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Solving New Type of Linear Equations by Using New Transformation

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Abstract:

In this paper, we introduce definition of new transformation which we call it Al-Zughair transform. Also, we introduce properties, theorems, proofs and transformations of the constant functions, logarithms functions and other functions. Also, we introduce how we can use this transformation and its inverse to solve new type of linear ordinary differential equations, which we have presented in this research. (This transformation is discovered by prof. Ali Hassan Mohammed).

Key words: Al-Zughair transform, Logarithm coefficients, Inverse of Al-Zughair transform, new type of ordinary linear differential equations, L.O.D.E with logarithms coefficients, Ali's Equation.

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INTRODUCTION:

Laplace transformation [1]is considered as one of the important transformations which is known to solve the L.O.D.E. with constants coefficients and subject to some initial conditions and which has the general formula:

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = f(x)$$

Where $a_0, a_1 \dots, a_n$ are constants. With one condition that Laplace transformation of the function $f(x)$ is defined. In this paper we define a new transformation which is work to solve the L.O.D.E with logarithms coefficients (new type of linear ordinary differential equations) which has the general form:

$$a_0 \, (lnx)^n \frac{d^n y}{dx^n} + a_1 \, (lnx)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \, lnx \frac{dy}{dx} + a_n \, y = f(x)$$

Where $a_0, a_1 \dots, a_n$ are constants. This transformation is defined for some functions for example constant functions, logarithm functions, and other functions.

Definition(1):[2]

Let f is defined function at interval (a, b) then the integral transformation for f whose it′s symbol $F(p)$ is defined as:

$$F(p) = \int_a^b k(p, x) \, f(x) \, dx$$

Where k is a fixed function of two variables, called the *kernel* of the transformation, and a, b are real numbers or ∓∞, such that the above integral converges.

Definition (2): Al-Zughair transform for the function $f(x)$, we denote by $Z[f(x)]$,where $x \in [1, e]$ is defined by the following integral:

$$Ꝣ[f(x)] = \int_1^e \frac{(lnx)^p}{x} f(x)dx = F(p)$$

Such that this integral is convergent , $p > -1$

Property (1) : (Linear property)

$Ꝣ[Af(x) \pm Bg(x)] = A\,Ꝣ[f(x)] \pm BꝢ[g(x)]$, where A and B are constants, the functions $f(x)$ and $g(x)$ are defined when $x \in [1, e]$

Proof:

$$Ꝣ[Af(x) \pm Bg(x)] = \int_1^e \frac{(lnx)^p}{x} [Af(x) \pm Bg(x)]dx$$

$$= \int_1^e \frac{(lnx)^p}{x} A\,f(x)\,dx \pm \int_1^e \frac{(lnx)^p}{x} B\,g(x)\,dx$$

$$= A\int_1^e \frac{(lnx)^p}{x} f(x)\,dx \pm B\int_1^e \frac{(lnx)^p}{x} g(x)\,dx$$

$$= A\,Ꝣ[f(x)] \pm BꝢ[g(x)]$$

Transformations for some functions:

We are going to find the Ꝣ-transformation for some functions, like the fixed functions, logarithm functions, polynomial functions and other functions.

1- If $f(x) = 1$, $p > -1$, then

$$Ꝣ[1] = \frac{1}{p+1}$$

Proof:

$$Ꝣ[1] = \int_1^e \frac{(lnx)^p}{x}.(1)dx = \left.\frac{(lnx)^{p+1}}{p+1}\right|_1^e = \frac{1}{p+1}$$

2- If $f(x) = k$, $p > -1$ and k is constant , then

$$Ɀ[k] = \frac{k}{p+1}$$

Proof:

$$Ɀ[k] = \int_1^e \frac{(lnx)^p}{x}.kdx = k\int_1^e \frac{(lnx)^p}{x}dx = k\left.\frac{(lnx)^{p+1}}{p+1}\right|_1^e = \frac{k}{p+1}$$

