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Model of Discovery Learning with the Help of GeoGebra

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Abstract:

This paper presents a model for discovery learning with the help of GeoGebra that provide opportunities for students to engage in mathematical activities such as exploration, conjecturing, explanation, and generalization. This model has been primarily developed to suit the use of discovery learning method in dynamic environment – GeoGebra. This model was used to design an example which aims at consolidating students' understandings on the concepts of center and radius and developing the concepts of locus and perpendicular bisector.

Key words: Model of the discovery learning with GeoGebra; dynamic mathematic software; guide discovery learning

1 INTRODUCTION

In some recent years, Ministry of Education and Training Viet Nam has required and encouraged the use of information technology in learning and teaching at secondary high schools. In Viet Nam, more and more schools provide students with a computer of their own. The availability of different kinds of technologies in mathematics classrooms is increasing.

GeoGebra is dynamic mathematics software designed for teaching and learning mathematics in secondary school and college level. The question was raised that how to use GeoGebra efficiently.

In this paper we introduce model teaching for GeGebra which foster students' capability to explore, conjecture, verify, explain and make generalizations.

2 THEORETICAL BACKGROUND

Discovery learning

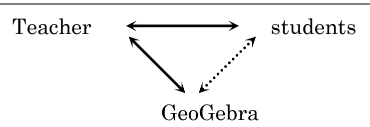
"Discovery learning" is a label that has been high profile in discussions about education, including mathematical education, since at least the 1940s. It has a long history in education (Dewey, 1938; Bruner,1961). Discovery learning is a kind of instructional method in which the teacher guided students using some questions to help them in exploring, conjecturing and constructing their knowledge. Bicknell-Holmes and Hoffman (2000) describe the three primary attributes of discovery learning. There are 1) exploring and problem-solving to create, integrate, and generalize knowledge, 2) student driven, interest-based activities in which the student determines the sequence and frequency, and 3) activities to encourage the integration of new knowledge into the learner's existing knowledge base. Since technology allows for more student-centered approaches including active learning, mathematical experiments, or discovery learning.

What is GeoGebra

GeoGebra is an excellent platform for experimentation, which supports the development of mathematical concepts and the abilities to explain geometric properties. It combines the ease of use of a dynamic geometry software with certain features of a computer algebra system and therefore, allows for bridging the gap between the mathematical disciplines of geometry, algebra, and even calculus (Hohenwarter and Preiner, 2007).

GeoGebra can be used to visualize mathematical concepts as well as to create instructional materials. On the other hand, GeoGebra has the potential to foster active and student-centered learning by allowing for mathematical experiments, interactive explorations, as well as discovery learning. Moreover, researchers agree that one of the most appreciated affordances provided by GeoGebra is the possibility to make investigations. Le Viet Minh Triet (2014) introduced two different levels of use of GeoGebra in learning and teaching mathematic (see Table 1)

Table 1: Level's integrating GeoGebra

Level	**Teacher**	**Student**
	Control GeoGebra	Observe, make conjectures, find the solution
	Give supporting if the student	Self using GeoGebra to explore, make conjectures and find the solution

3 METHOD

Our study consists of two phases.

In the first step, we developed a model for teaching based on the fundamental principles identified in our theoretical background. The first step is described in detail in Sect. 4.1. To illustrate and examine the model a concrete example of a task situation was developed. Each component task in the example was examined, and predictions about student performance were made. The second phase is described in detail in Sect. 4.2

