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A New Approach of Ziegler-Nichols Rules for Tuning PID Controllers

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Abstract:

This paper presents a new and efficient technique for the Ziegler–Nichols tuning rule which is too simply a heuristic method dedicated to tune a PID (Proportional-Integral-Derivative) controller. This later is applied to a closed-loop control system to improve the performance of digital systems and particularly to improve their stabilities. On the other hand, and in some specific cases, the PID faces

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problems to enhance the stability of digital systems. To confront the instability problem, we propose the insertion of a derivative controller (D) in the minor feedback path. This new method allows the recovery of the system stability and simultaneously reduces the maximum overshoot in the unit-step response and the response time respectively.

Key words: Control system, Closed-loop control, PID Controller, Zeigler-Nichols rules.

1. INTRODUCTION

The concept of automatic control has found during last decades a wide and essential place in many domains, especially in modern physics. Systems being controlled automatically played nowadays a major role in the advancement and improvement of engineering capabilities and contribute efficaciously in many other fields. Including our daily life activities, which are frequently influenced by some systems or devices controlled automatically, where we can find a numerous application based on it, like computerized control systems, transportation systems, power systems, temperature limiting systems, robotics, etc [Kalman 1961] and [Rivera 1986].

A control system is basically, the interconnection of physical components in some or other manners to provide a desired function, including sometimes the control of the system itself. Each component is represented by a block and the complete block diagram which represents the system is formed by different blocks once connected. A block diagram explains the cause and explains the relationship existing between input and output of the system, through the functionality of different blocks [Gabel 1987] and [Skogestad 2000].

Essentially, a control system includes two main blocks:
Plant: Or the process represents the element of the system to be controlled or regulated

Controller: Is the component used to control the plant, it can be implanted inside or outside the system.

Concretely, a controller is a control loop feedback mechanism, used basically to calculate the accuracy of steady-state (error between the input and the output), then to correct the error based on the relative stability and finally to reduce the maximum overshoot. In other word, a controller is used to improve the performance of the control system [Caminos 2002], [Hang 2002] and [Åström 2004].

In order to achieve the best performance of the system, the controller parameters, should be carefully selected through some process known as controller tuning [Meshram 2012]. In this context, Ziegler-Nichols suggested some rules to tune controllers [Cho 2014] and [Lee 2014]. But still, some limitations reappear when the order n of the system is odd and the coefficient a_1 in the characteristic equation is zero. In these cases, controllers using Zeigler-Nichols process proved to be unable to recover the stability of the control system. To tackle this problem, we propose throughout this paper the insertion of a derivative (D) controller in the minor feedback path. That will enable both, firstly the recovery of the coefficient a_1 and secondly the regaining of the system stability again.

The rest of this paper is organized as follows. The mathematical modeling of control systems is presented in section 2. Then, the design of control systems and the PID controller are explained in section 3. Section 4 presents the Zeigler-Nichols rules for tuning of PID controllers. And finally, the new approach of the Zeigler-Nichols rule will be proposed in section 5.

2. MATHEMATICAL MODELING OF CONTROL SYSTEMS

One of the most important tasks in the analysis and design of control systems is the mathematical modeling of the systems. There are a number of mathematical representations to describe a controlled process.

a. Transfer function: It is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, with all zero initial conditions.

b. Block diagram: It is used to represent all types of systems. It can be used, together with transfer functions, to describe the cause and effect relationships throughout the system.

2.1. Transfer Function

The transfer function of a linear time-invariant system is defined as the Laplace transform of the impulse response, with all the initial conditions set to zero [Gabel 1987].

Let $G(s)$ denote the transfer of a single-input, single output system with input $r(t)$, output $c(t)$, and impulse response $g(t)$.

