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The Cyclic Decomposition of the Factor Group $cf(Q_{2m}\times D_5,Z)/\overline{R}(Q_{2m}\times D_5)$ when m = p, p>2, is prime number

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Abstract:

Let Q_{2m} be the quaternion group of order 4m when $m=p>2$ is an odd number, and D_5 be the dihedral group of order 10. let $cf(Q_{2m}\times D_5,Z)$ be the abelain group of Z–valued class function of the group $(Q_{2m}\times D_5)$. The intersection of $cf(Q_{2m}\times D_5,Z)$ with the group of generalized characters of $(Q_{2m}\times D_5)$.which is denoted by $R(Q_{2m}\times D_5)$ is a normal subgroup of the group $cf(Q_{2m}\times D_5,Z)$ denoted by $\overline{R}(Q_{2m}\times D_5)$, the factor group $cf(Q_{2m}\times D_5,Z)/\overline{R}(Q_{2m}\times D_5)$ is a finite abelian group denoted by $K(Q_{2m}\times D_5)$.the purpose of this paper to find $K(Q_{2p}\times D_5)=\overset{3}{\underset{n=1}{\oplus}}K(Q_{2m})\oplus C_4\overset{2(r+1)}{\underset{n=1}{\oplus}}C_{5^{s+1}}\oplus C_5\oplus C_8$ when m = p, p>2 is prime number.

Key words: cyclic decomposition, factor group, prime number

1. INTRODUCTION

Let G be a finite group, two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G, this relation is an equivalence relation on G. and this equivalence relation called Γ- classes.

The Z-valued class function on the group G, which is constant on the Γ- classes forms a finitely generated abelian group cf(G,Z) of a rank equal to the number of Γ- classes . the intersection of cf(G,Z) with the group of all generalized characters of G, R(G) is a normal subgroup of cf(G,Z) denoted by $\overline{R}$(G) each element in $\overline{R}$(G) can be written as $u_1\theta_1+u_2\theta_2+\ldots\ldots+u_i\theta_i$, where i is the number of Γ- classes, $u_1,u_2,\ldots,u_i \in Z$ and $\theta_i=\sum_{\sigma\in Gal(Q(\chi_i)/Q)}\sigma(\chi_i)$ where χ_i is an irreducible character of the group G and σ is any element in Galios group $Gal(Q(\chi_i)/Q)$. let ≡*(G) denotes the i×i matrix which corresponds to the θ_i's and columns correspond to the Γ-classes of G the matrix expressing $\overline{R}$(G) basis in terms of the cf(G,Z) basis is ≡*(G) In 1995 N. R. Mahmood [6]

Studied the factor group cf(Q_{2m},Z) / $\overline{R}$(Q_{2m}). The aim of this paper is to find $\equiv^*(Q_{2m}\times D_5)$ and the factor group cf($Q_{2m}\times D_5$,Z) / $\overline{R}$ ($Q_{2m}\times D_5$) when m = p, p>2 is prime number.

2. PRELIMINARIES

We are review in this section some definitions and results:

<u>Definition (2.1)</u>: [1]

Let F be a field .The general linear group GL(n,F) is a multiplicative group of all non-singular $n\times n$ matrices over F.

<u>Definition (2.2)</u>:[1]

Let F be a field .A matrix representation of G is homomorphism T: G→GL (n, F), n is called ***the degree of representation*** T .T is called a unit representation (principal) if T(g)= 1 for all g ∈ G.

Definition (2.3): [2]

A matrix representation T: G→GL (n,F) is said to be ***reducible*** if there exists a non-singular matrix A over F such that:

$$A^{-1}\,T(g)\,A=\begin{bmatrix} T_1(g) & T(g) \\ 0 & T_2(g) \end{bmatrix},\ \text{for all } g\in G.$$

Where $T^1(g)$ and $T^2(g)$ are matrices representations over F of the dimensions r×r, s×s respectively and E(g) is a matrix of the dimensions r×s such that $0< r < n$ and $r+s= n$.

