On Artin cokernel of the Group($Q_{2m} \times C_5$) where $m = p_1 \cdot p_2$, $\gcd(p_1, p_2) = 1$, $p_1, p_2 > 2$ and $p_1, p_2$ are primes numbers

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Abstract:

The main purpose of This paper is to find Artin s character table $\text{Ar}(Q_{2m} \times C_5)$ when $m$ is an odd number such that $m = p_1 \cdot p_2$, $p_1, p_2 > 2$ where $\gcd(p_1, p_2) = 1$ and $p_1, p_2$ are primes numbers; where $Q_{2m}$ is denoted to Quaternion group of order $4m$, time is said to have only one dimension and space to have three dimension, the mathematical quaternion partakes of both these elements; in technical language it may be said to be "time plus space", or "space plus time" and in this sense it has, or at least involves a reference to four dimensions, and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols., 1882, 1885, 1889)), and $C_5$ is Cyclic group of order 5. In 1962, C. W. Curits & I. Reiner studied Representation Theory of finite groups, In 1976, I. M. Isaacs studied Character Theory of finite groups, In 1982, M. S. Kirdar studied The Factor Group of the $Z$-Valued class function modulo the group of the Generalized Characters, In 1995, N.R. Mahmood studies The Cyclic Decomposition of the factor Group $\text{cf}(Q_{2m}, Z)/\bar{K} (G) (Q_{2m})$, In 2002, K-Sekiguchi studies Extensions and the Irreducibilities of the Induced
Naserr Rasool Mahmood, Salah Hsaaoun Jihadi - On Artin cokernel of the Group($Q_{2m} \times C_5$) where $m=p_1.p_2$. g.c.d($p_1,p_2$) = 1 , $p_1, p_2 > 2$ and $p_1, p_2$ are primes numbers

Characters of Cyclic P-Group , In 2008, A.H.Abdul-Munem studied Artin Cokernel of The Quaternion group $Q_{2m}$ when $m$ is an Odd number, In 2006, A.S. Abed found the Artin characters table of dihedral group $D^n$ when $n$ is an odd number.

Key words: odd number, prime number, Quaternion group, and Cyclic group

1. INTRODUCTION:

Representation Theory is a branch of mathematics that studies abstract algebra structures by Representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication in which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication, Moreover, representation and character theory provide applications, not only in other branches of mathematics but also in physics and chemistry.

Fore a finite group $G$, The factor group $\overline{R}$($G$)/$T(G)$ is called the Artin cokernel of $G$ denoted $AC(G)$, $\overline{R}(G)$ denoted the abelian group generated by Z-valued characters of $G$ under the operation of point wise addition, $T(G)$ is a sub group of $\overline{R}$($G$) which is generated by Artin's characters.

2-PRELIMINARS: (3,1) :[1]

The Generalized Quaternion Group $Q_{2m}$: For each positive integer $m\geq2$ ,The generalized Quaternion Group $Q_{2m}$ of order $4m$ with two generators $x$ and $y$ satisfies $Q_{2m} = \{x^h y^k, 0 \leq h \leq 2m-}$
Naserr Rasool Mahmood, Salah Hsaaoun Jihadi. On Artin cokernel of the Group($Q_{2m} \times C_5$) where $m=p_1.p_2$, $g.c.d(p_1,p_2) = 1$ , $p_1, p_2 > 2$ and $p_1, p_2$ are primes numbers

1,k=0,1)Which has the following properties \{x^m=y^4=I,yx^my^{-1}=x^{-m}\}.Let G be a finite group ,all the characters of group G induced from a principal character of cyclic subgroup of G are called Artin characters of G . Artin characters of the finite group can be displayed in a table called Artin characters table of G which is denoted by Ar(G); The first row is $\Gamma$-conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G (CL_\alpha)|$ and other rows contains the values of Artin characters.

**Theorem: (3,2): [2]**
The general form of Artin characters table of $C_p^s$ When p is a prime number and s is a positive integer number is given by :-

Ar($C_p^s$)=

<table>
<thead>
<tr>
<th>$\Gamma$-classes</th>
<th>(1)</th>
<th>(x^{p^m}^{-1})</th>
<th>(x^{p^m}^{-1})</th>
<th>(x^{p^m}^{-1})</th>
<th>...</th>
<th>(x^s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[CL_\alpha]$. (</td>
<td>C_p^s(CL_\alpha)</td>
<td>)</td>
<td>$p^s$</td>
<td>$p^s$</td>
<td>$p^s$</td>
<td>$p^s$</td>
</tr>
<tr>
<td>$\phi_{1}$</td>
<td>$p^s$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{2}$</td>
<td>$p^{s-1}$</td>
<td>$p^{s-1}$</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{3}$</td>
<td>$p^{s-2}$</td>
<td>$p^{s-2}$</td>
<td>$p^{s-2}$</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{4}$</td>
<td>$p^{s-3}$</td>
<td>$p^{s-3}$</td>
<td>$p^{s-3}$</td>
<td>$p^{s-3}$</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{5}$</td>
<td>$p^{s-4}$</td>
<td>$p^{s-4}$</td>
<td>$p^{s-4}$</td>
<td>$p^{s-4}$</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{6}$</td>
<td>$p^{s-5}$</td>
<td>$p^{s-5}$</td>
<td>$p^{s-5}$</td>
<td>$p^{s-5}$</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>

Table (3,1)

**Example : (3,3):**-

We can write Artin characters tables of the groups $C_{p_1}$ and $C_{p_2}$ ,$p_1, p_2>2$ and $p_1, p_2$ are primes numbers
On Artin cokernel of the Group $\mathbb{Q}_{2m} \times C_5$ where $m=p_1.p_2$, g.c.d($p_1.p_2$) = 1, $p_1$, $p_2$ $> 2$ and $p_1$, $p_2$ are primes numbers

$$\text{Ar}(C_{p_2}) =$$

<table>
<thead>
<tr>
<th>$\Gamma$-classes</th>
<th>$[1]$</th>
<th>$[x]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{c_2}$(CL$\alpha$)</td>
<td>$p_1$</td>
<td>$p_1$</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>$p_1$</td>
<td>0</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table (3,2)

$$\text{Ar}(C_{p_1}) =$$

<table>
<thead>
<tr>
<th>$\Gamma$-classes</th>
<th>$[1]$</th>
<th>$[x]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{c_5}$(CL$\alpha$)</td>
<td>$p_2$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>$p_2$</td>
<td>0</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table (3,3)