The transfer function $G(s)$ is defined as:

$$G(s) = Laplace\ transform\ of\ g(t) = \mathcal{L}[g(t)] \quad (1)$$

The transfer function $G(s)$ is related to the Laplace transform of the input and the output through the following relation:

$$G(s) = \frac{C(s)}{R(s)} \quad (2)$$

With all the initial conditions set to zero, and $C(s)$ and $R(s)$ are the Laplace transform of $c(t)$ and $r(t)$, respectively. The variable $s = jw$ is referred to as the Laplace operator.

In general, the transfer functions between $r(t)$ and $c(t)$ takes the form:

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \quad (3)$$

The roots of the numerator are called poles of the system and the roots of the denominator are called zeros of the system. By setting the denominator function to zero, we obtain what is referred to as the characteristic equation:

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0 \quad (4)$$

The stability of linear, single-input, single-output systems is completely governed by the roots of the characteristic equation.

2.2. Block Diagram

Because of their simplicity and versatility, block diagrams are often used by control engineers to describe all types of systems. A block diagram can be used simply to represent the composition and interconnection of a system. Also, it can be used, together with transfer functions, to represent the cause-and-effect relationships throughout the system. Transfer Function is defined as the relationship between an input signal and an output signal to a device. Fig.1 shown an example of a block diagram of a closed-loop system [Skogestad 2000].

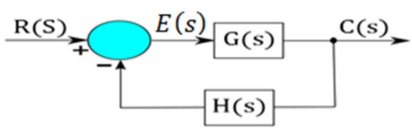

Figure 1: Block diagram of a closed-loop system

The elements of an open-loop control system $G(s)$ can usually be divided into two parts: the controller $G_c(s)$ and the controlled process $G_p(s)$.

The output $C(s)$ is fed back to the summing point, where is compared with the reference input $R(s)$ (the output of the process is constantly monitored by the sensor $H(s)$). The closed-loop nature of the system is clearly indicated by Fig1. The output of the block, $C(s)$ in this case, is obtained by multiplying the transfer function $G(s)$ by the input to the block, $E(s)$. Any linear control system may be represented by a block diagram consisting of blocks, summing points, and branch points.

We note that, the closed-loop transfer function is given by:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad (5)$$

All the foundations of analysis that we have arranged in this section led to the ultimate goal of design of control systems.

3. Design of Control Systems

The principal objective of this section is to present the design of control systems. Starting with the controlled process such as that shown by the block diagram in Fig. 2, control system design involves the following three steps [Stogestad 2000]:

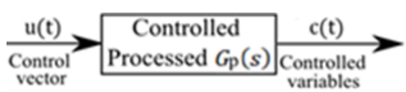

Figure 2: Controlled process

a. Determine what the system should do and how to do it (design specifications).

b. Determine the controller or system configuration, relative to how it is connected to the controlled process.

c. Determine the parameter values of the controller to achieve the design goals.

3.1. Design Specifications

We often use design specifications to describe what the system should do and how it is done. These specifications are unique to each individual application, and often include specifications about **relative stability, steady-state accuracy (error),** the

transient-response and **frequency-response characteristics** [Caminos 2002].

The design of linear control systems can be carried out in either the time domain or the frequency domain. In this paper, we are interested by the design of linear control system in the time domain. For instance, **steady-state accuracy** is often specified with respect to a step input, a ramp input, or a parabolic input, and the design to meet a certain requirement is more conveniently carried out in the domain. Other specifications such as maximum overshoot, rise time, and settling time, are all defined for a unit-step input, and therefore are used specifically for time-domain design. We should be quick to point out, however, that in most cases, time-domain specifications such as stability, steady-state error, **maximum overshoot**, **rise time** and **settling time** are usually used as the final measure of system performance.

3.2. Controller Configurations

In general, the dynamics of a linear controlled process can be represented by the block diagram shown in Fig. 1 The design objective is to have the controlled variables, represented by the output vector $c(t)$, behave in certain desirable ways [Caminos 2002].

Fig. 3 illustrates several commonly used system configurations with controller compensation. These are described briefly as follows:

a. Series (cascade) compensation: Fig. 3-a shows the most commonly used system configuration with the controller placed in series with the controlled process, and the configuration is referred to as series or cascade compensation.

b. Feedback compensation: In Fig. 3-b, the controller is placed in the minor feedback path, and the scheme is called feedback compensation.