If no such reducible matrix exists, then T(g) is called ***an irreducible matrix representation***.

Definition (2.4):[3]

The trace of an $n\times n$ matrix A is the sum of main diagonal elements, denoted by tr(A).

Definition (2.5): [4]

Let T be a matrix representation of G over the field F. The ***character*** χ of a matrix representation T is the mapping χ: G→F defined by $\chi(g)=tr(T(g))$ for all $g\in G$ .The degree of T is called the degree of χ .

Remark (2.6): [5]

(I) A finite group G has a finite number conjugacy classes and a finite number of distinct K- irreducible characters, the group characters of a group representation is constant on a conjugacy class, the values of the characters can be written as a table known the characters table which is denoted by ≡(G).

(II) If $G=C_n=<r>$ is the cyclic group of the order n generated by r, and $\omega = e^{2\pi i/n}$ is primitive n-th root of unity, then ≡(C_n) is:

CL_α	[I]	$[r]$	$[r^2]$	…	$[r^{n-1}]$
$\|CL_\alpha\|$	1	1	1	…	1
$\|C_G(CL_\alpha)\|$	n	n	n	…	n
χ_1	1	1	1	…	1
χ_2	1	ω	ω^2	…	ω^{n-1}
χ_3	1	ω^2	ω^4	…	ω^{n-2}
⋮	⋮	⋮	⋮	⋱	⋮
χ_n	1	ω^{n-1}	ω^{n-2}	…	ω

Table (2.1)

Definition (2.7): [6]

For each positive integer m, ***The generalized Quaternion Group Q_{2m}*** is a non- abelain group of order 4m with two generators x and y can write it as:

$$Q_{2m} = \{x^k y^t , 0 \le k \le 2m\text{-}1, t = 0,1\}$$

which has the following properties:{ $x^{2m} = y^4 = I$, $yx^m y^{-1} = x^{-m}$ }

Remark (2.8):

The group Q_{2m}×D_5 is the direct product group of the quaternion group Q_{2m} and the group D_5 is the dihedral group of order 10.

The character table of The Quaternion Group Q_{2m} when m is an odd Number(2.9) [6]

There are two types of irreducible characters one of them is the character of the linear representation which are denoted by Ψ_1, Ψ_2, Ψ_3 and Ψ_4 respectively as in the following table:

	x^k	x^ky
Ψ_1	1	1
Ψ_2	1	-1
Ψ_3	$(-1)^k$	$i\,(-1)^k$
Ψ_4	$(-1)^k$	$i\,(-1)^{k+1}$

Table (2.2)

The other characters of irreducible representations of degree 2 are denoted by χ_h such that:

$\chi_h\,(x^k) = \omega^{hk} + \omega^{-hk} = e^{\pi ihk/m} + e^{-\pi ihk/m} = 2\cos(\pi hk/m)$, $\chi_h\,(x^k y) = 0$

where $0 \le k \le 2m\text{-}1$, $1 \le h \le m\text{-}1$ and $\omega = e^{2\pi i/2m}$.

so there are *m+3* irreducible characters of Q_{2m} .then , the general form of the character table of Q_{2m} when m is an odd number is given in the following table:

