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Super Face Anti-magic Labeling of Subdivided Prism Graph

SHAHID ZAMAN
PhD Scholar, Faculty of Mathematics and Statistics
Central China Normal University, Wuhan, P.R. China
NOOR REHMAN
PhD Scholar, Faculty of Mathematics and Statistics
Central China Normal University, Wuhan, P.R. China
JAWAD ALAM
Lecturer in Department of Mathematics
University of Sialkot, Sialkot, Punjab, Pakistan
ASMA JABEEN
Post Graduate Student, Department of Mathematics
Govt. College Women University, Sialkot, Pakistan

Abstract

Suppose $G = (V, E, F)$ is a finite plane graph with vertex set *V(G),* edge set *E(G)* and face set *F(G).* A bijection $\eta : V \cup E \cup F \to \{1,2,3,.....|V(G)| + |E(G)| + |F(G)|\}$ is known as labeling of type (1, 1, 1). The labeling of type (1, 1, 1) is called face magic (or face anti-magic resp.), if all faces of a graph have same (or different resp.) weight. A face anti-magic labeling is called super, if the smallest possible labels appear on the vertices. In this paper, we deal with super face anti-magic labeling of type (1, 1, 1) for the subdivided prism graphs and we show that general graph admits super face anti-magic labeling.

Key words: Plane graph, Subdivided prism graph, SFAMT-labeled graph

INTRODUCTION

We consider all graphs are finite, simple, undirected and plane. The plane graph $G = (V, E, F)$ has vertex set $V(G)$, edge set *E(G),* face set

F(G). We follow either Wallis [1] or West [2] for most of the graph theory terminology and notation used in this paper.

Graph labeling is one major research area in graph theory. Most of the graph labeling methods trace their origin to the concept of β-valuation introduced by Rosa [3]. The same concept was introduced by Golomb who called it a graceful labeling [4]. Various types of graph labelings such as graceful labeling, harmonious labeling, equitable labeling, cordial labeling, arithmetic labeling, Skolem graceful labeling, magic labeling, antimagic labeling, set-magic labeling, multiplicative and strongly multiplicative labeling, prime labeling, mean labeling and orthogonal labeling have been investigated by several authors.

The concept of graph labeling has a wide range of applications to other branches of science such as X-ray crystallography, coding theory, cryptography, astronomy, circuit design and communication networks design.

A labeling of a graph is any mapping that sends some set of graph elements to a set of numbers (usually positive integers). If the domain is the vertex set or edge set or the face set, the labeling is called respectively vertex labeling or edge labeling or face labeling. A bijection $\eta : V \cup E \cup F \to \{1,2,3,.....|V(G)|+|E(G)|+|F(G)|\}$ is called a labeling of type (1, 1, 1). The weight of a face under a labeling is the sum of the labels (if present) carried by that face and the edges and vertices surrounding it. A labeling of type (1, 1, 1) is called face magic (or face anti-magic resp.), if all faces of a graph have same (or different resp.) weight.

Lih [5] describes magic labeling of type (1, 1, 0) for wheels, friendship graphs and prisms. The notion of anti-magic labeling was introduced by Hartsfield and Ringel [14] in 1990. In graph labeling, various types of anti-magic labeling such as (a, d)-antimagic labeling, (a, d)-edge anti-magic vertex labeling, (a, d)-edge anti-magic total labeling, super (a, d)-edge anti-magic total labeling, (a, d)-vertex anti-magic total labeling, super (a, d)-vertex anti-magic total labelling and face anti-magic labeling of planar graphs have been studied by several authors. The 0-antimagic labelings of type (1, 1, 1) for grid graphs and honeycomb are given in [6] and [7], respectively. The concept of *d*-antimagic labeling of plane graphs was defined in [8]. The *d*-antimagic labeling of type (1, 1, 1) for the hexagonal planar maps, generalized Petersen graph $P(n, 2)$ and grids can be found in [9–11], respectively. Lin et al. [12] showed that prism D_n, $n \geq 3$ admits d-antimagic labeling of type (1, 1, 1) for $d \in \{2, 4, 5, 6\}$. The d-antimagic labeling of type (1, 1, 1) for D_n and for several $n \geq 7$ are described in [13].

