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Motivating Master Students and Pre-service Teachers of Mathematics Education to Use Analogous Reasoning in Teaching Mathematics

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Abstract:

Analogous reasoning has some important roles in mathematics instruction, such as constructing the meaning of knowledge, formulating hypotheses, solving math problems, predicting, and preventing and correcting student errors. Because of these roles, teachers are encouraged to use the analogy in their mathematics teaching strategies. This study was intended to promote master students and pre-service teachers to utilize analogous reasoning in teaching mathematics. The sample included 61 pre-service teachers and 47 master students in mathematics education. They were taught theoretical foundations of analogy, teaching with the analogy, and models of teaching by analogy, then they were instructed to analyze textbooks and design lesson plans that used analogous reasoning in teaching specific contents in high schools. The data collected included their worksheets and analyzed mainly qualitatively, including using a scale to assess how well they used analogous reasoning in teaching mathematics. The results revealed that the participants made progress in analyzing textbooks about analogy inference. As a result, they had effective orientations in the design of student-guided lessons using inference to find new knowledge, thus promoting learners' activeness. Additionally, some suggestions were drawn for teachers and students when using analogous reasoning.

Keywords: master students, pre-service teachers, analogous reasoning, mathematics education.

1. INTRODUCTION

When faced with a new situation, students tend to compare and contrast it with previous similar problems, thereby figuring out how to solve it. The use of analogous reasoning in the teaching process requires students to work on old knowledge to discover new knowledge by themselves. Therefore, students are active people to form new hypotheses. This process promotes thinking development because it requires learners to think, analyze, compare, contrast, and generalize knowledge; thereby, encouraging a passion for learning and promoting students' independent thinking, critical thinking and creative thinking. Therefore, many authors have been interested in similarity, similarity inference, and analogy inference in teaching. The analogy is used in various fields such as science, technology, physics, and mathematics (Guler, P. D. (2008); Yener, D. (2012)).

The famous educator, Polya, G. (1997) studied analogous reasoning in mathematics and suggested that analogy can provide a source for new problems and improve performance and problem-solving ideas. He introduced analogy inference and its relation to generalization and specialization in solving mathematical problems. Holyoak (1997) developed research on the use of analogy inference in problem-solving and suggested that the process of matching should be goal-oriented: the coherence of using analogy depends on a unified structure, semantics and purposes. So, there should be as many similarity relationships and properties as possible between the source and the destination, and it helps solve proximity (cited by Richland et al. (2004)). Also, Richland, Holyoak and Stigler (2004) studied and considered the following issues: student-teacher participation, similar sources, setting goals and similar appearance contexts. Similar data from 103 appeared in 25 randomly selected 8th-grade math classes in the United States, showing that teachers frequently use similar instructional mechanisms to teach concepts. Building resources and goals are also related to meeting students' learning needs under the control and help of teachers. Besides, Salih, M. (2008) recommended a proposed analogous reasoning model for the translation concept.

Atkins (2004) focused on students making analogies in science and provided a model for this understanding. The author provides evidence of similarity classification and the bases of similarity, thereby suggesting that similarity is generated based on schemas and cognitive models. Research by Lee et al. (2007) focused on discussing two issues: how to get math gifted sixth and 8th graders to use induction, analogy, and imagery in the learning process of problem-solving and the role of induction, analogy, and imagery in mathematical exploration.

Author Chung, H. (1994) has defined analogy inference as" is an inference based on some similar properties of two objects, to conclude other similar properties of those two objects" with the same diagram, illustrative examples and conditions to ensure the reliability of the analogy inference. Also, author Chau, L. T. H. (2004) has introduced the exploitation of analogy reasoning in teaching spatial geometry: the first is the similarity of properties of plane geometry and spatial geometry; the second is to use the analogy in solving two problems when there is the similarity of factors given in hypothesis and conclusion. Author Tam (2007) emphasized the need to "focus on giving students the same thought manipulation between teaching planar geometry and spatial geometry" and pointed out mistakes when using analogous reasoning. For the content of the coordinate method, the author analyzed the characteristics, then pointed out the similarity between the knowledge in the plane and space.