CL_α	[I]	$[x^2]$	$[x^4]$	⋯	$[x^{m-1}]$	$[x^m]$	$[x]$	$[x^3]$	⋯	$[x^{m-2}]$	[y]	[xy]
$\lvert CL_\alpha \rvert$	1	2	2	⋯	2	1	2	2	⋯	2	m	m
$\lvert C_{Q2m}(CL_\alpha) \rvert$	4m	2m	2m	⋯	2m	4m	2m	2m	⋯	2m	4	4
Ψ_1	1	1	1	⋯	1	1	1	1	⋯	1	1	1
χ_2	2	$\omega^4+\omega^{2m-4}$	$\omega^8+\omega^{2m-8}$	⋯	$\omega^{2(m-1)}+\omega^2$	2	$\omega^2+\omega^{2(m-1)}$	$\omega^6+\omega^{2m-6}$	⋯	$\omega^{2(m-2)}+\omega^4$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
$\chi_{(m-1)}$	2	$\omega^{2(m-1)}+\omega^2$	$\omega^{4(m-1)}+\omega^4$	⋯	$\omega^{m+1}+\omega^{m-1}$	2	$\omega^{m-1}+\omega^{m+1}$	$\omega^{m-3}+\omega^{m+3}$	⋯	$\omega^2+\omega^{2(m-1)}$	0	0
Ψ_2	1	1	1	⋯	1	1	1	1	⋯	1	-1	-1
χ_1	2	$\omega^2+\omega^{2(m-1)}$	$\omega^4+\omega^{4(m-1)}$	⋯	$\omega^{m-1}+\omega^{m+1}$	-2	$\omega+\omega^{2m-1}$	$\omega^3+\omega^{2m-3}$	⋯	$\omega^{m-2}+\omega^{m+2}$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
$\chi_{(m-2)}$	2	$\omega^{2m-4}+\omega^4$	$\omega^{2m-8}+\omega^8$	⋯	$\omega^2+\omega^{2(m-1)}$	-2	$\omega^{m-2}+\omega^{m+2}$	$\omega^{m-6}+\omega^{m+6}$	⋯	$\omega^{(m-2)^2}+\omega^{m^2-4}$	0	0
Ψ_3	1	1	1	⋯	1	-1	-1	-1	⋯	-1	i	-i
Ψ_4	1	1	1	⋯	1	-1	-1	-1	⋯	-1	-i	i

Table (2.3)

Theorem (2.10):[7]

1. The sum of characters is a character.
2. The product of characters is a character.

Theorem (2.11):[1]

Let T_1: $G_1 \rightarrow GL(n,F)$ and T_2: $G_2 \rightarrow GL(m,F)$ are two irreducible representations of the groups G_1 and G_2 with characters χ_1 and χ_2 respectively , then $T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with the character $\chi_1 \cdot \chi_2$.

Definition (2.12):[8]

A rational valued character θ of G is a character whose values are in Z, which is θ(g) ∈ Z, for all g ∈ G .

Proposition (2.13):[9]

The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form basis for $\bar{R}$(G), where χ*i* are the irreducible characters of G .and their numbers are equal to the number of all distinct Γ- classes of G.

3. *The Factor Group K(G)*

We will study the factor group K(C_n) and K(Q_{2m})

Definition (3.1):[10]

A k-th order minor is the determinant of the sub matrix obtained by taking k rows and k columns of A.

Divisor over principal ideal domain, we can form the greatest common divisor (g.c.d) of all the k-th order minors of A, it is called ***the k-th determinant divisor of A*** and denoted by D_k(A).

Definition (3.2):[7]

Let M be a matrix with entries in a principal ideal domain R,be equivalent to a matrix D = diag {d_1, d_2, … ,d_r, 0, 0, … , 0} such that $d_j \mid d_{j+1}$ for $1 \le j < r$.

We call D ***the invariant factor matrix of*** M and d_1, d_2, … ,d_r the invariant factors of M.

Theorem (3.3):[11]

Let M $\in M_{n\times m}$(A) be a matrix with entries in a principle ideal domain. Then there exist two invertible matrices L $\in GL_n$(A) , W $\in GL_m$(A) and a quasi-diagonal matrix D $\in M_{n\times m}$(A) (that is , d_{ij} =0 for $i \neq j$)such that

1- M=LDW .
2- $d_1 | d_2$,…,$d_i | d_{i+1}$,…, where the d_j are the diagonal entries of D and then, D_k(LDW)=D_k(M) modulo the group of unites of A .

Proposition(3.4):[12]

Let A and B be two non-singular matrices of the rank n and m respectively, over a principal domain R . and let L_1AW_1 =D(A) = diag{d_1(A),d_2(A),…….,d_n(A)}, L_2BW_2 =D(B) = diag{d_1(B),d_2(B),…….,d_m(B)} be the invariant factor matrices of A and B, then (L_1⊗L_2).(A⊗B).(W_1⊗W_2)=D(A)⊗D(B) and from this we can write down the invariant factor matrix of A⊗B.