3- If $f(x) = (lnx)^n$, $p > -(n+1)$, then

$$Ɀ[(lnx)^n] = \frac{1}{p+(n+1)}$$

Proof:

$$Ɀ[(lnx)^n] = \int_1^e \frac{(lnx)^p}{x}(lnx)^n dx = \int_1^e \frac{(lnx)^{p+n}}{x}dx$$

$$= \left.\frac{(lnx)^{p+n+1}}{p+n+1}\right|_1^e = \frac{1}{p+(n+1)}$$

4- If $f(x) = lnlnx$, $p > -1$, then

$$Ɀ[lnlnx] = \frac{-1}{(p+1)^2}$$

Proof:

$$Ɀ[lnlnx] = \int_1^e \frac{(lnx)^p}{x} lnlnx\, dx$$

Integrate by part,

Let $u = lnlnx \implies du = x^{-1}(lnx)^{-1}dx$, and $dv = \frac{(lnx)^p}{x}dx \implies v = \frac{(lnx)^{p+1}}{p+1}$

$$so, \int_1^e \frac{(lnx)^p}{x} lnlnx\, dx = lnlnx.\left.\frac{(lnx)^{p+1}}{p+1}\right|_1^e - \frac{1}{p+1}\int_1^e \frac{(lnx)^p}{x}dx$$

$$= \frac{-1}{(p+1)^2}$$

5- If $f(x) = (lnlnx)^n$, $p > -1, n = 1,2,3,\ldots$, then

$$Ɀ[(lnlnx)^n] = \frac{(-1)^n\, n!}{(p+1)^{n+1}}$$

Proof:

if n=1

$$\Rightarrow \qquad Ƶ[lnlnx] = \frac{-1}{(p+1)^2} \qquad \text{from(4)}$$

if n=2

$$Ƶ[(lnlnx)^2] = \int_1^e \frac{(lnx)^p}{x}(lnlnx)^2\,dx$$

Integrate by part ,

Let $u = (lnlnx)^2 \Rightarrow du = 2x^{-1}\,(lnx)^{-1}lnlnx\;\,dx$,

and , $dv = \frac{(lnx)^p}{x}\,dx \quad \Rightarrow \quad v = \frac{(lnx)^{p+1}}{p+1}$

$$\int_1^e \frac{(lnx)^p}{x}(lnlnx)^2dx$$

$$= (lnlnx)^2\,\frac{(lnx)^{p+1}}{p+1}\Bigg|_1^e - \frac{2}{p+1}\int_1^e \frac{(lnx)^p}{x}lnlnx\,dx$$

$$= \frac{-2}{p+1}\cdot\frac{-1}{(p+1)^2} = \frac{2}{(p+1)^3}$$

if n=3

$$Ƶ[(lnlnx)^3] = \int_1^e \frac{(lnx)^p}{x}(lnlnx)^3\,dx$$

Integrate by part ,

Let $u = (lnlnx)^3 \Rightarrow du = 3x^{-1}\,(lnx)^{-1}(lnlnx)^2\,dx$,

and $dv = \frac{(lnx)^p}{x} \Rightarrow v = \frac{(lnx)^{p+1}}{p+1}$

$$\int_1^e \frac{(lnx)^p}{x}(lnlnx)^3dx$$

$$= (lnlnx)^3\,\frac{(lnx)^{p+1}}{p+1}\Bigg|_1^e - \frac{3}{p+1}\int_1^e \frac{(lnx)^p}{x}(lnlnx)^2\,dx$$

$$= \frac{-3}{p+1}\cdot\frac{(-2)(-1)}{(p+1)^3} = \frac{-3.(-2)(-1)}{(p+1)^4} = \frac{-3!}{(p+1)^4}$$