Author Loc, N. P (2015, 2016) mentioned the theoretical basis of similarity inference, two types of similarity inference by relation and attribute. Besides, author Nguyen Phu Loc mentioned two models TWA and FAR, using analogous reasoning in teaching to explore the concepts of exponential, derivative and limit of series. Furthermore, they conducted a study on 18 math teachers in high schools in the Mekong Delta provinces and 83 final year math pre-service teachers at Can Tho University. They pointed out that high school math teachers and math pedagogy students have not prioritized using analogy reasoning to help students discover new knowledge. In a few cases, teachers and students only use analogous reasoning to help students review old knowledge and lead into new lessons but have not created a connection and correspondence between knowledge and knowledge, old knowledge and new knowledge. Besides, teachers and students have not yet noticed the mistakes that students may make when using analogous reasoning in learning new knowledge, especially in solving math problems. It was necessary to provide a theoretical basis and apply analogous reasoning in teaching for teachers and pedagogical students. For this purpose, we research and propose measures to guide graduate students and students of mathematics pedagogy to practice teaching with analogous reasoning; thereby, improving the teaching practice for master students and pre-service teachers of mathematics education. The process of teaching concept discovery with analogy inference (improved from the TWA model) can be divided into five steps:

Step 1: Suggest starting motivation and target direction;

Step 2: Arousing students' memories of source knowledge;

Step 3: Ask students to point out the corresponding signs between source and destination;

Step 4: Point out incorrect conclusions, identify specific signs of the new concept;

Step 5: Ask students to state the definition of the new concept;

Step 6: The teacher corrects the new concept and gives illustrative examples and exercises.

Objectives of the study

The main purpose of this study is to address the following:

i) Promote master students and pre-service teachers of mathematics education to exploit and improve analogous reasoning activities presented in textbooks to promote students' activeness.
ii) Motivate master students and pre-service teachers of mathematics education to apply teaching processes with analogous reasoning to typical situations in mathematics.

Research questions

This study hopes to elucidate two questions:

i) How can master students and pre-service teachers of mathematics education analyze and improve the activities using analogous reasoning in textbooks towards promoting the activeness of students?.
ii) From the analysis of activities using analogous reasoning in textbooks, how do master students and pre-service teachers of mathematics education design lessons explore new concepts, theorems and solve math problems according to teaching processes with analogous reasoning?.

2. METHODOLOGY

2.1. Participants

The experiment was conducted for 61 third-year pre-service maths teachers, course 40 and 47 master students majoring in Theory and Teaching Methods of Mathematics, Course 23 of Can Tho University. Math pre-service teachers were studying Maths teaching and learning activities. Before that, the graduate students had learned modules on teaching methods such as Principles of teaching Mathematics, Methods of teaching Mathematics, Educational activities in high schools, Math program design. Master students were studying the topic Trends in Math Teaching. Most of the students were math teachers who were teaching at high schools in the Mekong Delta provinces.

2.2. The process of the study

The content of the experiment was carried out as follows.

Step 1: The teacher introduced the theoretical basis of analogous inference and analyzed an activity using analogous inference in the textbook to present the concept of the coordinate system in space (Advanced Geometry 12th Textbook) on a similar basis to the in-plane coordinate system. Besides, the researcher also introduced teaching concept discovery with analogous reasoning, teaching process of theorem discovery with similar reasoning in 45 minutes.

Purpose of Step 1: This step aimed to help Master students and pre-service teachers have a basic understanding of cognitive literacy, characteristics, and role of cognitive literacy in teaching. Moreover, the researcher also guided them through analyzing an activity using mathematical reasoning in textbooks and understand teaching with mathematical reasoning in some typical mathematics situations.