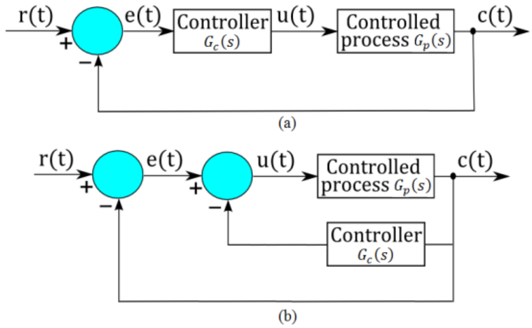

Figure 3: Various controller configurations in control-system compensation. (a) Series or cascade compensation. (b) Feedback compensation

In next subsections, we will describe briefly the dynamic characteristics of the PI (Proportional-Integral) controller, PD (Proportional Derivative) controller and PID (Proportional-Integral-Derivative) controller. We will present the effect of each controller to improve the system performance.

3.3. Design with PD Controller

In this part, we will introduce a simple yet multipurpose feedback compensator structure, the Proportional-Derivative (PD) controller. The PD controller is characterized by the transfer function [Cho 2014]:

$$G_c(s) = K_p(1 + T_d s) \qquad (6)$$

Where K_p and T_d are the proportional and derivative constants, respectively.

The PD controller is a lead compensator. The transfer function involves one zero, but no pole. The value of K_p is usually determined to satisfy the steady-state requirement.

Though it is not effective with lightly damped or initially unstable systems, a properly designed PD controller can affect the performance, on the time domain, of a control system in the following ways:

a. Improving damping and reducing maximum overshoot.
b. Reducing rise time and settling time.

3.4. Design with PI Controller

We see that the PD controller can improve the damping and rise time of a control system but the steady-state error is not affected.
The Proportional-Integral (PI) controller is characterized by the transfer function [Cho 2014]:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i}s\right) \qquad (7)$$

The PI controller is a lag compensator. It possesses a zero $s = -1/T_i$ and a pole at $s = 0$. A properly designed PI controller can affect the performance, on the time domain, of a control system in the following ways:
a. Improving the relative stability.
b. Improving the steady-state error.

3.5. Design with PID Controller

From the preceding discussions, we see that the PD controller could add damping to a system, but the steady-state response is not affected. The PI controller could improve the relative stability and improve the steady-state error at the same time, but the rise time is increased. This leads to the motivation of using a PID controller so that the best features of each of the PI and PD controllers are utilized. Consider that the PID controller consists of a PI portion connected in cascade with a PD portion. The transfer function of the PID controller is written as [Lee 2014]:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \qquad (8)$$

Thereafter, we will present, in the next section, the Ziegler-Nichols's method. This method allows the determination of the controller's parameter values to improve the system performance, i.e, to improve the relative stability, the steady-state error and to reduce the maximum overshoot, the rise time and the settling time.

4. ZIEGLER-NICHOLS RULES FOR TUNING OF PID CONTROLLERS

Most of the conventional design of a linear controlled process using the Ziegler-Nichols rules can be represented by the block diagram shown in Fig. 4 [Hang 2002] and [Åström 2004].

In this Figure, the controller PID is placed in series with the controlled process (plant).

If a mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining parameters of the controller that will meet the transient and steady-state specifications of the closed- loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then an analytical approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to the tuning of PID controllers.