if n=4

$$ʑ[(lnlnx)^4] = \int_1^e \frac{(lnx)^p}{x} (lnlnx)^4 \, dx$$

Integrate by part ,

Let $u = (lnlnx)^4 \implies du = 4x^{-1} (lnx)^{-1} (lnlnx)^3 dx$,

and $dv = \frac{(lnx)^p}{x} \implies v = \frac{(lnx)^{p+1}}{p+1}$

$$\int_1^e \frac{(lnx)^p}{x} (lnlnx)^4 dx$$

$$= (lnlnx)^4 \left. \frac{(lnx)^{p+1}}{p+1} \right|_1^e - \frac{4}{p+1} \int_1^e \frac{(lnx)^p}{x} (lnlnx)^4 \, dx$$

$$= \frac{-4}{p+1} \cdot \frac{(-1)(-2)(-3)}{(p+1)^3} = \frac{4!}{(p+1)^5}$$

Thus ,

$$ʑ[(lnlnx)^n] = \frac{(-1)^n \, n!}{(p+1)^n} \quad , n = 1,2,3, \ldots$$

6- If $f(x) = si\,n(alnlnx)$, $p > -1$, $a \in \mathbb{R}$, then

$$ʑ[sin(alnlnx)] = \frac{-a}{(p+1)^2 + a^2}$$

Proof:

$$ʑ[sin(alnlnx)] = \int_1^e \frac{(lnx)^p}{x} sin(alnlnx) \, dx$$

$$= \int_1^e \frac{(lnx)^p}{x} \left(\frac{e^{ialnlnx} - e^{-ialnlnx}}{2i} \right) dx$$

$$= \int_1^e \frac{(lnx)^p}{x} \left(\frac{(lnx)^{ia} - (lnx)^{-ia}}{2i} \right) dx$$

$$= \frac{1}{2i} \left(\int_1^e \frac{(lnx)^{p+ia}}{x} dx - \int_1^e \frac{(lnx)^{p-ia}}{x} dx \right)$$

$$= \frac{1}{2i} \left(\int_1^e \frac{(lnx)^{p+ia}}{x} dx - \int_1^e \frac{(lnx)^{p-ia}}{x} dx \right)$$

$$= \frac{1}{2i}\left[\frac{(lnx)^{p+ia+1}}{p+ia+1} - \frac{(lnx)^{p-ia+1}}{p-ia+1}\right]_1^e$$

$$= \frac{1}{2i}\left[\frac{1}{p+ia+1} - \frac{1}{p-ia+1}\right]$$

$$= \frac{-a}{(p+1)^2 + a^2}$$

7- If $f(x) = cos(alnlnx),\ p > -1\ , a \in \mathbb{R}$, then

$$Ꝣ[cos(alnlnx)] = \frac{p+1}{(p+1)^2 + a^2}$$

Proof:

$$Ꝣ[cos(alnlnx)] = \int_1^e \frac{(lnx)^p}{x} cos(alnlnx)\, dx$$

$$= \int_1^e \frac{(lnx)^p}{x}\left(\frac{e^{ialnlnx} + e^{-ialnlnx}}{2}\right) dx$$

$$= \int_1^e \frac{(lnx)^p}{x}\left(\frac{(lnx)^{ia} + (lnx)^{-ia}}{2}\right) dx$$

$$= \frac{1}{2}\left(\int_1^e \frac{(lnx)^{p+ia}}{x} dx + \int_1^e \frac{(lnx)^{p-ia}}{x} dx\right)$$

$$= \frac{1}{2}\left[\frac{(lnx)^{p+ia+1}}{p+ia+1} + \frac{(lnx)^{p-ia+1}}{p-ia+1}\right]_1^e$$

$$= \frac{1}{2}\left[\frac{1}{p+ia+1} + \frac{1}{p-ia+1}\right]$$

$$= \frac{p+1}{(p+1)^2 + a^2}$$

8- If $f(x) = sinh(alnlnx)\ ,\ |p+1| > a\ ,\ a \in \mathbb{R}$, then

$$Ꝣ[sinh(alnlnx)] = \frac{-a}{(p+1)^2 - a^2}$$

Proof:

