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Properties of Special Ellipsoids Family

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Abstract

This article describes the variability of the ellipsoid of the second order class of deformations relative to the plane. Also, a group of linear expressions, which reflects these faces, is shown.

Key words: ellipsoid, tangent, tangent plane, Linear Reflection Equation, surface, volume.

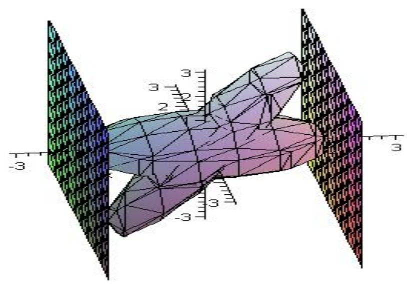
INTRODUCTION

Unlike movement in the Euclidean region, there are linear substitutions, they represent the ellipsoid again in ellipsoid but the basic geometric characteristic of the resulting ellipsoid varies. However, some variations in ellipsoid may be preserved in such a linear transformation. Naturally, the matrix orthogonality and symmetry conditions are not fulfilled in the linear substitution.

MATERIALS AND METHODS

Our goal is to determine the amount of invariant residues that can be preserved when linear deformation, while maintaining parallel sections of an optional ellipsoid plane. We use analytical geometry and linear algebra. Let's take $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=$

1 (1) ellipsoid given together with canonic equation. Let's look at the family of such ellipsoids with center of symmetry center at the beginning of coordinates, with x = 0 plane, with fractional ellipsoid tangent $\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (2) and every ellipsoid belonging to the family can be tangent on plane $x=\pm a$. This defines ellipsoid family with { Ψ }, Let's take h_1 and h_2 as the distance until OXZ and OXY plane in proportion from the point of tangent x=a to plane.



Lemma: Ellipsoids belonging to family {Ψ} shall be in the following equation

$$\left(1+\frac{h_1^2}{b^2}+\frac{h_2^2}{c^2}\right)\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-\frac{2h_1}{ab^2}xy-\frac{2h_2}{ac^2}xz=1 \qquad (3).$$

Proof: The general equation of ellipsoids from {Ψ} family shall be in the following equation.

$$a_{11}x^2+\frac{y^2}{b^2}+\frac{z^2}{c^2}+2a_{12}xy+2a_{13}xz=1 \qquad (4)$$

Because, a beginning of coordinates for centre of ellipsoids belonging to this family and tangent with x=0 plane shall be an ellipse $\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

Now let's define a_{ij}. For this we shall use in (a,h_1,h_2) point (4) the tangent of ellipsoid in x=a plane. Equation of tangent plane

$$(a_{11}a+a_{12}h_1+a_{13}h_2)x+\left(a_{12}a+\frac{1}{b^2}h_1\right)y+\left(a_{13}a+\frac{1}{c^2}h_2\right)z=1 \qquad (5)$$

We shall form the following system due to necessity of this plane coming one by one with plane x=a.

$$\begin{cases} a_{11}a+a_{12}h_1+a_{13}h_2=\frac{1}{a} \\ a_{12}a+\frac{1}{b^2}h_1=0 \\ a_{13}a+\frac{1}{c^2}h_2=0 \end{cases} \Rightarrow \begin{cases} a_{11}=\frac{1}{a^2}-\frac{a_{12}h_1}{a}-\frac{a_{13}h_2}{a} \\ a_{12}=-\frac{1}{ab^2}h_1 \\ a_{13}=\frac{1}{ac^2}h_2 \end{cases}$$

$$a_{11}=\frac{1}{a^2}+\frac{{h_1}^2}{a^2b^2}+\frac{{h_2}^2}{a^2c^2}$$

all coefficients of a_{ij} are expressed with a, b, c, h_1, h_2. Thus, general equation of ellipsoids family is

$$\left(1+\frac{h_1^2}{b^2}+\frac{h_2^2}{c^2}\right)\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-\frac{2h_1}{ab^2}xy-\frac{2h_2}{ac^2}xz=1.$$

Lemma is proven.

Result:

Theorem: 1*.This reflection shall reflect*

$$\begin{cases} x` = x \\ y` = -\frac{h_1}{a}x+y \\ z` = -\frac{h_2}{a}x+z \end{cases} \qquad A=\begin{pmatrix} 1 & 0 & 0 \\ -\frac{h_1}{a} & 1 & 0 \\ -\frac{h_2}{a} & 0 & 1 \end{pmatrix}$$

ellipsoid from {Ψ} family to ellipsoid (1) with the same value.

Proof: $\left(1+\frac{h_1^2}{b^2}+\frac{h_2^2}{c^2}\right)\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-\frac{2h_1}{ab^2}xy-\frac{2h_2}{ac^2}xz=1$

Let's simplify the equation.

$$\frac{x^2}{a^2}+\frac{x^2}{a^2}\cdot\frac{h_1^2}{b^2}+\frac{x^2}{a^2}\cdot\frac{h_2^2}{c^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-\frac{2h_1}{ab^2}xy-\frac{2h_2}{ac^2}xz=1$$

Let's bring it in the form of full square.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-2\cdot\frac{y}{b}\cdot\frac{h_1}{ab}x+\frac{x^2}{a^2}\cdot\frac{h_1^2}{b^2}+\frac{z^2}{c^2}-2\cdot\frac{z}{c}\cdot\frac{h_2}{ac}x+\frac{x^2}{a^2}\cdot\frac{h_2^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\left(\frac{y}{b}-\frac{h_1}{ab}x\right)^2+\left(\frac{z}{c}-\frac{h_2}{ac}x\right)^2=1 \qquad \frac{x^2}{a^2}+\frac{(y-\frac{h_1}{a}x)^2}{b^2}+\frac{(z-\frac{h_2}{a}x)^2}{c^2}=1$$

(6)

Hence, if we enter the definition of

$$\begin{cases} x = x` \\ y-\frac{h_1}{a}x = y` \\ z-\frac{h_2}{a}x = z` \end{cases}.$$

we will get an ellipsoid of $\frac{x`^2}{a^2}+\frac{y`^2}{b^2}+\frac{z`^2}{c^2}=1$ form[1]. Theorem is proven.