Let H_1 and H_2 be P_1-group and P_2-group respectively, whereP_1 and P_2 are distinct primes. We know that ≡ (H_1 ×H_2) = ≡ (H_1) ⊗ ≡ (H_2)

(P_1,P_2) =1, so we have ≡*(H_1 × H_2) = ≡*(H_1) ⊗≡*(H_2) .

Remark (3.5):[9]

Suppose cf(G,Z) is of the rank l , the matrix expressing the $\bar{R}$ (G) basis in terms of the cf(G,Z) = Z^l basis is ≡*(G).

Hence by theorem (3.3),we can find two matrices L and W with a determinant ±1 such that L.≡*(G).W=D(≡*(G))= diag{d_1,d_2,………, d_l }, d_i = ± D_i (≡*(G))/ ± D_{i-1} (≡*(G)) .

Theorem (3.6):[9]

K(G) = $\oplus \sum C_{d_i}$ Such that d_i = ± D_i (≡*(G))/ ± D_{i-1} (≡*(G)) .

Proposition (3.7):[9]

The ***rational valued character tables of the cyclic group*** C_{p^s} of the rank s+1 where p is a prime number which is denoted by (≡*(C_{p^s})), is given as follows:

Γ-classes	*[r]*	$[r^{P}]$	$[r^{p^2}]$	…	$[r^{p^{s-3}}]$	$[r^{p^{s-2}}]$	$[r^{p^{s-1}}]$	*[I]*
θ_1	1	1	1	…	1	1	1	1
θ_2	-1	p-1	p-1	…	p-1	p-1	p-1	p-1
θ_3	0	-p	p(p-1)	…	p(p-1)	p(p-1)	p(p-1)	p(p-1)
…	…	…	…	⋱	…	…	…	…
θ_{s-1}	0	0	0	…	$-p^{s-3}$	$p^{s-3}(p-1)$	$p^{s-3}(p-1)$	$p^{s-3}(p-1)$
θ_s	0	0	0	…	0	$-p^{s-2}$	$p^{s-2}(p-1)$	$p^{s-2}(p-1)$
θ_{s+1}	0	0	0	…	0	0	$-p^{s-1}$	$p^{s-1}(p-1)$

Table (3.1)

Where its rank s+1 which represents the number of all distinct Γ-classes.

Proposition (3.8):[9]

If p is a prime number ,then D(≡*(C_{p^s})) = diag{p^s ,p^{s-1} ,……,p,1}.

Theorem(3.9) :[9]

Let p be a prime number, then: K(C_{p^s}) = $\oplus \sum_{i=1}^{s} C_{p^i}$.

Proposition (3.10): [6]

The rational valued character table of Q_{2m} when m is an odd number is given as follows:

Γ- classes of C_{2m}			
	x^{2r}	x^{2r+1}	[y]
θ_1	1 1 … 1 ≡*(C_m)	1 1 … 1 ≡*(C_m)	1
θ_2			0
⋮			⋮
$\theta_{(I/2)-1}$			0
$\theta_{(I/2)}$			0
$\theta_{(I/2)+1}$	1 1 … 1 ≡*(C_m)	1 1 … 1 H	-1
$\theta_{(I/2)+2}$			0
⋮			⋮
θ_{I-1}			0
θ_{I}			0
θ_{I+1}	2 2 … 2	-2 -2 … -2	0

Table (3.2)

Where 0< r < m-1 , I is the number of Γ- classes of C_{2m} , θ_j such that 1< j < I+1 are the rational valued characters of group Q_{2m} and if we denote C_{ij} the elements $\overset{*}{\equiv}(C_m)$ and h_{ij} the elements of H as defined by :

$$h_{ij} = \begin{cases} C_{ij} & if \quad i=1 \\ -C_{ij} & if \quad i \neq 1 \end{cases} ,$$

and where I is the number of Γ-classes of C_{2m}.

Theorem(2.11): [6]

If m is an odd number, then: $K(Q_{2m}) = K(C_{2m}) \oplus C_4$.