**Corollary : (3,4): [2]**

Let $m = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_n^{\alpha_n}$ where g.c.d($p_i, p_j$) = 1, if $i \neq j$ and $p_i$’s are primes numbers, and $\alpha_n$ any positive integers,

$$\text{Ar}(C_m) = \text{Ar}(C_{p_1^{\alpha_1}}) \bigotimes \text{Ar}(C_{p_2^{\alpha_2}}) \bigotimes \cdots \bigotimes \text{Ar}(C_{p_n^{\alpha_n}}).$$

Then;

**Example (3.5):-**

Consider the cyclic group $C_2 \cdot p_1.p_2$, $p_1$, $p_2$ $> 2$, Where g. c. d($p_1.p_2$) = 1 and $p_1$, $p_2$ are primes numbers. To find Artin characters table for it, we use corollary (3,4) as the following: $\text{Ar}(C_2.p_1.p_2 ) = \text{Ar}(C_2) \bigotimes \text{Ar}(C_{p_1}) \bigotimes \text{Ar}(C_{p_2})$, by using theorem (3,2) to find $\text{Ar}(C_2)$ is given as follows :

$$\text{Ar}(C_2) =$$

<table>
<thead>
<tr>
<th>$\Gamma$-classes</th>
<th>$[1]$</th>
<th>$[x]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{c_2}$(CL$\alpha$)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table (3,4)

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$$\text{Ar}(C_2 \cdot p_1, p_2) =$$

| $\Gamma$-classes | $|CL_\alpha|$ | $[1]$ | $[x^2]$ | $[x^{2p_1}]$ | $[x^{2p_2}]$ | $[x^{p_1}p_2]$ | $[x^{p_1}]$ | $[x^{p_2}]$ | $[x]$ |
|------------------|-------------|-------|----------|-------------|-------------|--------------|----------|----------|------|
| $C_{C_2 \cdot p_1, p_2}$ ($CL_\alpha$) | $2p_1 p_2$ | $p_1 p_2$ | $p_1 p_2$ | $2p_1 p_2$ | $p_1 p_2$ | $p_1 p_2$ | $p_1 p_2$ | $p_1 p_2$ |
| $\varphi_1$ | $2p_1 p_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\varphi_2$ | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 |
| $\varphi_3$ | 2p_1 | 0 | 2p_1 | 0 | 0 | 0 | 0 | 0 |
| $\varphi_4$ | 2p_1 | 0 | 0 | 2p_1 | 0 | 0 | 0 | 0 |
| $\varphi_5$ | $P_1 p_2$ | 0 | 0 | $P_2$ | 0 | 0 | $P_2$ | 0 |
| $\varphi_6$ | $P_1$ | 0 | $P_1$ | 0 | 0 | $P_1$ | 0 | 0 |
| $\varphi_7$ | $P_2$ | 0 | 0 | $P_2$ | 0 | 0 | $P_2$ | 0 |
| $\varphi_8$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Table (3.5)

**Theorem (3.6):[1]**

The Artin characters table of the Quaternion group $\mathbb{Q}_{2m}$ when $m$ is an odd number is given as follows:

$$\text{Ar}(\mathbb{Q}_{2m}) =$$

| $\Gamma$-Classes | $|C_{\mathbb{Q}_{2m}}(CL_\alpha)|$ | $x^{2r}$ | $x^{2r+1}$ | $|y|$ |
|------------------|-----------------------------|--------|-------------|------|
| $|C_{\mathbb{Q}_{2m}}(\mathbb{C}_4)|$ | $4m$ | 2 | ... | 2 | 2m | 2 |
| $\Phi_1$ | $2m$ | 2 | ... | 2m | 4m | 2m | 2m | 2m | 2 |
| $\Phi_{l+1}$ | $m$ | 0 | ... | 0 | $m$ | 0 | ... | 0 | 1 |

Table (3.7)

where $0 \leq r \leq m-1$, $l$ is the number of $\Gamma$-classes of $\mathbb{C}_{2m}$ and $\Phi_j$ are the Artin characters of the quaternion group $\mathbb{Q}_{2m}$, for all $1 \leq j \leq l+1$.

**Example (3.6):**

To construct $\text{Ar}(\mathbb{Q}_2 \cdot p_1, p_2)$, $p_1, p_2 > 2$ Where $\text{g.c.d}(p_1, p_2) = 1$ and $p_1, p_2$ are primes numbers. By using theorem (3.5):
On Artin cokernel of the Group\((Q_{2m} \times C_5)\) where \(m=p_1 \cdot p_2,\) g.c.d\((p_1, p_2) = 1, p_1, p_2 > 2\) and \(p_1, p_2\) are primes numbers

\[
\text{Ar}(Q_2 \cdot p_1, p_2) =
\]

| \(\Gamma\)-Classes | \(1\) | \(x^2\) | \(x^{p_1}\) | \(x^{p_2}\) | \(x^{p_1, p_2}\) | \(x^{p_i}\) | \(x\) | \(y\) |
|---------------------|-------|--------|--------|--------|--------|--------|-------|
| \(|CL\alpha|\) \(|Cl\alpha|\) \(|\Phi_i\) \(|\Phi_i|\) | \(4p_1p_2\) | \(2p_1p_2\) | \(2p_1p_2\) | \(2p_1p_2\) | \(2p_1p_2\) | \(2\) |
| \(\Phi_1\) \(4p_1, p_2\) | \(2\) | \(0\) | \(0\) | \(0\) | \(0\) | \(0\) | \(0\) |
| \(\Phi_2\) \(4\) | \(4\) | \(4\) | \(4\) | \(0\) | \(0\) | \(0\) | \(0\) |
| \(\Phi_3\) \(4p_1\) | \(0\) | \(0\) | \(0\) | \(0\) | \(0\) | \(0\) | \(0\) |
| \(\Phi_4\) \(4p_2\) | \(0\) | \(0\) | \(4p_2\) | \(0\) | \(0\) | \(0\) | \(0\) |
| \(\Phi_5\) \(2p_1p_2\) | \(0\) | \(0\) | \(0\) | \(2p_1p_2\) | \(0\) | \(0\) | \(0\) |
| \(\Phi_6\) \(2p_1\) | \(0\) | \(2p_1\) | \(0\) | \(0\) | \(2p_1\) | \(0\) | \(0\) |
| \(\Phi_7\) \(2p_2\) | \(0\) | \(2p_2\) | \(0\) | \(0\) | \(2p_1\) | \(0\) | \(0\) |
| \(\Phi_8\) \(2\) | \(2\) | \(2\) | \(2\) | \(2\) | \(2\) | \(2\) | \(0\) |
| \(\Phi_9\) \(p_1, p_2\) | \(0\) | \(0\) | \(0\) | \(p_1, p_2\) | \(0\) | \(0\) | \(1\) |