This paper concentrate the super face anti-magic total (SFAMT)-labeling of type (1, 1, 1) for the subdivision prism graph.

SFAMT-LABELING FOR $G_{p,n}$

Consider a general sub-divided prism graph $G_{p,n}$, where $n \geq 3$ and $p \geq 1$ with all the vertices of degree 2. We define the vertex set $V(G_{p,n})$, edge set $E(G_{p,n})$ and face set $F(G_{p,n})$ of sub-divided prism graph as follows (Fig. 1)

$$\begin{aligned}
V(G_{p,n}) &= \{x_t; 1 \leq t \leq n\} \cup \{y_t; 1 \leq t \leq n\} \cup \{u_{tk}; 1 \leq t \leq n, 1 \leq k \leq p\} \\
&\cup \{v_{tk}; 1 \leq t \leq n, 1 \leq k \leq p\} \cup \{w_{tk}; 1 \leq t \leq n, 1 \leq k \leq p\} \\
E(G_{p,n}) &= \{x_t u_{tk}; 1 \leq t \leq n, k = 1\} \cup \{y_t w_{tk}; 1 \leq t \leq n, k = 1\} \\
&\cup \{x_t v_{tk}; 1 \leq t \leq n, k = 1\} \cup \{x_{t+1} u_{tk}; 1 \leq t \leq n, k = p\} \\
&\cup \{y_t v_{tk}; 1 \leq t \leq n, k = p\} \cup \{y_{t+1} w_{tk}; 1 \leq t \leq n, k = p\} \\
&\cup \{u_{tk} u_{tk+1}; 1 \leq t \leq n, 1 \leq k \leq p-1\} \\
&\cup \{v_{tk} v_{tk+1}; 1 \leq t \leq n, 1 \leq k \leq p-1\} \\
&\cup \{w_{tk} w_{tk+1}; 1 \leq t \leq n, 1 \leq k \leq p-1\} \\
F(G_{p,n}) &= \{f_t; 1 \leq t \leq n\}
\end{aligned}$$

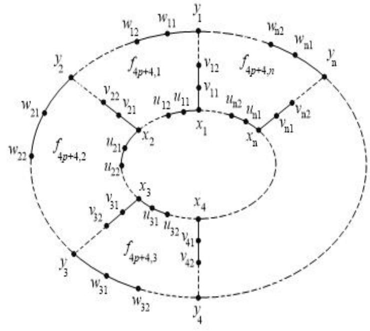

Fig 1: The graph $G_{p,n}$

THEOREMS OF SFAMT-LABELING

Theorem 1: The subdivided prism $G_{p,n}, n \geq 4$ and $p = 1$ admits SFAMT-labeling of type (1, 1, 1), where *n* and *p* are positive integers.

Proof: For $n \geq 4$ and $p = 1$, we define labeling $\eta : V \cup E \cup F \rightarrow \{1, 2, 3, ...|V(G)| + |E(G)| + |F(G)|\}$ as follows

$$\eta(x_t) = \{\ 2t - 1, \qquad 1 \leq t \leq n$$

$$\eta(y_t) = \begin{cases} 2t, & t = 1 \\ 2(1-t) + 4, & 2 \leq t \leq n \end{cases}$$

$$\eta(u_{tk}) = \begin{cases} 4n - 2t - 1, & 1 \leq t \leq n-1 \\ 4n - 1, & t = n \end{cases}$$

$$\eta(w_{tk}) = \begin{cases} 4n, & t = 1 \\ 2t + 2n - 2, & 2 \leq t \leq n \end{cases}$$

$$\eta(v_{tk}) = \begin{cases} 4n + 2, & t = 1 \\ 4n + 1, & t = 2 \\ 5n - t + 3, & 3 \leq t \leq n \end{cases}$$

$$\eta(x_t u_{tk}) = \begin{cases} 9n - t + 2, & 2 \leq t \leq n \\ 8n + 1, & t = 1 \end{cases}$$

$$\eta(y_t w_{tk}) = \begin{cases} 8n, & t = 1 \\ 7n + t - 1, & 2 \leq t \leq n \end{cases}$$

