0 0 0 0 0 0 0
Φ(1,3)	144 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	48 0 0 0 0 0 0 0 0 0
Φ(2,3)	72 72 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	24 24 0 0 0 0 0 0 0 0
Φ(3,3)	36 36 36 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	12 12 12 0 0 0 0 0 0 0
Φ(4,3)	18 18 18 18 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	6 6 6 6 0 0 0 0 0 0
Φ(5,3)	48 0 0 0 48 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0	16 0 0 0 16 0 0 0 0 0
Φ(6,3)	24 24 24 24 24 24 0 0 0 0	0 0 0 0 0 0 0 0 0 0	8 8 8 8 8 8 0 0 0 0
Φ(7,3)	12 12 12 0 12 12 12 0 0 0	0 0 0 0 0 0 0 0 0 0	4 4 4 0 4 4 4 0 0 0
Φ(8,3)	6 6 6 6 6 6 6 6 0 0	0 0 0 0 0 0 0 0 0 0	2 2 2 2 2 2 2 2 0 0
Φ(9,3)	36 36 0 0 0 0 0 0 6 0	0 0 0 0 0 0 0 0 0 0	12 12 0 0 0 0 0 0 2 0
Φ(10,3)	36 36 0 0 0 0 0 0 0 6	0 0 0 0 0 0 0 0 0 0	12 12 0 0 0 0 0 0 0 2

Table (3)

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