Table (3.8)

**Theorem (3.8): [4]**

Let \(H\) be a cyclic subgroup of \(G\) and \(h_1, h_2, \ldots, h_m\) are chosen representatives for the \(m\)-conjugate classes of \(H\) contained in \(CL(g)\) in \(G\), then :

\[
\phi'(g) = \begin{cases} 
\frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^{m} \phi(h_i) & \text{if } h_i \in H \cap CL(g) \\
0 & \text{if } H \cap CL(g) = \phi 
\end{cases}
\]

**Proposition (3.9). [3]**

The number of all distinct Artin characters on group \(G\) is equal to the number of \(\Gamma\)-classes on \(G\). Furthermore, Artin characters are constant on each \(\Gamma\)-classes.

3. THE MAIN RESULTS:

In this section we give the general form of Artin’s characters table of the group \((Q_{2m} \times C_5)\), When \(m=p_1, p_2, p_1, p_2 > 2\) Where g.c.d\((p_1, p_2) = 1\) and \(p_1, p_2\) are primes numbers. The group\((Q_{2m} \times C_5)\) is the direct product group of the quaternion group\(Q_{2m}\) of order \(4m\) and the cyclic group \(C_5\) of order 5, then the order of The group\((Q_{2m} \times C_5)\) is \(20m\).
Example: (4.1):-
Let \( m=15=3 \cdot 5 \), \( p_1 = 3 \), \( p_2 = 5 \), then \((Q_{2m}\times C_5) = (Q_{30}\times C_5) = (Q_{2.3.5}\times C_5)=\{(1,1)\), \((1, z)\), \((1, z^2)\), \((1, z^3)\), \((1, z^4)\) \}, \(x\), \(z\), \((x, z^2)\), \((x, z^3)\), \((x, z^4)\) \}, \((x, z^5)\), \((x, z^6)\), \((x, z^7)\), \((x, z^8)\), \((x, z^9)\), \((x, z^{10})\), \((x, z^{11})\), \((x, z^{12})\), \((x, z^{13})\), \((x, z^{14})\), \((x, z^{15})\) \}, \((x, z^{16})\), \((x, z^{17})\), \((x, z^{18})\), \((x, z^{19})\), \((x, z^{20})\), \((x, z^{21})\), \((x, z^{22})\), \((x, z^{23})\), \((x, z^{24})\), \((x, z^{25})\), \((x, z^{26})\), \((x, z^{27})\), \((x, z^{28})\), \((x, z^{29})\), \((x, z^{30})\) \}. to find Artin’s characters for this group, there are 18 cyclic subgroups, which are:

\(<1, 1>, <x^2, 1>, <x^4, 1>, <x^{10}, 1>, <x^{15}, 1>, <x^3, 1>, <x^5, 1>, <x, 1>, <y, 1>, <1, z>, <x^2, z>, <x^4, z>, <x^{10}, z>, <x^{15}, z>, <x^3, z>, <x^5, z>, <x, z>, <y, z>\),

then there are 18 \( \Gamma \)-Classes, we have 18 distinct Artin’s characters, Let \( g \in (Q_{30}\times C_5), g=(q, 1) \) or \( g=(q, z), q \in Q_{10} \), \( z \in C_5 \) and let \( \varphi \) the principal character of \( H \), \( \Phi_j \) Artin characters of \( Q_{10}, 1 \leq j \leq 9 \), then by using theorem (3.8):

\[
\Phi_j(g) = \begin{cases} 
\frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \varphi(h_i) & \text{if } h_i \in H \cap CL(g) \\
0 & \text{if } H \cap CL(g) = \phi
\end{cases}
\]

Case (I): If \( H \) is a cyclic subgroup of \( (Q_{2m}\times \{1\}) \) then:

\( H = <1,1> \), If \( g=(1,1) \)

\( \Phi_{(1,1)}(1,1) = \frac{|Q_{2m}\times C_5|}{|C_G(g)|} \varphi((1,1)) = \frac{300}{1} = 300 \) since \( H \cap CL(g) = \{1, (1,1)\} \)

\( \varphi(g) = 1 \)

Otherwise \( \Phi_{(1,1)}(g) = 0 \)

\( H = <x^2, 1> \), If \( g=(1,1) \)

\( \Phi_{(2,1)}(g) = \frac{|Q_{2m}\times C_5|}{|C_G(g)|} \varphi(g) = \frac{300}{15} = 20 \) since \( H \cap CL(g) = \{1, (1,1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^2, 1) \), \( \Phi_{(2,1)}(x^2, 1) = \frac{|Q_{2m}\times C_5|}{|C_G(g)|} \varphi(g) + \varphi(g^{-1}) = \frac{150}{15} (1 + 1) = 20 \) since \( H \cap CL(g) = \{g, g^{-1}\} \) and \( \varphi(g) = \varphi(g^{-1}) = 1 \)
If \( g = (x^6, I), \Phi_{(6,1)}((x^6, I)) = \frac{|C_{2x^6I}|}{|C_{gI}|} \varphi \left( g \right) + \varphi \left( g^{-1} \right) = 1 \) since \( H \cap CL(g) = (g, g^{-1}) \) and \( \varphi \left( g \right) = \varphi \left( g^{-1} \right) = 1 \)

If \( g = (x^{10}, I), \Phi_{(6,1)}((x^{10}, I)) = \frac{|C_{2x^{10}I}|}{|C_{gI}|} \varphi \left( g \right) + \varphi \left( g^{-1} \right) = 1 \) since \( H \cap CL(g) = (g, g^{-1}) \) and \( \varphi \left( g \right) = \varphi \left( g^{-1} \right) = 1 \)

Otherwise \( \Phi_{(6,1)}(g) = 0 \) since \( H \cap CL(g) = \phi \)