Example(2.12):

The cyclic decomposition $K(Q_{14})$can be written as follows :

$K(Q_{14}) = K(Q_{2.7}) = K(C_{2.7}) \oplus C_4$

$$= \overset{(2)}{C_7} \oplus \overset{(2)}{C_2} \oplus C_4 = \bigoplus_{i=1}^{2} C_7 \bigoplus_{i=1}^{2} C_2 \oplus C_4 \quad .$$

3. THE MAIN RESULTS

In this section we find the rational valued character table of the group (Q_{2m}×D_5) and K(Q_{2m}×D_5).

Theorem(4.1) :

Let m is an odd number, the rational valued character table $\equiv^*$(Q_{2m}×D_5) of the group Q_{2m}×D_5 is: $\equiv^*(Q_{2m}\times D_5) = (\equiv^*(Q_{2m}) \otimes \equiv^*(D_5))$

Proof:

The character table of D_5 is :

CL_α	$[d_1]$	$[d_2]$	$[d_3]$	$[d_4]$
X'_1	1	1	1	1
X'_2	1	1	1	-1
X'_3	2	τ_1	τ_2	0
X'_4	2	τ_2	τ_1	0

Table (4.1)

Where $[d_1]=\{(e)\}$, $[d_2]=\{ a , a^4\}$, $[d_3]=\{ a^2, a^3 \}$, $[d_4]=\{b, ba , ba^2, ba^3, ba^4\}$, $\tau_1 = \omega+\omega^4$, $\tau_2 = \omega^2+\omega^3$, $\omega = e^{\frac{2\pi i}{5}}$.

The rational valued character table of D_5 is equal to:

$\equiv^* D_5$ =

Cl_a	$[d'_1]$	$[d'_2]$	$[d'_3]$
θ'_1	1	1	1
θ'_2	1	1	-1
θ'_3	4	-1	0

Table (4.2)

Where $[d'_1]=\{(e)\}$, $[d'_2]=\{a , a^2 , a^3, a^4 \}$, $[d'_3] =\{b, ba , ba^2, ba^3, ba^4\}$, Then,

$\chi'_1(d_1)=\chi'_1(d_2) = \chi'_1(d_3) = \chi'_1(d_4) = \theta'_1(d'_1) =\theta'_1(d'_2) = \theta'_1(d'_3) = 1$

$\chi'_2(d_1) =\chi'_2(d_2)) =\chi'_2(d_3) = \theta'_2(d'_1) =\theta'_2(d'_2) = 1$, $\chi'_2(d_4) = \theta'_2(d'_3) = -1$

$\chi'_3(d_1) + \chi'_4(d_1) = \theta'_3(d'_1) = 4$, $\chi'_3(d_2) +\chi'_4(d'_2) = \chi'_3(d_3)+ \chi'_4(d_3)= \theta'_3(d'_2) = -1$

$\chi'_3(d_4) + \chi'_4(d_4) = \theta'_3(d'_3) = 0$

From the definition of $Q_{2m}\times D_5$ and theorem (2.11)

$\equiv Q_{2m}\times D_5 = (\equiv Q_{2m}) \otimes (\equiv D_5)$

each element in $Q_{2m}\times D_5$

Let $t\in(Q_{2m}\times D_5)$, $t=(q,d)$ $\forall q\in Q_{2m}$ and $d\in D_5$, $d\in\{e, a , a^2 , a^3, a^4, b, ba , ba^2, ba^3, ba^4\}$.