$$\eta(x_{t+1} u_{tk}) = \{5n + t, \qquad 1 \leq t \leq n$$

$$\eta(y_{t+1} w_{tk}) = \{6n, \qquad t = n$$

$$\eta(x_t v_t) = \begin{cases} 10n + t + 1, & 1 \leq t \leq n-1 \\ 10n + 1, & t = n \end{cases}$$

$$\eta(y_t v_t) = \begin{cases} 12n - t, & 1 \leq t \leq n-1 \\ 12n, & t = n \end{cases}$$

$$\eta(f_t) = \{9n + t, \qquad 1 \leq t \leq n$$

The weight of 8-sided faces $f_{8,t}$ for each *t* = 1, 2, 3,....., *n*, is calculated as follows

$$W(f_{8,t}) = \eta(x_t) + \eta(y_t) + \eta(u_{tk}) + \eta(w_{tk}) + \eta(v_{tk}) + \eta(x_t u_{tk}) + \eta(y_t w_{tk})$$

$$+ \eta(x_{t+1} u_{tk}) + \eta(y_{t+1} w_{tk}) + \eta(x_t v_t) + \eta(y_t v_t) + \eta(f_t)$$

$$W(f_{8,t}) = 100n - t + 11, \quad t = \{1, 2\}$$

$$W(f_{8,t}) = 100n - t + 15, \quad t = \{3, 4, 5, ..., n\}$$

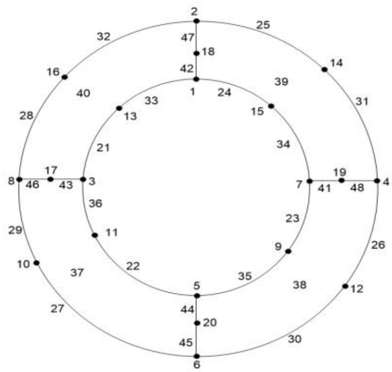

Fig 2: SFAMT-Labeling for $G_{1,4}$

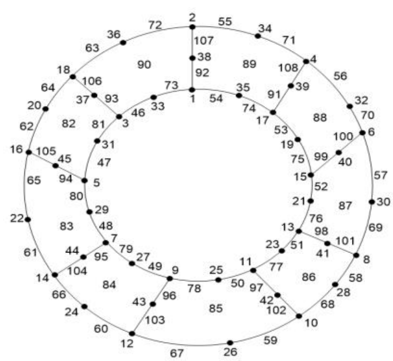

Fig 3: SFAMT-Labeling for $G_{1,9}$

Theorem 2: The subdivided prism $G_{p,n}$, $n = 3$ and $p \geq 1$ admits SFAMT-labeling of type (1, 1, 1), where *n* and *p* are positive integers.

Proof: For $n = 3$ and $p \geq 1$, we define labeling $\eta : V \cup E \cup F \rightarrow \{1, 2, 3, ...|V(G)| + |E(G)| + |F(G)|\}$ as follows

$$\eta(x_t)=\left\{\ 2t-1, \quad 1\le t\le 3\right.$$

$$\eta(y_t)=\begin{cases} 2t, & t=1\\ 10-2t, & 2\le t\le 3\end{cases}$$

$$\eta(u_{tk})=\begin{cases} 11-2t, & 1\le t\le 2, \quad k=1\\ 11, & t=3, \quad k=1\\ 3k+3p+8, & t=1, \quad 2\le k\le p\\ 3k+3p+9, & t=2, \quad 2\le k\le p\\ 3k+3p+7, & t=3, \quad 2\le k\le p\end{cases}$$

$$\eta(w_{tk})=\begin{cases} 12, & t=1, \quad k=1\\ 2t+4, & 2\le t\le 3, \quad k=1\\ 6p+3k+t+4, & 1\le t\le 2, \quad 2\le k\le p\\ 6p+3k+4, & t=3, \quad 2\le k\le p\end{cases}$$

$$\eta(v_{tk})=\begin{cases} 14, & t=1, \quad k=1\\ 2t+9, & 2\le t\le 3, \quad k=1\\ 3k+t+10, & 1\le t\le 2, \quad 2\le k\le p\\ 3k+10, & t=3, \quad 2\le k\le p\end{cases}$$