If \( H = \langle x^6, I \rangle \), If \( g = (1, I), \Phi_{(6,1)}((1, I)) = \frac{|C_{2x^{6}I}|}{|C_{gI}|} \varphi \left( g \right) = \frac{300}{5} = 60 = 5.12 = 5. \Phi_3 (1) \) since \( H \cap CL(g) = (1, I) \) and \( \varphi \left( g \right) = 1 \)

If \( g = (x^6, I), \Phi_{(6,1)}((x^6, I)) = \frac{|C_{2x^6I}|}{|C_{gI}|} \varphi \left( g \right) + \varphi \left( g^{-1} \right) = \frac{150}{5} (1 + 1) = 60 = 5.12 = 5. \Phi_3 (x^6) \) since \( H \cap CL(g) = (g, g^{-1}) \) and \( \varphi \left( g \right) = \varphi \left( g^{-1} \right) = 1 \)

Otherwise \( \Phi_{(6,1)}(g) = 0 \) since \( H \cap CL(g) = \phi \)

If \( H = \langle x^{10}, I \rangle \), If \( g = (1, I), \Phi_{(6,1)}((1, I)) = \frac{|C_{2x^{10}I}|}{|C_{gI}|} \varphi \left( g \right) = \frac{300}{5} = 100 = 5.20 = 5. \Phi_4 (1) \) since \( H \cap CL(g) = (1, I) \) and \( \varphi \left( g \right) = 1 \)

If \( g = (x^{10}, I), \Phi_{(6,1)}((x^{10}, I)) = \frac{|C_{2x^{10}I}|}{|C_{gI}|} \varphi \left( g \right) + \varphi \left( g^{-1} \right) = \frac{150}{5} (1 + 1) = 100 = 5.20 = 5. \Phi_4 (x^{10}) \) since \( H \cap CL(g) = (g, g^{-1}) \) and \( \varphi \left( g \right) = \varphi \left( g^{-1} \right) = 1 \)

Otherwise \( \Phi_{(6,1)}(g) = 0 \) since \( H \cap CL(g) = \phi \)

If \( H = \langle x^{15}, I \rangle \), If \( g = (1, I), \Phi_{(6,1)}((1, I)) = \frac{|C_{2x^{15}I}|}{|C_{gI}|} \varphi \left( g \right) = \frac{300}{10} = 30 = 5.6 = 5. \Phi_5 (1) \) since \( H \cap CL(g) = (1, I) \) and \( \varphi \left( g \right) = 1 \)

Otherwise \( \Phi_{(6,1)}(g) = 0 \) since \( H \cap CL(g) = \phi \)

If \( H = \langle x^6, I \rangle \), If \( g = (1, I), \Phi_{(6,1)}((1, I)) = \frac{|C_{2x^6I}|}{|C_{gI}|} \varphi \left( g \right) = \frac{300}{10} = 30 = 5.6 = 5. \Phi_6 (x^6) \) since \( H \cap CL(g) = (1, I) \) and \( \varphi \left( g \right) = 1 \)

If \( g = (x^6, I), \Phi_{(6,1)}((x^6, I)) = \frac{|C_{2x^6I}|}{|C_{gI}|} \varphi \left( g \right) + \varphi \left( g^{-1} \right) = \frac{150}{10} (1 + 1) = 30 = 5.6 = 5. \Phi_6 (x^6) \) since \( H \cap CL(g) = (g, g^{-1}) \) and \( \varphi \left( g \right) = \varphi \left( g^{-1} \right) = 1 \)

If \( g = (x^{15}, I), \Phi_{(6,1)}((x^{15}, I)) = \frac{|C_{2x^{15}I}|}{|C_{gI}|} \varphi \left( g \right) + \varphi \left( g^{-1} \right) = \frac{150}{10} (1 + 1) = 30 = 5.6 = 5. \Phi_6 (x^{15}) \) since \( H \cap CL(g) = (g, g^{-1}) \) and \( \varphi \left( g \right) = \varphi \left( g^{-1} \right) = 1 \)

Otherwise \( \Phi_{(6,1)}(g) = 0 \) since \( H \cap CL(g) = \phi \)
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Group\( \mathbb{Q}_{2n} \times \mathbb{C}_3 \) where \( m=p_1, p_2, \) \( \text{g.c.d}(p_1, p_2) = 1, p_1, p_2 > 2 \) and \( p_1, p_2 \) are primes numbers

\[
\text{H}_7 = \langle x^6, I \rangle, \text{If } g = (1, I) \Phi(g, I)(1, I) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g)) = 300 = 5.10 = 5. \phi_5 (1) \text{ since } \bigcap \text{CL(g)} = (1, I), \text{and } \varphi(g) = 1
\]

If \( g = (x^{10}, I) \Phi(g, (x^{10}, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{6} (1 + 1) = 50 = 5.10 = 5. \phi_5 \text{ (x^{10})}
\]

since \( \bigcap \text{CL(g)} = \{ g, g^{-1} \} \) and \( \varphi(g) = \varphi(g^{-1}) = 1 \)

If \( g = (x^{15}, I) \Phi(g, (x^{15}, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g)) = 300 = 1 = 50 = 5.10 = 5. \phi_5 \text{ (x^{15})} \text{ since } \bigcap \text{CL(g)} = (x^{15}, I), \text{and } \varphi(g) = 1
\]

Otherwise \( \Phi(g, (1)) = 0 \text{ since } \bigcap \text{CL(g)} = \phi \)

\( \text{H}_8 = \langle x^3, I \rangle, \text{If } g = (1, I) \Phi(g, (3, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g)) = \frac{300}{30} = 10 = 5.2 = 5. \phi_5 (1) \text{ since } \bigcap \text{CL(g)} = (1, I) \text{ and } \varphi(g) = 1 \)

If \( g = (x^3, I) \Phi(g, (3, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{30} (1 + 1) = 10 = 5.2 = 5. \phi_5 \text{ (x^3)} \text{ since } \bigcap \text{CL(g)} = \{ g, g^{-1} \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1
\]

If \( g = (x^7, I) \Phi(g, (7, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{30} (1 + 1) = 10 = 5.2 = 5. \phi_5 \text{ (x^7)} \text{ since } \bigcap \text{CL(g)} = \{ g, g^{-1} \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1
\]