4 MODEL OF THE DISCOVERY LEARNING WITH GEOGEBRA

4.1 Discovery learning model with the help of GeoGebra

Table 2: Discovery learning model with the help of GeoGebra

Phases	Tasks for the teacher (a)	Tasks for the students (b)
[i] present the problem and Make the motivation	[1a] Create the learning situation to lead the student to the discovery. The problem presented which is motivating and inspiring through various methods like the demonstration, narration, questioning, etc.	[1b]. Analyze to understand the problems
[ii]Do experiment with GeoGebra	[2a] Use GeoGebra as a tool to represent the problem; Make an appropriate construction; study different cases	[2b] Use GeoGebra as a tool to represent the problem; Make an appropriate construction; study different cases
[iii] Formulate and explain—formulate rules or explanation	[3a] Making conjectures: Ask students to make the predictions	[3b] Explore and make the conjectures via different tool provided by GeoGebra
	[4a] Verifying conjectures: Using GeoGebra to verify or reject the inferences. Are you assured of the truth of your conjectures? If not, try to use the GeoGebra to support your theory. When you are convinced, go to the next task.	[4b] Using GeoGebra to verify or reject the conjectures.
	[5a] explaining conjectures: Explain in your words why your theory is correct.	[5b] explain why the conjecture is true
[iv] Construct a proof.	[6a] Making formal proof and checking the solution: Construct a proof and check the solution	[6b] write and test the solution
[v] Closure	[7a] Generalizing or extending the problem: Investigate if your conjecture can be generalized or extended.	[7b] Perform the tasks above with new premises, by using appropriate techniques, such as posting what if? or what if not? questions.

4.2 An example

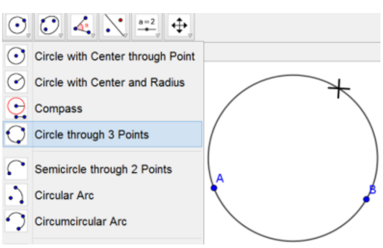
To demonstrate how the model could be utilized, we introduce a concrete example. This case focuses on the following task which was mentioned in Grade 9th Geometry textbook.

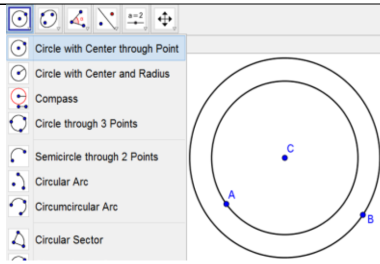
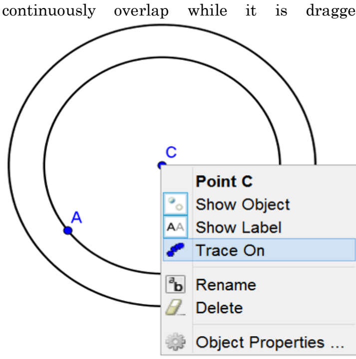
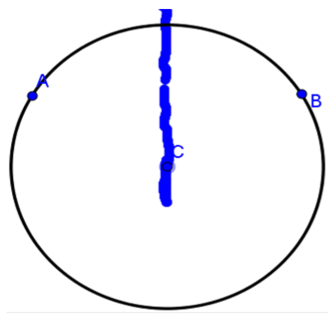
The question (translated by me): "Given two points A and B:
a) Draw a circle passing through the two points
b) How much such circles are there? What is the line that contains all the centers of those circles" (p.98)

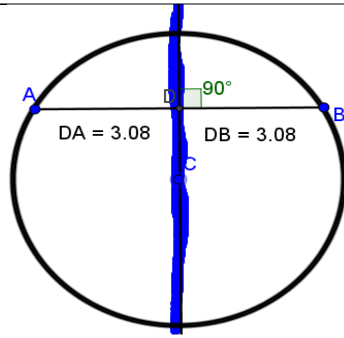
To solve this question with the help of GeoGebra, we investigated students with the problem (*) which was restated as follows: "*Let A and B be two fixed points. How many circles can be constructed through A and B*"

In the following table, the tasks are elaborated, and predictions about student performances are made.

Table 3: An example for discovery learning model

Activities	**Tasks for the teacher (a)**	**Tasks for the students (b)**
[i] Present the problem and Make the motivation	1a. Given the problem 1: " Let A and B be two fixed points. How many circles can be constructed through A and B?"	1b. Listen and read the asking
[ii] Do experiment with GeoGebra	[2a] Use GeoGebra as a tool to represent the problem - Use GeoGebra as a tool to represent two points A and B - use the command "Circle through Three Points" and click on points A and B then move the cursor away from the points without clicking (see Fig.2).  Fig 1: Apply the Circle Through 3 Points tool on two points A and B - How many circles through A and B - Investigate the relationship among these circles.	[2b] Observe data to find out relationships among observed data - Student: by dragging the circle randomly, there are infinitely many circles passing through A and B. Furthermore, the size of the circle looks to assume any values. - Thinks
[iii] Formulate and explain—formulate rules or explanation	[3a] Ask students to make prediction - Use the Circle With Center Through Point tool; - Select an arbitrary point as the center point C and the given point A as a point on circle;	[3b] Formulate a conjecture. - It is most likely that the circle produced does not pass through another given point B