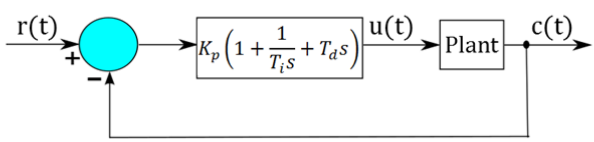

Figure 4: Closed-loop system with a PID controller

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values K_p, T_i and T_d) based on experimental step response or based on the value of K_p, that results in marginal stability when only proportional control action is used. Ziegler-Nichols rules, which are briefly presented in the following, are useful when mathematical models of plants are not known. Such rules suggest a set of values of K_p, T_i and T_d that will give a stable operation of the system. However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable. In such, a case we need series of fine tunings until an acceptable result is obtained. In fact, the Ziegler-Nichols tuning rules give an educated guess for the parameter values and provide a starting point for the tuning rather than giving the final settings for K_p, T_i and T_d in a single shot.

Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant. We shall give a brief presentation of the method's Ziegler and Nichols [Meshram 2012] and [Lee 2014].

We first set $T_i \to \infty$ and $T_d = 0$. Using the proportional control action only (see Fig. 5), increase K_p from 0 to critical value K_{cr} at which the output first exhibits sustained oscillations. Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined. If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.

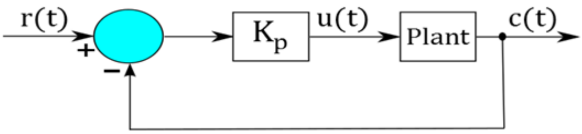

Figure 5: Closed-loop system with a proportional controller

Thus, Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i and T_d according to the formula shown in Tab. 1.

Table 1: Ziegler-Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr}

Type of Controller	K_p	T_i	T_d
P	$0.5\,K_{cr}$	∞	0
PI	$0.45\,K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6\,K_{cr}$	$0.5\,P_{cr}$	$0.125\,P_{cr}$

Notice that the PID controller tuned by this method of Ziegler-Nichols rules gives:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right)$$
$$= 0.075K_{cr}P_{cr}\frac{\left(s+\frac{4}{P_{cr}}\right)^2}{s} \quad (9)$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -\frac{4}{P_{cr}}$.

4.1. Example

Consider the control system shown in Fig. 6 in which a PID controller is used to control the system.

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler-Nichols tuning rule for the determination of the values of parameters K_p, T_i and T_d.

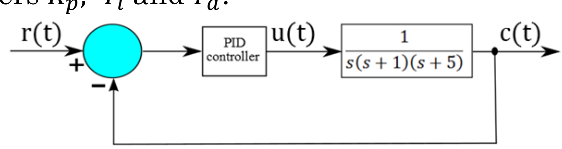

Figure 6: PID-controlled system

Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

By setting $T_i \to \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5)+K_p} \qquad (10)$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can obtained by use of Routh's stability criterion (Appendix A). Since the characteristic equation for the closed-loop system is:

$$s^3 + 6s^2 + 5s + K_p = 0 \qquad (11)$$

The Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & 0 \\ s^0 & K_p & 0 \end{array} \qquad (12)$$

Examining the coefficients of the first column of the Routh table, we find sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to $K_{cr} (= 30)$, the characteristic equation becomes:

$$s^3 + 6s^2 + 5s + 30 = 0 \qquad (13)$$

To find the frequency of the sustained oscillation, we substitute $s = jw$ into this characteristic equation as follows:

$$(jw)^3 + 6(jw)^2 + 5(jw) + 30 = 0 \quad (14)$$

or

$$6(5 - w^2) + jw(5 - w^2) = 0$$

from which we find the frequency of the sustained oscillation to be $w^2 = 5$ or $w = \sqrt{5}$.
Hence, the period of sustained oscillation is:

$$P_{cr} = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Tab. 1, we determine K_p, T_i and T_d as follows:

$$\begin{aligned} K_p &= 0.6K_{cr} = 18 \\ T_i &= 0.5P_{cr} = 1.405 \\ T_d &= 0.125P_{cr} = 0.35124 \end{aligned} \quad (15)$$

The transfer function of the PID controller is thus:

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right) \end{aligned} \quad (16)$$

A block diagram of the control system with the designed PID controller is shown in Fig. 7.