If \( g = (x^{10}, I) \Phi(g, (10, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{30} (1 + 1) = 10 = 5.2 = 5. \phi_5 \text{ (x^{10})} \text{ since } \bigcap \text{CL(g)} = \{ g, g^{-1} \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1
\]

If \( g = (x^3, I) \Phi(g, (3, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{30} (1 + 1) = 10 = 5.2 = 5. \phi_5 \text{ (x^3)} \text{ since } \bigcap \text{CL(g)} = \{ g, g^{-1} \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1
\]

If \( g = (x^5, I) \Phi(g, (5, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{30} (1 + 1) = 10 = 5.2 = 5. \phi_5 \text{ (x^5)} \text{ since } \bigcap \text{CL(g)} = \{ g, g^{-1} \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1
\]

If \( g = (x^7, I) \Phi(g, (7, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{150}{30} (1 + 1) = 10 = 5.2 = 5. \phi_5 \text{ (x^7)} \text{ since } \bigcap \text{CL(g)} = \{ g, g^{-1} \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1
\]

If \( g = (x^{10}, I) \Phi(g, (10, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g)) = \frac{300}{4} = 1 = 75 = 5.15 = 5. \phi_5 \text{ (x^{10})} \text{ since } \bigcap \text{CL(g)} = (x^{15}, I) \text{ and } \varphi(g) = 1
\]

Otherwise \( \Phi(g, (1)) = 0 \text{ since } \bigcap \text{CL(g)} = \phi \)

\( \text{H}_9 = \langle y^2, I \rangle, \text{If } g = (1, I) \Phi(g, (3, I)) = \frac{|\mathbb{Q}_{2n} \times \mathbb{C}_3(g)|}{|\mathfrak{g}(g)|} (\varphi(g)) = \frac{300}{4} = 1 = 75 = 5.15 = 5. \phi_5 \text{ (y^2)} \text{ since } \bigcap \text{CL(g)} = (1, I) \text{ and } \varphi(g) = 1
\]
If \( g=(x^{10}, I) \), then \( \Phi_{(10)}((x^{10}, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(10)}|} \) and \( \varphi(g) = 1 = 60 = \Phi_1(1) \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(y, I) \), then \( \Phi_{(1)}((y, I)) = \frac{|Q_{2m} \times C_5(y)|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{\phi, g^{-1}\} \)

Otherwise \( \Phi_{(1)}(g) = 0 \) since \( H \cap \text{CL}(g) = \phi \)

Case (II): If \( H \) is a cyclic subgroup of \( (Q_{2m} \times \{\phi\}) \), then:

\( H_2 = \phi \), if \( g=(1, I) \), then \( \Phi_{(1, 2)}((1, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

Otherwise \( \Phi_{(1, 2)}(g) = 0 \) since \( H \cap \text{CL}(g) = \phi \)

\( H_2 = \phi \), if \( g=(1, I) \), then \( \Phi_{(2, 3)}((x, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^2, I) \), then \( \Phi_{(2, 3)}((x^2, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^3, I) \), then \( \Phi_{(2, 3)}((x^3, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^4, I) \), then \( \Phi_{(2, 3)}((x^4, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^5, I) \), then \( \Phi_{(2, 3)}((x^5, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^{10}, I) \), then \( \Phi_{(2, 3)}((x^{10}, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

If \( g=(x^{10}, I) \), then \( \Phi_{(2, 3)}((x^{10}, I)) = \frac{|Q_{2m} \times C_5|}{|G_{(1)}|} \) and \( \varphi(g) = \varphi(g^1) = 1 \) since \( H \cap \text{CL}(g) = \{(1, 1)\} \) and \( \varphi(g) = 1 \)

Otherwise \( \Phi_{(2, 3)}(g) = 0 \) since \( H \cap \text{CL}(g) = \phi \)
If $g = (1, I), \Phi_{5, 2}((1, I)) = \frac{|C_{Q_{2^a} \times C_5}|}{|C_{\Phi}(g)|} \phi_5 = 30 = \Phi_5 (1)$ since $H \cap CL(g) = (1, I)$ and $\Phi (g) = 1$

If $g = (1, z), \Phi_{5, 2}((1, z)) = \frac{|C_{Q_{2^a} \times C_5}|}{|C_{\Phi}(g)|} \phi_5 = 30 = \Phi_5 (1)$ since $H \cap CL(g) = (1, z)$ and $\Phi (g) = 1$
Naserr Rasool Mahmood, Salah Hsaaoun Jihadi. On Artin cokernel of the Group($\mathbb{Q}_{10} \times C_5$) where $m=p_1.p_2$. g.c.d($p_1,p_2$) = 1, $p_1, p_2 > 2$ and $p_1, p_2$ are primes numbers

If $g=(1,z), \Phi_{(6,2)}((1,z))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{50}.1=6=\phi_5 (1)$ since $H \cap \text{CL}(g)\{1, z\}$ and $\varphi(g)=1$

If $g=(x^6,l), \Phi_{(6,2)}((x^6,l))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g) + \varphi(g^-))=\frac{150}{50}.(1+1)=6=\phi_6 (x^6)$ since $H \cap \text{CL}(g)\{g, g^-\}$ and $\varphi(g)=\varphi(g^-)=1$

If $g=(x^6, z), \Phi_{(6,2)}((x^6,z))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g) + \varphi(g^-))=\frac{150}{50}.(1+1)=6=\phi_6 (x^6)$ since $H \cap \text{CL}(g)\{g, g^-\}$ and $\varphi(g)=\varphi(g^-)=1$

If $g=(x^{15}, l), \Phi_{(6,2)}((x^{15}, l))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{50}.1=6=\phi_6 (x^{15})$ since $H \cap \text{CL}(g)\{1, (x^{15})\}$ and $\varphi(g)=1$

If $g=(x^{15}, z), \Phi_{(6,2)}((x^{15}, z))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{50}.1=6=\phi_6 (x^{15})$ since $H \cap \text{CL}(g)\{1, (x^{15})\}$ and $\varphi(g)=1$

If $g=(x^3, l), \Phi_{(6,2)}((x^3, l))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g) + \varphi(g^-))=\frac{150}{50}.(1+1)=6=\phi_6 (x^3)$ since $H \cap \text{CL}(g)\{g, g^-\}$ and $\varphi(g)=\varphi(g^-)=1$