$$Ꝣ[sinh(alnlnx)] = \int_1^e \frac{(lnx)^p}{x} sinh(alnlnx)\, dx$$

$$= \int_1^e \frac{(lnx)^p}{x} \left(\frac{e^{alnlnx} - e^{-alnlnx}}{2} \right) dx$$

$$= \int_1^e \frac{(lnx)^p}{x} \left(\frac{(lnx)^a - (lnx)^{-a}}{2} \right) dx$$

$$= \frac{1}{2} \left(\int_1^e \frac{(lnx)^{p+a}}{x} dx - \int_1^e \frac{(lnx)^{p-a}}{x} dx \right)$$

$$= \frac{1}{2} \left(\int_1^e \frac{(lnx)^{p+a}}{x} dx - \int_1^e \frac{(lnx)^{p-a}}{x} dx \right)$$

$$= \frac{1}{2} \left[\frac{(lnx)^{p+a+1}}{p+a+1} - \frac{(lnx)^{p-a+1}}{p-a+1} \right]_1^e$$

$$= \frac{1}{2} \left[\frac{1}{p+a+1} - \frac{1}{p-a+1} \right]$$

$$= \frac{-a}{(p+1)^2 - a^2}$$

9- If $f(x) = cosh(alnlnx)$, $|p+1| > a$, $a \in \mathbb{R}$, then

$$Ɀ[cosh(alnlnx)] = \frac{p+1}{(p+1)^2 - a^2}$$

Proof:

$$Ɀ[cosh(alnlnx)] = \int_1^e \frac{(lnx)^p}{x} cosh(alnlnx)\, dx$$

$$= \int_1^e \frac{(lnx)^p}{x} \left(\frac{e^{alnlnx} + e^{-alnlnx}}{2} \right) dx$$

$$= \int_1^e \frac{(lnx)^p}{x} \left(\frac{(lnx)^a + (lnx)^{-a}}{2} \right) dx$$

$$= \frac{1}{2} \left(\int_1^e \frac{(lnx)^{p+a}}{x} dx + \int_1^e \frac{(lnx)^{p-a}}{x} dx \right)$$

$$= \frac{1}{2}\left(\int_1^e \frac{(lnx)^{p+a}}{x} dx + \int_1^e \frac{(lnx)^{p-a}}{x} dx \right)$$

$$= \frac{1}{2}\left[\frac{(lnx)^{p+a+1}}{p+a+1} + \frac{(lnx)^{p-a+1}}{p-a+1} \right]_1^e$$

$$= \frac{1}{2}\left[\frac{1}{p+a+1} + \frac{1}{p-a+1} \right]$$

$$= \frac{p+1}{(p+1)^2 - a^2}$$

Theorem(1):

If $Z[f(x)] = F(p)$ and a is constant, then $Z[(lnx)^a f(x)] = F(p+a)$

Proof:

$$Z[(lnx)^a] = \int_1^e \frac{(lnx)^p}{x} (lnx)^a f(x)\ dx$$

$$= \int_1^e \frac{(lnx)^{p+a}}{x} f(x)\ dx$$

$$= F(p+a)$$

For example:

$$1 - Z\,[(lnx)^4 \cosh(6lnlnx)] = \frac{p+5}{(p+5)^2 - 36}$$

$$2 - Z[lnx\,(lnlnx)^9] = \frac{-\,9!}{(p+2)^{10}}$$

Definition(3):

Let $f(x)$ be a function where $, x \in [1, e]$ and $Z[f(x)] = F(p)$, $f(x)$ is said to be an inverse for the Al-Zughair transform and written as:

$Z^{-1}[F(p)] = f(x)$ where Z^{-1} returns the transformation to the original function.