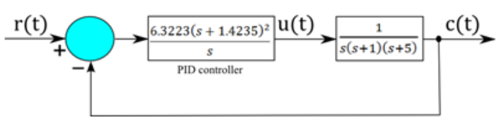

Figure 7: Block diagram of the system with PID controller designed by use of the Ziegler-Nichols tuning rule

Next, let us examine the unit-step response of the system. The closed-loop transfer function $\frac{C(s)}{R(s)}$ is given by:

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2+18s+12.811}{s^4+6s^3+11.3223s^2+18s+12.811} \text{N} \quad (17)$$

The unit step response of this system can be obtained easily with MATLAB. The resulting unit-step response curve is shown in Fig. 8.

5. A NEW APPROACH OF THE ZIEGLER-NICHOLS'S RULES

In this section, we are going to define a new approach of Zeigler-Nichols's rule. To do so, let us firstly consider the control system shown in Fig. 9. From this design, we can derive a general case for our proposition concerning this rule.

We notice directly that the controlled process (plant) is unstable without the existence of a PID controller, which from the preceding discussions; we concluded that, the main task of PID, is to improve the performance of plant or process.

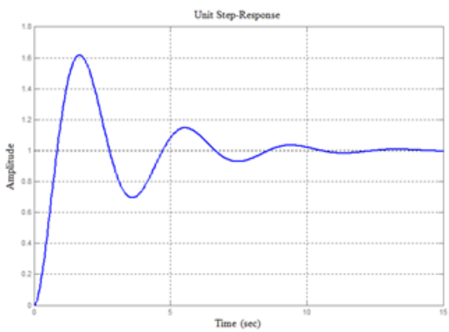

Figure 8: Unit step-response with PID controller by use of Ziegler-Nichols rules

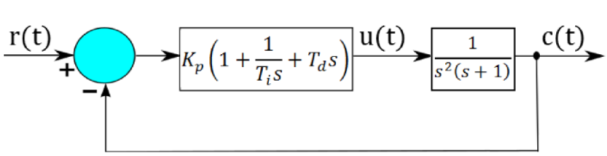

Figure 9: Block diagram of the system with PID controller designed by use of the Ziegler-Nichols tuning rule

Let us now test the stability of the system by applying the Ziegler-Nichols tuning rule for the determination of parameters K_p, T_i, and T_d, and then to check if this method is able to improve the performance of the plant presented by figure 9 or not?

By setting $T_i \to \infty$ and $T_d = 0,$ we obtain the closed-loop transfer function as following:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s^3 + s^2 + K_p} \qquad (18)$$

Since the characteristic equation for the closed-loop system is:

$$s^3 + s^2 + K_p = 0 \qquad (19)$$

The coefficient a_1 of the characteristic equation vanishes and equal to zero, which let this system marginally unstable. In this case, we can conclude that the Zeigler-Nichols's rule is incapable to improve the performance of the system.

To solve the instability problem, we propose, in this paper, the insertion of a derivative (D) controller in the minor feedback path as shown in Fig. 10.

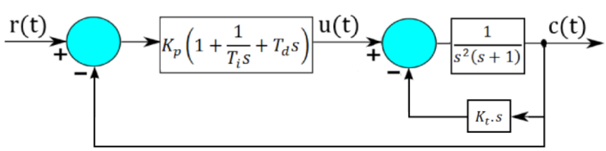

Figure 10: The proposed block diagram of the system with PID controller designed by use of the Ziegler-Nichols tuning rule

This addition enables the recovery of the coefficient a_1 in the characteristic equation. The new characteristic equation for the minor closed-loop is:

$$\frac{C(s)}{U(s)} = \frac{1}{s^3+s^2+K_t s} \qquad (20)$$

By setting now $T_i \to \infty$ and $T_d = 0,$ we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s^3+s^2+K_t s+K_p} \qquad (21)$$

Since the characteristic equation for the closed-loop system is:

$$s^3 + s^2 + K_t s + K_p = 0 \quad (22)$$

The Routh array becomes as indicated:

$$\begin{matrix} s^3 & 1 & K_t \\ s^2 & 1 & K_p \\ s^1 & K_t - K_p & 0 \\ s^0 & K_p & 0 \end{matrix} \quad (23)$$