$$\eta(x_t v_{tk})=\begin{cases} 9p+t=22, & 1\le t\le 2\\ 9p-t+25, & t=3, \quad k=1\end{cases}$$

$$\eta(x_t u_{tk})=\begin{cases} 9p+2t+14, & 1\le t\le 2, \quad k=1\\ 9p+17, & t=3, \quad k=1\end{cases}$$

$$\eta(v_{tk} v_{tk+1})=\begin{cases} 3k+9p-t+21, & 1\le t\le 2, \quad 2\le k\le p\\ 3k+9p+21, & t=3, \quad 2\le k\le p\end{cases}$$

$$\eta(u_{tk} u_{tk+1})=\begin{cases} 9p+t+6, & 1\le t\le 3, \quad k=1\\ 12p+3k-t+21, & 1\le t\le 2, \quad 2\le k\le p\\ 15p+21, & t=3, \quad 2\le k\le p\end{cases}$$

$$\eta(y_t w_{tk})=\begin{cases} 9p-2t+17, & 1\le t\le 2, \quad k=1\\ 9p+14, & t=3, \quad k=1\end{cases}$$

$$\eta(w_{tk} w_{tk+1})=\begin{cases} 9p-t+13, & 1\le t\le 3, \quad k=1\\ 15p+3k-t+18, & 1\le t\le 2, \quad 2\le k\le p\\ 15p+3k+18, & t=3, \quad 2\le k\le p\end{cases}$$

$$\eta(f_t)=\begin{cases} 9p-2t+23, & 1\le t\le 2\\ 9p+20, & t=3\end{cases}$$

The weight of (4*p*+4)-sided faces $f_{4p+4,t}$ for each $t=\{1,2,3\}$ is calculated as follows

$$W(f_{4p+4,t})=\eta(x_t)+\eta(y_t)+\eta(u_{tk})+\eta(w_{tk})+\eta(v_{tk})+\eta(x_t v_{tk})+\eta(v_{tk}v_{tk+1})$$

$$+\eta(x_t u_{tk})+\eta(u_{tk}u_{tk+1})+\eta(y_t w_{tk})+\eta(w_{tk}w_{tk+1})+\eta(f_t)$$

$$W(f_{4p+4,t})=355p-t-44, \quad t=\{1,2\}$$

$$W(f_{4p+4,t})=355p-t-41, \quad t=\{3\}$$

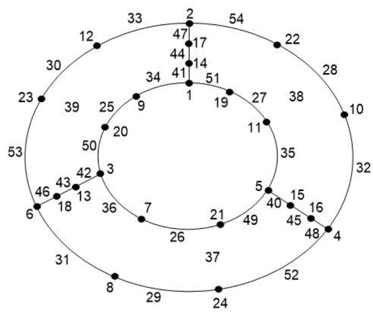

Fig 4: SFAMT-Labeling for $G_{2,3}$

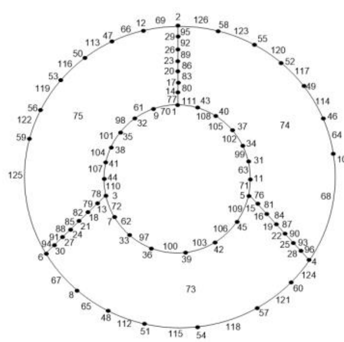

Fig 5: SFAMT-Labeling for $G_{6,3}$

CONCLUSION AND FUTURE WORK

In this paper, we have studied the SFAMT-labeling of type (1, 1, 1) of the graphs $G_{p,n}$ which are obtained by subdivided prism. We have seen that these graphs admit SFAMT-labeling for $n \geq 4$ & $p=1$ and

$n=3\ \&\ p\geq 1$. We close this section by raising the following open problems.

Future Problem 1: For every $n\geq 3, p\geq 1,$ the graph $G_{p,n}$ is obtained by *m* disjoint copies of the graph $G_{p,n}$, then $mG_{p,n}$ admits super face magic total (SFMT)-labeling of type (1, 1, 1).

Future Problem 2: For every $n\geq 3, p\geq 1,$ the graph $G_{p,n}$ is obtained by *m* disjoint copies of the graph $G_{p,n}$, then $mG_{p,n}$ admits super face anti-magic total (SFAMT)-labeling of type (1, 1, 1).

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