For example:

$$Z^{-1}\left[\frac{k}{p+1}\right] = k \qquad ; \qquad p > -1\,,\ \text{since} \quad Z[k] = \frac{k}{p+1}$$

$$Ⴚ^{-1}\left[\frac{1}{p+(n+1)}\right]=(lnx)^n \ ; \ p>-(n+1)$$

$$\text{since } \ Ⴚ[(lnx)^n]=\frac{1}{p+(n+1)}$$

$$Ⴚ^{-1}\left[\frac{(-1)^n\, n!}{(p+1)^n}\right]=(lnlnx)^n \ ; \ p>-1\,, \ \text{since } \ Ⴚ[(lnlnx)^n]$$

$$=\frac{(-1)^n\, n!}{(p+1)^{n+1}}$$

$$Ⴚ^{-1}\left[\frac{-a}{(p+1)^2+a^2}\right]=sin(alnlnx) \qquad ; \ p>-1$$

$$\text{since}, \qquad Ⴚ[sin(alnlnx)]=\frac{-a}{(p+1)^2+a^2}$$

$$Ⴚ^{-1}\left[\frac{p+1}{(p+1)^2-a^2}\right]=cosh(alnlnx) \qquad ; \ |p+1|>a$$

$$\text{since}, \qquad Ⴚ[cosh(alnlnx)]=\frac{p+1}{(p+1)^2-a^2}$$

$Ⴚ^{-1}$ has the linear property as it is for the transformation Ⴚ .i.e

$$Ⴚ^{-1}[a_1F_1(p)\pm a_2F_2(p)\pm\cdots\pm a_nF_n(p)]$$
$$=a_1Ⴚ^{-1}[F_1(p)]\pm a_2Ⴚ^{-1}[F_2(p)]\pm\cdots\pm a_nႺ^{-1}[F_n(p)]$$
$$=a_1f_1(x)\pm a_2f_2(x)\pm\cdots\pm a_nf_n(x)$$

Where $a_1, a_2, \dots, a_n$ are constants, the functions $f_1(x), f_2(x), \cdots, f_n(x)$ are defined when $x\in[1,e]$.

Theorem (2): If $Ⴚ^{-1}[F(p)]=f(x)$, then $Ⴚ^{-1}[F(p+a)]=(lnx)^a f(x)$

Where a is constant.

Proof:

$$Ⴚ^{-1}[F(p+a)]=(lnx)^a f(x)=(lnx)^a\, Ⴚ^{-1}[F(p)]$$

For example:

$$1- \ Ⴚ^{-1}\left[\frac{1}{(p-6)^2-16}\right]=\frac{-1}{4}(lnx)^{-7}sinh(4lnlnx)$$

$$2- \ Ⴚ^{-1}\left[\frac{p+4}{(p+3)^2+25}\right]=Ⴚ^{-1}\left[\frac{p+3}{(p+3)^2+25}+\frac{1}{(p+3)^2+25}\right]$$

$$= (lnx)^2 \, cos(5lnlnx) - \frac{1}{5}(lnx)^2 \, sin(5lnlnx)$$

New Type of Linear Ordinary Differential Equations

Definition (4): The equation

$$a_0(lnx)^n \frac{d^n y}{dx^n} + a_1(lnx)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}\, lnx \frac{dy}{dx} + a_n\, y = f(x)$$

Where $a_0, a_1 \ldots, a_n$ are constants and $f(x)$ is a function of x , is called ***Ali's Equation***

Solving the Linear Differential Equations with Logarithm Coefficients

One of the most important applications of the Ɀ-transform is solving the linear differential equations with logarithm coefficients. Transforming the L.O.D.E. into algebraic equation in $y\,(p)$ is done by transforming the derivations and their coefficients and the function $f(x)$ to the new formulas .After transforming the equation

$$a_0(lnx)^n \frac{d^n y}{dx^n} + a_1(lnx)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1}\, lnx \frac{dy}{dx} + a_n\, y = f(x)$$

into an algebraic equation, we are going to find the inverse transformation for this algebraic equation and the result will be the solution of the differential equation .