If $g=(x^3, z), \Phi_{(6,2)}((x^3,z))= \frac{\mid \mathbb{Q}_{10} \times C_5 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g) + \varphi(g^-))=\frac{150}{50}.(1+1)=6=\phi_6 (x^3)$ since $H \cap \text{CL}(g)\{g, g^-\}$ and $\varphi(g)=\varphi(g^-)=1$

Otherwise $\Phi_{(6,2)}(g)=0$ since $H \cap \text{CL}(g)=\phi$ $H:=<x^3, z>$, If $g=(1, l), \Phi_{(7,2)}((1, l))= \frac{\mid \mathbb{Q}_{2} \times C_3 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{30}.1=10=\phi_5 (1)$ since $H \cap \text{CL}(g)\{1, (1, l)\}$ and $\varphi(g)=1$

If $g=(1, z), \Phi_{(7,2)}((1, z))= \frac{\mid \mathbb{Q}_{2} \times C_3 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{30}.1=10=\phi_5 (1)$ since $H \cap \text{CL}(g)\{1, (1, z)\}$

If $g=(x^{10}, l), \Phi_{(7,2)}((x^{10}, l))= \frac{\mid \mathbb{Q}_{2} \times C_3 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g) + \varphi(g^-))=\frac{150}{30}.(1+1)=10=\phi_5 (x^{10})$ since $H \cap \text{CL}(g)\{g, g^-\}$ and $\varphi(g)=\varphi(g^-)=1$

If $g=(x^{10}, z), \Phi_{(7,2)}((x^{10}, z))= \frac{\mid \mathbb{Q}_{2} \times C_3 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g) + \varphi(g^-))=\frac{150}{30}.(1+1)=10=\phi_5 (x^{10})$ since $H \cap \text{CL}(g)\{g, g^-\}$ and $\varphi(g)=\varphi(g^-)=1$

If $g=(x^{15}, l), \Phi_{(7,2)}((x^{15}, l))= \frac{\mid \mathbb{Q}_{2} \times C_3 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{30}.1=10=\phi_5 (x^{15})$ since $H \cap \text{CL}(g)\{1, (x^{15})\}$ and $\varphi(g)=1$

If $g=(x^{15}, z), \Phi_{(7,2)}((x^{15}, z))= \frac{\mid \mathbb{Q}_{2} \times C_3 \mid}{\mid \mathbb{Q}_{5} \mid} (\varphi(g))=\frac{300}{30}.1=10=\phi_5 (x^{15})$ since $H \cap \text{CL}(g)\{1, (x^{15})\}$ and $\varphi(g)=1$
If $g=(x^i,z), \Phi_{(7,2)}((x^i,z)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{150}{30} (1+1) = 10 = \phi_x(x^i)$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

Otherwise $\Phi_{(7,2)}(g) = 0$ since $H \cap CL(g) = \phi$

If $g=(1,1), \Phi_{8,2}(1,1) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) = \frac{300}{150} = 2 = \phi_b(1)$ since $H \cap CL(g) = \{(1,1)\}$ and $\varphi(g) = 1$

If $g=(x^2,1), \Phi_{8,2}((x^2,1)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{300}{150} (1+1) = 2 = \phi_b(x^2)$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

If $g=(x^2, z), \Phi_{8,2}((x^2, z)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{300}{150} (1+1) = 2 = \phi_b(x^2)$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

If $g=(x^4,1), \Phi_{8,2}((x^4,1)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{300}{150} (1+1) = 2 = \phi_b(x^4)$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

If $g=(x^4, z), \Phi_{8,2}((x^4, z)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{300}{150} (1+1) = 2 = \phi_b(x^4)$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

If $g=(x^{10},1), \Phi_{8,2}((x^{10},1)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{300}{150} (1+1) = 2 = \phi_b(x^{10})$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

If $g=(x^{10}, z), \Phi_{8,2}((x^{10}, z)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) + \varphi(g^{-1}) = \frac{300}{150} (1+1) = 2 = \phi_b(x^{10})$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(g^{-1}) = 1$

If $g=(x^{15},1), \Phi_{8,2}((x^{15},1)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) = \frac{300}{150} (1) = 2 = \phi_b(x^{15})$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(1) = 1$

If $g=(x^{15}, z), \Phi_{8,2}((x^{15}, z)) = \frac{|\mathbb{Q}_{2a} \times \mathbb{C}_5|}{|\mathbb{H}(g)|} (\varphi(g) = \frac{300}{150} (1) = 2 = \phi_b(x^{15})$ since $H \cap CL(g) = \{ g \} \text{ and } \varphi(g) = \varphi(1) = 1$
Naserr Rasool Mahmood, Salah Hsaaoun Jihadi. On Artin cokernel of the Group \((\mathbb{Q}_{2m} \times \mathbb{C}_5)\) where \(m=p_1, p_2\), \(g.c.d(p_1, p_2) = 1\), \(p_1, p_2 > 2\) and \(p_1, p_2\) are primes numbers

If \(g=(x^5, z), \Phi_{(9, 2)}((x^5, z)) = \frac{\zeta_{30} \times \zeta_5(g)}{\zeta_2(g)} \left( \varphi(g) + \varphi(g^{-1}) \right) = -\frac{300}{150} (1 + 1) = 2 = \varphi_9(x^5)\) since \(H \bigcap CL(g)\)

Otherwise \(\Phi_{(9, 2)}(g) = 0\) since \(H \bigcap CL(g) = \phi\)

Then, the Artin characters table of \((\mathbb{Q}_{30} \times \mathbb{C}_5)\) is given in the following Table:

\[\text{Ar}(\mathbb{Q}_{30} \times \mathbb{C}_5) = \]
Naserr Rasool Mahmood, Salah Hsaaoun Jihadi. **On Artin cokernel of the Group**$\left(\mathbb{Q}_{2m}\times \mathbb{C}_5\right)$** where** $m=p_1p_2$ , $g.c.d(p_1,p_2)=1$ , $p_1,p_2>2$ **and** $p_1,p_2$ **are primes numbers**

| Table (4.1) |

**Theorem (4.2):**

The Artin’s character table of the group $\left(\mathbb{Q}_{2m}\times \mathbb{C}_5\right)$ where $m=p_1p_2$ , $p_1,p_2>2$ and $p_1,p_2$ are primes numbers; is given as follows:

$$\text{Ar}(\mathbb{Q}_{2m}\times \mathbb{C}_5)=$$

| Table (4.2) Which is $18\times18$ matrix square. |

**Proof:**

Let $g \in \left(\mathbb{Q}_{2m} \times \mathbb{C}_5\right)$ ; $g=(q,I)$ or $g=(q,z)$ , $q \in \mathbb{Q}_{2m}, I,z \in \mathbb{C}_5$