By examining the coefficients of the first column of the Routh table, we find a continuous oscillation will occur if $K_t = K_p$. Thus, the critical gain K_{cr} is

$$K_{cr} = K_t \qquad (24)$$

With gain K_p set equal to $K_{cr}(= K_t)$, the characteristic equation becomes:

$$s^3 + s^2 + K_t s + K_t = 0 \qquad (25)$$

This condition gives $K_t > 0$. Let now for example, setting$K_t = 1$, to find the frequency of the sustained oscillation, we substitute $s = jw$ into this characteristic equation as follows:

$$(jw)^3 + (jw)^2 + (jw) + 1 = 0 \qquad (26)$$

or

$$(1 - w^2) + jw(1 - w^2) = 0 \qquad (27)$$

from which we find the frequency of the sustained oscillation to be $w^2 = 1$ or $w = 1$. Hence, the sustained period of oscillation is:

$$P_{cr} = \frac{2\pi}{w} = \frac{2\pi}{1} = 6.28318$$

Referring to Tab. 1, we determine K_p, T_i, and T_d as follows:

$$K_p = 0.6K_{cr} = 0.6$$
$$T_i = 0.5P_{cr} = 3.1416$$
$$T_d = 0.125P_{cr} = 0.7853$$

The transfer function, $G_c(s)$, of the PID controller is thus:

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 0.6\left(1 + \frac{1}{3.1416s} + 0.7853s\right)$$
$$= 0.4711\,\frac{(s+0.3)^2}{s} \qquad (28)$$

A block diagram of the control system with the proposed PID controller is shown in Fig. 11.

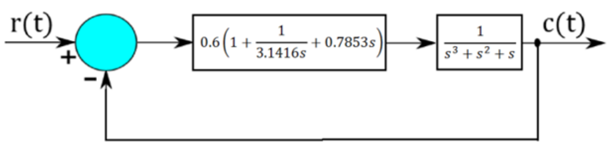

Figure 11: New block diagram of the system with PID controller designed by use of the Ziegler-Nichols tuning rule

Next, let us examine the unit-step response of the two systems (with and without a derivative controller in the minor feedback. The closed-loop transfer function $\frac{C(s)}{R(s)}$ is given by:

$$\frac{C(s)}{R(s)} = \frac{1.48s^2+1.89s+0.6}{3.1416(s^4+s^3+s^2)+1.89s+0.6} \qquad (29)$$

The unit step response of this system can be obtained easily with MATLAB. The resulting unit-step response curve is shown in Fig. 12.

We noticed that the unit-step response of the system without using derivative controller is unstable, but by using of it in the minor feedback, our system became stable. However, the maximum overshoot in the unit-step response is approximately 50%. The amount of maximum overshoot is excessive. It can be reduced by find that by keeping $K_p = 0.6$ and moving the double zero of the PID controller $s = -0.15$, that is, using the PID controller

$$G_c(s) = 0.6\left(1 + \frac{1}{22.22s} + 0.15s\right) \quad (30)$$
$$= 2\frac{(s + 0.15)^2}{s}$$

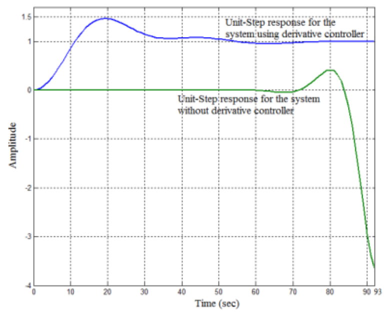

Figure 12: Unit step-response of the system shown in Figures 9 and 11

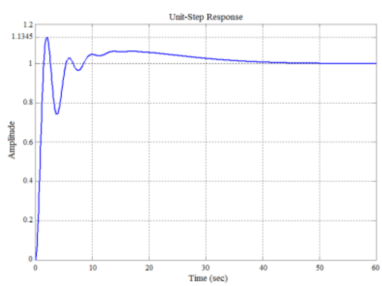

Figure 13: Unit step-response of the system shown in Figure 10 with PID controller having parameters $K_p = 0.6, T_i = 22.22$, and $T_d = 0.15$

The maximum overshoot in the unit-step response and the response time have reduced to approximately 13.5% and to 60 sec respectively (see fig. 13).