Theorem(4):

If the function $y(lnx)$ is defined for $x \in [1, e]$ and its derivatives $y^{(1)}(lnx)\,, y^{(2)}(lnx)\,, \ldots, y^{(n)}(lnx)$ are exist then:

$$Ɀ\big[(lnx)^n y^{(n)}(lnx)\big] = y^{(n-1)}(1) + (-1)^n (p+n) y^{(n-2)}(1) + (-1)^{n-1}(p+n)\big(p+(n-1)\big) y^{(n-3)}(1) + \cdots + (p+n)\big(p+(n-1)\big) \ldots (p+2) y(1) + (-1)^n (p+n)!\, Ɀ[y(lnx)]$$

Proof:

If n=1 ,

$$Ƶ[lnx\, y'(lnx)] = \int_1^e \frac{(lnx)^p}{x} lnx\, y'(lnx)\, dx = \int_1^e \frac{(lnx)^{p+1}}{x} y'(lnx)\, dx$$

$$\text{Let } u = (lnx)^{p+1} \Rightarrow du = (p+1)\frac{(lnx)^p}{x} dx$$

$$dv = \frac{y'(lnx)}{x} dx \Rightarrow \quad v = y(lnx)$$

$$\int_1^e \frac{(lnx)^{p+1}}{x} y'(lnx)\, dx$$

$$= (lnx)^{p+1}. y(lnx)|_1^e - (p+1)\int_1^e \frac{(lnx)^p}{x} lnx\, y(lnx)\, dx$$

$$= y(1) - (p+1)\, Ƶ[y(lnx)]$$

If n=2 ,

$$Ƶ[(lnx)^2\, y''(lnx)] = \int_1^e \frac{(lnx)^{p+2}}{x} y''(lnx)\, dx$$

$$\text{Let } u = (lnx)^{p+2} \Rightarrow du = (p+2)\frac{(lnx)^{p+1}}{x} dx$$

$$dv = \frac{y''(lnx)}{x} dx \Rightarrow \quad v = y'(lnx)$$

$$\int_1^e \frac{(lnx)^{p+2}}{x} y''(lnx)\, dx$$

$$= (lnx)^{p+2}. y'(lnx)|_1^e - (p+2)\int_1^e \frac{(lnx)^{p+1}}{x} y(lnx)\, dx$$

$$= y'(1) - (p+2)\, Ƶ[lnx\, . y'(lnx)]$$

$$= y'(1) - (p+2)y(1) + (p+2)(p+1)\, Ƶ[y(lnx)]$$

If n=3 ,

$$Ƶ[(lnx)^3\, y'''(lnx)] = \int_1^e \frac{(lnx)^{p+3}}{x} y'''(lnx)\, dx$$

$$\text{Let } u = (lnx)^{p+3} \Rightarrow du = (p+3)\frac{(lnx)^{p+2}}{x}dx$$

$$dv = \frac{y'''(lnx)}{x}\,dx \Longrightarrow \quad v = y''(lnx)$$

$$\int_1^e \frac{(lnx)^{p+3}}{x}\,y'''(lnx)\,dx$$

$$= (lnx)^{p+3}.\,y''(lnx)|_1^e - (p+3)\int_1^e \frac{(lnx)^{p+2}}{x}\,y''(lnx)\,dx$$

$$= y''(1) - (p+3)\,Ⱬ[(lnx)^2\,.\,y''(lnx)]$$

$$= y''(1) - (p+3)y'(1) + (p+3)(p+2)\,y(1) - (p+3)\,(p+2)\,(p+1)\,Ⱬ[y(lnx)]$$

Then ,

$$Ⱬ\left[(lnx)^n y^{(n)}(lnx)\right] = y^{(n-1)}(1) + (-1)^n(p+n)y^{(n-2)}(1) + (-1)^{n-1}(p+n)\big(p+(n-1)\big)y^{(n-3)} + \cdots + (p+n)\big(p+(n-1)\big)\ldots(p+2)y(1) + (-1)^n(p+n)!\,Ⱬ[y(lnx)]$$