**Case (I): If H is a cyclic subgroup of (Q_{2m}× {I}), then:**

1. $H=\langle (x,I) \rangle$
2. $H=\langle (y,I) \rangle$

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Naserr Rasool Mahmood, Salah Hsaaoun Jihadi. On Artin cokernel of the Group\((\mathbb{Q}_{2m} \times C_5)\) where \(m=p_1p_2\), \(\text{g.c.d}(p_1, p_2) = 1\), \(p_1, p_2 > 2\) and \(p_1, p_2\) are primes numbers

and \(\varphi\) the principal character of \(H\), \(\Phi_j\) Artin characters of \(\mathbb{Q}_{2m}\), \(1 \leq j \leq l + 1\) then by using theorem (3,8):

\[
\Phi_j(g) = \begin{cases} 
\frac{|C_G(g)|}{|C_H(g)|} \sum_{i=1}^m \varphi(h_i) & \text{if } h_i \in H \cap \text{CL}(g) \\
0 & \text{if } H \cap \text{CL}(g) = \phi
\end{cases}
\]

(i) If \(g = (1, I)\), \(\Phi_{(1,1)}(g) = \frac{|C_{\mathbb{Q}_{2m} \times C_5}(\text{CL}, 0)|}{|C_H(g)|} \varphi((1,1)) = \frac{20m}{|C_H((1,1))|} . 5 \Phi_j(1)\) since \(H \cap \text{CL}(1,1) = \{(1,1)\}\) and \(\varphi(g) = 1\)

(ii) If \(g = (x^m, I)\), \(g \in H\), \(\Phi_{(1)}(g) = \frac{|C_{\mathbb{Q}_{2m} \times C_5}(\text{CL}, 0)|}{|C_H(g)|} \varphi(g) = \frac{20m}{|C_H(x^m, I)|}.1 = \frac{5|\mathbb{Q}_{2m}(x^m)|}{|C_{<\times>(x^m)|}.1 = 5 \Phi_j(x^m)\) since \(H \cap \text{CL}(g) = \{g\}\) and \(\varphi(g) = 1\)

(iii) If \(g \neq (x^m, I)\) and, \(g \in H\), \(\Phi_{(1)}(g) = \frac{|C_{\mathbb{Q}_{2m} \times C_5}(\text{CL}, 0)|}{|C_H(g)|} (\varphi(g) + \varphi(g^{-1})) = \frac{10m}{|C_H(g)|}(1 + 1) = \frac{20m}{|C_H(g)|} = \frac{5|\mathbb{Q}_{2m}(q)|}{|C_{<\times>(q)|} = 5 \Phi_j(q)\) since \(H \cap \text{CL}(g) = \{g, g^{-1}\}, g = (q, I), q \in \mathbb{Q}_{2m}, q \neq x^m\) and \(\varphi(g) = \varphi(g^{-1}) = 1\)

(iv) If \(g \notin H\), \(\Phi_{(1)}(g) = 0 = 5 \Phi_j(q)\) since \(H \cap \text{CL}(g) = \phi\) and \(q \in \mathbb{Q}_{2m}\)

2-IF \(H = \langle y, I \rangle = \{(1, I), (y, I), (y^2, I), (y^3, I)\}\)

(i) If \(g = (1, I)\), \(\Phi^{(l+1)}(l_1) = \frac{|C_{\mathbb{Q}_{2m} \times C_5}(\text{CL}, 0)|}{|C_H(g)|} \varphi(g) = \frac{20m}{4}.1 = 5.m = 5\Phi_j(1)\) since \(H \cap \text{CL}(1, I) = \{(1, I)\}\) and \(\varphi(g) = 1\)
(ii) If \( g = (x^m, I) = (y^2, I) \) and \( g \in H \), then
\[
\Phi^{(i+1,1)}(g) = \left| \frac{C_{Q_{2m} \times C_5}(CL,g)}{C_H(g)} \right| \Phi(g)
\]
\[
= \frac{20m}{4} \cdot 1 = 5m = 5 \Phi_{(i+1)}(x^m)
\]
since \( H \cap CL(g) = \{g\} \) and \( \Phi(g) = 1 \).

(ii) If \( g \neq (x^m, I) \) and \( g \in H \), i.e. \( \{g = (y, I) \) or \( g = (y^3, I)\} \), then
\[
\Phi^{(i+1,1)}(g) = \left| \frac{C_{Q_{2m} \times C_5}(CL,g)}{C_H(g)} \right| \left( \Phi(g) + \Phi(g^{-1}) \right)
\]
\[
= \frac{10}{4} \cdot (1 + 1) = \frac{20}{4} = 5.1 = 5 \Phi_{(i,1)}(y)
\]
since \( H \cap CL(g) = \{g, g^{-1}\} \) and \( \Phi(g) = \Phi(g^{-1}) = 1 \).
Otherwise \( \Phi^{(i+1,1)}(g) = 0 \) since \( H \cap CL(g) = \phi \).

Case (II): If \( H \) is a cyclic subgroup of \( (Q_{2m} \times \{z\}) \) then:
\[
1 \cdot H = \langle (x, z) \rangle \quad 2 \cdot H = \langle (y, z) \rangle
\]
and \( \Phi \) the principal character of \( H \), \( \Phi_j \) Artin characters of \( Q_{2m} \), \( 1 \leq j \leq l + 1 \) then by using theorem (3,8):
\[
\Phi_j(g) = \left\{
\begin{array}{ll}
\left| \frac{C_G(g)}{C_H(g)} \right| \sum_{i=1}^{m} \phi(h_i) & \text{if } h_i \in H \cap CL(g) \\
0 & \text{if } H \cap CL(g) = \phi
\end{array}
\right.
\]
\[
1 \cdot H = \langle (x, z) \rangle
\]
(i) If \( g = (1, I), (1, z) \), then
\[
\Phi_{(i,2)}(g) = \left| \frac{C_{Q_{2m} \times C_5}(CL,g)}{C_H(g)} \right| \Phi(g)
\]
\[
= \frac{20m}{|C_H((1, I))|} \cdot 1 = \frac{5}{5} \cdot \frac{|Q_{2m}(1)|}{|C_{<x>(1)|} \Phi(1) = \Phi_j(1)
\]
since \( H \cap CL(g) = \{1, (1, z)\} \) and \( \Phi(g) = 1 \).