	 **Fig 2: Two circles with center C that pass through A and B, respectively** - Drag point C so that point B appears to lie on the circle; - Let us produce one more circle by applying the Circle With Center Through Point command again with the same point C as the center point and then point B as a point on circle. - Since we want to find a circle passing through both point A and point B, we drag point C so that the two circles overlap; - What happens to the point C if the two circles continuously overlap while it is dragged?  **Fig 3: Activating tracing for point C with the Trace On contextual menu option** - activate the trace of center point C by right-clicking on the point then choosing the Trace On option from the pop-up contextual menu (see Fig 3). - drag the point by keeping the two circles coincident (see Fig 4). - What is so unique to the trace of point C?	- point C should be located somewhere in the midway between A and B to keep the two circles coincide  **Fig 4: Trace of point C generated by attempting to keep two circles coincident while dragging C** - Student: C lies on the line - Student: C lies on the line which through a midpoint of A and B - Student: conjectured C lies on the perpendicular of AB - Student: C lies on the perpendicular bisector of AB
	[4a] Are you assured of the truth of your conjectures? If not, try to use the GeoGebra to support your theory. When you are convinced, go to the next task (see Fig 5). - construct the segment AB; - created directly by the GeoGebra Perpendicular Bisector menu option - create an arbitrary point C on the perpendicular bisector of AB - form a circle with center C through A by using the Circle With Center Through Point tool - drag point C to test the conjecture "How many circles can be constructed through A and B which A and B are two fixed points?	[4b] Use GeoGebra to support or refute the conjectures

	"What can we say about the centers of these circle?"	 **Fig 5: A line to fit the trace of C with accompanying measurements to encourage and support student conjectures** - see that points A and B remain on the circle - an infinite number of circles which can pass through two point Conjecturing: if the two circles continuously overlap, C lies on the d which is perpendicular bisector of AB while it is dragged
	[5a] Explain in your words why your conjecture is right. What can we say if C is the center and points A and B lie on the circle? Inverse, what can we say if C lies in the perpendicular bisector of AB?	[5b] the length of CA equals the length of CB because these two line segments are the radii of the same circle Therefore, if C is the center of an arbitrary circle passing through points A and B, then C must be lying on the perpendicular bisector of AB - If C lies in the perpendicular bisector of AB, then the length of CA equals the length of CB. So that, C is the center of the circle passing through points A and B.
[iv] Construct a proof.	[6a] Construct a proof.	[6b] Construct a proof. Phase 1: Let C be the center of the circles passing through points A and B. Then, $CB = CA$ Implies that P lies on the perpendicular bisector of AB. Phase 2: Let d be the perpendicular bisector of AB and C be the dynamic point on the d. We also see that, $CB = CA$ Implies that, C is the center of the circle passing through points A and B.
	[6a] Check the solution	[6b] Check the solution
[v] Closure	[7a]. Investigate if your conjecture can be generalized. How many circles can be constructed through A, B and C: a) if A, B and C are the nonlinear points? b) if A, B and C are the straight points?	[7b]. Perform the tasks above with new premises, by using appropriate techniques, such as posting what if? or what if not? questions. Conclusion if C is the center of an arbitrary circle passing through points A and B, then C must be lying on the perpendicular bisector of AB. With three non-collinear points, we can draw one and only one circle. Sketching a circle passing through three collinear points is impossible.

5. CONCLUSION

The above example demonstrates that the model can help the teacher to hold activities of learning for his students. It suits mathematical situations that provide possibilities for students to formulate conjectures that can be generalized.

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