According to task of the problem, an ellipse (2) has to belong to each ellipse of {Ψ} family. Therefore, if we cut each ellipsoid belonging to {Ψ} family by means of OYZ plane, we shall get the (2) ellipse.

Theorem: 2. *When we cut the ellipsoid from {Ψ} family parallel to plane OYZ and plane* $x = x_0$ *, ellipses made from the tangents shall be equal.*

Proof: We cut the ellipsoid of (1) form by means of plane $x = x_0$, that is

$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 - \frac{x_0^2}{a^2}$ $\quad\quad$ $\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{a^2 - x_0^2}{a^2}$

In this section/tangent, we will get the ellipse defined in the following equation. $\frac{y^2}{\left(\frac{b\sqrt{a^2-x_0^2}}{a}\right)^2} + \frac{z^2}{\left(\frac{c\sqrt{a^2-x_0^2}}{a}\right)^2} = 1$

Semi axes of this ellipse are equal to $\frac{b\sqrt{a^2-x_0^2}}{a}$ and $\frac{c\sqrt{a^2-x_0^2}}{a}$, surface of this ellipse is- $S_1 = \frac{bc(a^2-x_0^2)\pi}{a^2}$.[3] Now, let's learn what is tangent of the ellipsoid in the form of (3) or (6) by means of plane $x = x_0$. Hence, for equation in the form of (3)

$$\left(1 + \frac{h_1^2}{b^2} + \frac{h_2^2}{c^2}\right)\frac{x_0^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - \frac{2h_1}{ab^2}x_0 y - \frac{2h_2}{ac^2}x_0 z = 1$$

or for equation in the form of (6)

we shall form the following equation $\frac{x_0^2}{a^2} + \frac{(y-\frac{h_1}{a}x_0)^2}{b^2} + \frac{(z-\frac{h_2}{a}x_0)^2}{c^2} = 1$, $\frac{(y-\frac{h_1}{a}x_0)^2}{b^2} + \frac{(z-\frac{h_2}{a}x_0)^2}{c^2} = 1\text{-}\frac{x_0^2}{a^2}$. Naturally, tangents are comprised of ellipses. Their canonic equations can be defined by means of below given equation.

$$\frac{(y - \frac{h_1}{a}x_0)^2}{\left(\frac{b\sqrt{a^2 - x_0^2}}{a}\right)^2} + \frac{(z - \frac{h_2}{a}x_0)^2}{\left(\frac{c\sqrt{a^2 - x_0^2}}{a}\right)^2} = 1$$

In this equation, as h_1 and h_2 are variable, semi axes of ellipses formed in the tangent are equal, that is $\frac{b\sqrt{a^2-x_0^2}}{a}$ and $\frac{c\sqrt{a^2-x_0^2}}{a}$ so, they differ only with placement of their centre. Surface of this ellipse is equal to $S_2 = \frac{bc(a^2-x_0^2)}{a^2}$. So, we get $S_1 = S_2$. It comes out that, surfaces of such ellipses in corresponding tangents, their semi axes, their length shall be the similar. The theorem is proven.

Theorem 3. *Volumes of all ellipsoids belonging to {Ψ} family are equal and are calculated with* $V = \frac{4\pi}{3}abc$ *formula.*

Proof: Let's cut the ellipsoid in the form of (1) in interval [-a;a] with planes being perpendicular to axe - OX. There shall be formed ellipses in the tangent. We shall take the. S_1^1, S_2^1, S_3^1,... S_n^1 as its surfaces. These planes shall cut over the OX axe, let's its points as x_1, x_2, x_3, $...x_n$. So, the plane, cutting the surface - S_1^1 in the x_1 point from point x_2 shall form a surface of S_2^1 and other surfaces. If S_i^1 surfaces are given with values in point x_i with f(x) function, that is we create a sum by means of withdrawal of ξ_i point from interval of $f(x_i) = S_i^1$. (x_{i-1}, x_i) .$(i = \overline{1, n})$ [3]. $\sum_{i=1}^{n} f(\xi_i)\Delta x_i$, hence $\Delta x_i = x_i - x_{i-1}$.

$$\lim_{max\Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \int_{-a}^{a} f(x)dx = V_1$$

V_1-(1) gives a volume of ellipsoid. We use the same process for ellipsoids belonging to {Ψ} family. Here, planes cutting through the x_i points of any known ellipsoid from {Ψ} family shall form the tangents with surfaces of S_1^2, S_2^2, S_3^2,... S_n^2 respectively. According to the 2nd theorem, $S_i^1 = S_i^2$ equation is proper. Here, we may form the definite function of f(x). So, volume of any random ellipsoid belonging to this family is equal to $V_2 = \int_{-a}^{a} f(x)dx = V_1$. So, volume of ellipsoids belongings to this family and ellipsoids in the form of (1) are equal. As it is known from elementary mathematics, volume of ellipsoids of form (1) are calculated with formula $V_1 = \frac{4}{3}abc\pi$ [2] . As is $V_1 = V_2$, each volume of ellipsoids belonging to family {Ψ} shall be equal to $\frac{4}{3}abc\pi$. The theorem is proven.

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