(ii) \( g = (1, I), (x^m, I)(x^m, z), (1, z) \); \( g \in H \)

If \( g = (1, I), (1, z) \), then
\[
\Phi_{(i,2)}(g) = \left| \frac{C_{Q_{2m} \times C_5}(CL,g)}{C_H(g)} \right| \Phi(g)
\]
\[
= \frac{20m}{|C_H((1, I))|} \cdot 1 = \frac{5}{5} \cdot \frac{|Q_{2m}(1)|}{|C_{<x>(1)|} \Phi(1) = \Phi_j(1)
\]
since \( H \cap CL(g) = \{g\} \) and \( \Phi(g) = 1 \).
\( \text{If } g=(x^m,1), (x^m,z) \), \( \Phi_{(i,2)}(g) = \frac{|C_{Q_{2m}} \times C_5(C_{\omega})|}{|C_H(g)|} \varphi(g) \)

\( = \frac{|Q_{2m}(x^m)|}{5} \frac{|C_{< \omega}(x^m)|}{\varphi(x^m)} = \Phi_j(x^m) \) since \( H \cap CL(g) = \{g\} \) and \( \varphi(g) = 1 \)

(iii) If \( g \neq (x^m,1), (x^m,z) \) and, \( g \in H \)

\( \Phi_{(i,2)}(g) = \frac{|C_{Q_{2m}} \times C_5(C_{\omega})|}{|C_H(g)|} = (\varphi(g) + \varphi(g^{-1}) = \frac{10}{|C_H(g)|} (1 + 1) = \frac{5 |Q_{2m}(q)|}{5 |C_{< \omega}(q)|} \varphi(q) = \Phi_j(q) \) since \( H \cap CL(g) = \{g, g^{-1}\} \), \( \varphi(g) = \varphi(g^{-1}) = 1 \) and \( g=(q,z), q \in Q_{2m} ; q \neq x^m \)

(iv) If \( g \notin H \)

\( \Phi_{(i,2)}(g) = 0 = \Phi_j(q) \) since \( H \cap CL(g) = \phi \) and \( q \in Q_{2m} \)

2-IF \( H = \langle y, I \rangle = \{(1, I), (y, I), (y^2, 1), (y^3, I), (1, z), (y, z), (y^2, z), (y^3, z), (1, z^2), (y, z^2), (y^2, z^2), (y^3, z^3), (1, z^3), (y, z^3), (y^2, z^3), (y^3, z^4), (1, z^4), (y, z^4), (y^2, z^4), (y^3, z^5) \} \)

(i) If \( g = (1, I), (1, z) \), \( \Phi_{(i+1,2)}(g) = \frac{|C_{Q_{2m}} \times C_5(C_{\omega})|}{|C_H(g)|} \varphi(g) = \frac{20m}{20}. 1 = m \) = \( \Phi_{i+1}(g) \)

(ii) If \( g = (y^2, I) = (x^m, I), (y^2, z), (y^2, z^2), (y^2, z^3), (y^2, z) \) and \( g \in H \)

\( \Phi_{(i+1,2)}(g) = \frac{|C_{Q_{2m}} \times C_5(C_{\omega})|}{|C_H(g)|} \varphi(g) = \frac{20m}{20}. 1 = m = \Phi_{i+1}(g) \) since \( H \cap CL(g) = \{g\} \) and \( \varphi(g) = 1 \)

(ii) If \( g \neq (x^m, I) \) and \( g \in H \) i.e \( g = \{(y, I), (y, z), (y, z^2), (y, z^3), (y, z^4)\} \) or \( g = (y^3, I), (y^3, z), (y^3, z^2), (y^3, z^3), (y^3, z^4) \}

\( \Phi_{(i+1,2)}(g) = \frac{|C_{Q_{2m}} \times C_5(C_{\omega})|}{|C_H(g)|} = (\varphi(g) + \varphi(g^{-1}) = \frac{10}{20} (1 + 1) = 1 = \Phi_{i+1}(g) \) since \( H \cap CL(g) = \{g, g^{-1}\} \) and \( \varphi(g) = \varphi(g^{-1}) = 1 \)

Otherwise \( \Phi_{(i+1,2)}(g) = 0 \) since \( H \cap CL(g) = \phi \)
On Artin cokernel of the Group \( \mathbb{Q}_{2m} \times \mathbb{C}_5 \) where \( m = p_1.p_2 \), g.c.d\((p_1,p_2) = 1 \), \( p_1, p_2 > 2 \) and \( p_1, p_2 \) are primes numbers

**Example (4.3):**

To construct \( \text{Ar}(\mathbb{Q}_{66} \times \mathbb{C}_5) = \text{Ar}(\mathbb{Q}_{2.3.11} \times \mathbb{C}_5), p_1 = 3, p_2 = 11 \), we use theorem (3,5) as the following :

\[
\text{Ar}(\mathbb{Q}_{66}) =
\]

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<tr>
<th>( \ell )-Classes</th>
<th>([1])</th>
<th>([x^2])</th>
<th>([x^6])</th>
<th>([x^{22}])</th>
<th>([x^{33}])</th>
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<th>([x^{11}])</th>
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<td>0</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>( \phi_9 )</td>
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<td>0</td>
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</tr>
</tbody>
</table>

Table (4,3)

Then by using theorem (4,2) Artin characters table of the group \( \mathbb{Q}_{66} \times \mathbb{C}_5 \) is:-

\[
\text{Ar}(\mathbb{Q}_{66} \times \mathbb{C}_5) =
\]

<table>
<thead>
<tr>
<th>( \ell )-Classes</th>
<th>([1])</th>
<th>([2])</th>
<th>([3])</th>
<th>([4])</th>
<th>([5])</th>
<th>([6])</th>
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<th>([9])</th>
<th>([10])</th>
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<tr>
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</table>

Table (4,4)
On Artin cokernel of the Group($Q_{2m} \times C_5$) where $m=p_1p_2$, $g.c.d(p_1,p_2) = 1$, $p_1, p_2 > 2$ and $p_1, p_2$ are primes numbers

REFERENCES