From the above results, we can generalize our method on the control systems with the following characteristic equation:

$$s^n + s^{n-1} + \cdots + s^2 + c = 0 \quad (31)$$

The odd order of the system is designed by n and c is a constant.

6. CONCLUSION

Throughout this paper, a new and adventure method of designing a digital control system has been discussed and developed. The new approach overcomes the limitation presented by Ziegler–Nichols's rules when the order n of the system in question is odd, and allows the recovery of the coefficient a_1, together with regaining the stability of controlled system. All the necessary steps have been presented, and concrete examples have been solved, with and without the insertion of a derivative controller (D) in the minor feedback. Our approach, once conceived, has been simulated, and the unit step response of the system has been validated by using Matlab.

APPENDIX A

The most important problem in linear control systems concerns stability. That is, under what conditions will a system become unstable? If it is unstable, how should we stabilize the system? A control system is stable if and only if all closed-loop poles lie in the left-half s plane. Most linear closed-loop systems have closed-loop transfer functions of the form:

$$\frac{C(s)}{R(s)} = G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

Where the a's and b's are constants and $m \leq n$. A simple criterion, known as Routh's stability criterion, enables us to determine the number of closed-loop poles that lie in the right-half s plane without having to factor the denominator polynomial.

Routh's stability criterion tells us whether or not there are unstable roots in a polynomial equation without actually solving for them. This stability criterion applies to polynomials with only a finite number of terms. When the criterion is applied to a control system, information about absolute stability can be obtained directly from the coefficients of the characteristic equation.

The procedure in Routh's stability criterion is as follows:

a. Write the polynomial in s the following form:

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

where the coefficients are real quantities. We assume that $a_n \neq 0$; that is, any zero root has been removed.

b. If any of the coefficients are zero or negative in the presence of at least one positive coefficient, there is a root or roots that are imaginary or that have positive real parts.

c. If all coefficients are positive, arrange the coefficients of the polynomial in rows and columns according to the following pattern:

$$\begin{array}{llllll} s^n & a_n & a_{n-2} & a_{n-4} & a_{n-6} & \cdots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \cdots \\ s^{n-2} & b_1 & b_2 & b_3 & b_4 & \cdots \\ \vdots & \vdots & & & & \\ s^2 & c_1 & c_2 & & & \\ s^1 & d_1 & & & & \\ s^0 & e_1 & & & & \end{array}$$

The process of forming rows continues until we run out of elements. (The total number of rows is $n+1$). The coefficients b_1 , b_2 , b_3 and so on, are evaluated as follows:

$$b_1 = \frac{a_{n-1}.a_{n-2} - a_n.a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{a_{n-1}.a_{n-4} - a_n.a_{n-5}}{a_{n-1}}$$

$$b_3 = \frac{a_{n-1}.a_{n-6} - a_n.a_{n-7}}{a_{n-1}}$$

$$\vdots$$

This process is continued until the nth row has been completed. The complete array of coefficients is triangular. Note that in developing the array an entire row be divided or multiplied by a positive number in order to simplify the subsequent numerical calculation without altering the stability conclusion.

Routh's stability criterion states that the number of roots of characteristic equation with positive real parts is equal to the number of changes in sign of the coefficients of the first column of the array. It should be noted that the exact values of the terms in the first column need not be known; instead, only the signs are needed. The necessary and sufficient condition that all roots of characteristic equation lie in the left-half *s* plane is that all the coefficients of characteristic equation be positive and all terms in the first column of the array have positive signs.

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