$q= x^s y^k$, $0\leq s \leq 2m$, $k= 0, 1$

for every irreducible character of $Q_{2m}\times D_5$ is $\chi_{(i,j)} =\chi_i \cdot \chi'_j$

where χ_i and χ'_j are the irreducible character of Q_{2m} and D_5 respectively, then

$$\chi_{(i,j)}(t) = \chi_{(i,j)}(q,d) = \chi_i(q)\cdot\chi'_j(d) = \begin{cases} \chi_i(q) & if\ j=1\ and & d\in D_5 \\ \chi_i(q) & if\ j=2\ and & d\in\{e,a,a^2,a^3,a^4\} \\ -\chi_i(q) & if\ j=2\ and & d\in\{b,ba,ba^2,ba^3,ba^4\} \\ 4\chi_i(q) & if\ j=3\ and & d=\{e\} \\ -\chi_i(q) & if\ j=3\ and & d\in\{a,a^2,a^3,a^4\} \\ 0 & if\ j=3\ and & d\in\{b,ba,ba^2,ba^3,ba^4\} \end{cases}$$

From proposition (2.13)

$$\theta_{(i,j)} = \sum_{\sigma\epsilon Gal(Q(\chi_{(i,j)})/Q)}\sigma(\chi_{ij})$$

Where $\theta_{(i,j)}$ is the rational valued character of ($Q_{2m}\times D_5$) Then,

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_{(i,j)}(t))/Q)} \sigma(\chi_{ij}(t))$$

(A) If j=1 and $d \in D_5$

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q).\chi'_1(d)) = \theta_i(q)\cdot 1 = \theta_i(q)\cdot \theta'_j(d)$$

Where θ_i is the rational valued character of Q_{2m} .

(B) (I) If j=2 and $d \in \{ e, a , a^2 , a^3, a^4\}$

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q).\chi'_2(d)) = \theta_i(q)\cdot 1 = \theta_i(q)\cdot \theta'_j(d)$$

(II) If j=2 and $d \in \{ b, ba , ba^2, ba^3, ba^4\}$

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q).\chi'_2(d)) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q))[\sum_{\sigma \epsilon D_5} \sigma(\chi'_2(d)]$$

$$= \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q))\cdot(-1) = \theta_i(q)\cdot(-1) = \theta_i(q)\cdot \theta'_j(d)$$

(C) (I) If j=3 and $d \in \{e\}$

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q).\chi'_3(d)) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q)) [\sum_{\sigma \epsilon D_5} \sigma(\chi'_3(d)]$$

$$= \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q))[2+2] = \theta_i(q)\cdot 4 = \theta_i(q)\cdot \theta'_j(d)$$

(II) If j=3 and $d \in \{ a , a^2, a^3, a^4\}$

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q).\chi_3(d)) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q)) [\sum_{\sigma \epsilon D_5} \sigma(\chi'_3(d)]$$

$$= \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q))[\tau_1+\tau_2] = \theta_i(q)\cdot(-1) = \theta_i(q)\cdot \theta'_j(d)$$

(III) If j=3 and $d \in \{ b, ba , ba^2, ba^3, ba^4\}$

$$\theta_{(i,j)}(t) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q).\chi'_3(d)) = \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q)) [\sum_{\sigma \epsilon D_5} \sigma(\chi'_3(d)]$$

$$= \sum_{\sigma \epsilon Gal(Q(\chi_i(q))/Q)} \sigma(\chi_i(q))\cdot 0 = 0 = \theta_i(q)\cdot \theta'_j(d)$$

From [A] , [B] and [C] we get $\theta_{(i,j)} = \theta_i \cdot \theta'_j$

Then: $\equiv^*(Q_{2m}\times D_5) = \equiv^*(Q_{2m}) \otimes \equiv^*(D_5)$.

Example(4.2):

To calculate $\equiv^* Q_{22}\times D_5$, we can use theorem(3.1).

the rational valued character table of D_5 is:

Cl_a	$[d'_1]$	$[d'_2]$	$[d'_3]$
θ'_1	1	1	1
θ'_2	1	1	-1
θ'_3	4	-1	0

Table (4.3)

by proposition (3.7), and by proposition(3.10) the rational valued character table of Q_{22} is:

Γ-classes	$[I]$	$[x^2]$	$[x^{11}]$	$[x]$	$[y]$
θ_1	1	1	1	1	1
θ_2	10	-1	10	-1	0
θ_3	1	1	1	1	-1
θ_4	10	-1	-10	1	0
θ_5	2	2	-2	-2	0

Table (4.4)