Example(1): To find the solution of the differential equation
$lnx\, y' - 4y = \sinh(2\ln(lnx))$; $y(1) = -3$, $y = y(lnx)$
We take Ⱬ-transform to both sides of above equation we get :

$$Ⱬ[lnx\, y'(lnx)] - 4\,Ⱬ[y(lnx)] = Ⱬ[\sinh(2\ln(lnx))]$$

$$y(1) - (p+1)\,Ⱬ[y(lnx)] - 4\,Ⱬ[y(lnx)] = \frac{-2}{(p+1)^2-4}$$

$$-3 - (p+5)\,Ⱬ[y(lnx)] = \frac{-2}{(p+1)^2-4}$$

$$Ⱬ[y(lnx)] = \frac{-3p^2-6p+11}{(p+5)\,[(p+1)^2-4]}$$

By taking $Ⱬ^{-1}$-transform to both sides of above equation we get:

$$y = Ⱬ^{-1}\left[\frac{A}{(p+5)} + \frac{Bp+C}{(p+1)^2-4}\right]$$

$A + B = -3$ $\cdots(1)$
$2A + 5B + C = -6$ $\cdots(2)$
$-3A + 5C = 11$ $\cdots(3)$

Hence ,

$$A=\frac{-17}{6}\quad,\ B=\frac{-1}{6}\ ,\ C=\frac{1}{2}$$

$$\therefore y(lnx)=\frac{-17}{6}(lnx)^4-\frac{1}{6}cosh(2\,ln(lnx))-\frac{1}{3}sinh(2\,ln(lnx))$$

$$\Longrightarrow y(x)=\frac{-17}{6}x^4-\frac{1}{6}cosh(2\,ln\,x)-\frac{1}{3}sinh(2\,ln\,x)$$

Example (2): To find the solution of the differential equation

$$(lnx)^2\,y''(lnx)-lnx\,y'(lnx)+y(lnx)=ln(lnx)\qquad;\ y(1)=-1\ ,\ y'(1)=2\,,y=y(lnx)$$

We take Ꞩ-transform to both sides of above equation we get :

$$Ꞩ[(lnx)^2\,y''(lnx)]-Ꞩ[lnx\,y'(lnx)]+Ꞩ[y(lnx)]=Ꞩ[ln(lnx)]$$

$$y'(1)-(p+2)\,y(1)+(p+2)(p+1)Ꞩ[y(lnx)]-y(1)$$

$$+(p+1)Ꞩ[y(lnx)]+Ꞩ[y(lnx)]=\frac{-1}{(p+1)^2}$$

$$Ꞩ[y(lnx)]=\frac{-p^3-7p^2-11p-10}{(p+1)^2\,(p+2)^2}$$

By taking $Ꞩ^{-1}$-transform to both sides of above equation we get :

$$y=Ꞩ^{-1}\left[\frac{A}{p+1}+\frac{B}{(p+1)^2}+\frac{C}{p+2}+\frac{D}{(p+2)^2}\right]$$

$A+C=-1$ ⋯ (4)

$5A+B+4C+D=-7$ ⋯ (5)

$8A+4B+5C+2D=-11$ ⋯ (6)

$4A+4B+2C+D=-6$ ⋯ (7)

Hence ,

$$A=2\ ,\ B=-1\,,\ C=-3\ ,\ D=-4$$

$$y(lnx)=2+l\,n(lnx)-3lnx+4(lnx).(ln(lnx))$$

$$\Longrightarrow y(x)=2+l\,n\,x-3x+4x\,lnx$$

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