Step 2: The master students and pre-service teachers were divided into groups to discuss for 60 minutes. Students were mainly divided into 14 groups, each group from 4 to 5 pre-service teachers (denoted group nSV, where n was the number of the group). Master students were divided into seven groups, each with 6 to 7 Master students (denoted nCH, where n was the group number). The groups discussed and answered the following two questions (see Table 1):

Table 1. Experimental questions

Groups	Question 1	Question 2
1CH 1SV 8SV	Analyze how to use analogous reasoning in the lesson: Dot product of two vectors in the Geometry 10 textbook?	Using teaching processes with analogous reasoning to teach the dot product of two vectors and their properties in promoting students' activeness?
2CH 2SV 9SV	Analyze how to use analogy inference to present the concept of vectors in space in the basic Geometry 11 textbook?	Using teaching processes with analogous reasoning to teach the concept of vectors in space and vector operations to promote students' activeness?
3CH 3SV 10SV	Analyze how to use analogy inference to present parametric equations of straight lines in space in the basic Geometry 12 textbook?	Using teaching processes with analogous reasoning to teach the concept of parametric equations of straight lines in space to promote students' activeness?
4CH 4SV 11SV	Analyze how to use analogy inference to teach the formula for calculating the distance from a point to a plane in Advanced Geometry 12 textbooks?	Using the teaching process with analogous reasoning to teach the formula to calculate the distance from a point to a plane to promote students' activeness?
5CH 5CH 12SV	Analyze how to use analogous reasoning to teach the formula for calculating the general term of a geometric sequence in the Advanced Algebra and Calculus 11 textbook?	Using the teaching process with analogous reasoning to teach the theorem of the formula for calculating the general term of a geometric sequence in promoting students' activeness?
6CH 6SV 13SV	Analyze how to use the analogy to solve equations cosx=a in basic Algebra and Calculus 11 textbook?	Using the teaching process with analogous reasoning to teach how to solve equations cosx=a in promoting students' activeness?
7CH 7SV 14SV	Analyze how to use analogy reasoning to show how to solve quadratic equations with coefficients A, B, C being complex numbers in the Advanced 12th Calculus textbook?	Using the teaching process with analogous reasoning to teach how to solve quadratic equations with coefficients A, B, C being complex numbers to promote students' activeness?

Question 1 required students and graduate students to analyze activities using similar reasoning in textbooks in the questions posed to groups. Therefore, it was necessary to consider which source the textbook used for the knowledge to be taught, how the authors presented it, and what could be exploited from these activities; If necessary, how could it be improved to develop the positive character of students? In question 2, the researcher wanted to determine if students can practice designing a teaching activity for the content of knowledge just analyzed by applying teaching processes with analogy inferences or not.

Evaluation criteria

From the analogy inference classification in Table 2, it could be seen that there were different levels when using similar inference. It could be a simple similarity inference, specifically by saying that the source is the target similarity or a well-explained analogy inference that showed the corresponding signs between the source and the target. Therefore, it was necessary to build a ladder with criteria to evaluate the level of using analogous reasoning in the teaching process of teachers. Based on the analogy inference classifications and the steps in the TWA model, the researcher had proposed the criteria for evaluating the use of analogy inference to be divided into the following levels (see Table 2). This method was also the criterion to evaluate the design of teaching lessons with the same reasoning in question 2 in step 2.

Table 2. A rating scale for using analogous reasoning in teaching

Levels	The extent to which analogous reasoning is used
0	Do not use analogous reasoning
1	Only similar source names are listed.
2	Recall the characteristics of the source but have not yet established a correspondence with the target knowledge.
3	Correspondence between source and destination can be established.
4	Make good correspondence between source and target: point out similarities and differences; draw relevant conclusions by analogy.

2.4. Data collection and analysis

The worksheets of the groups were collected and classified according to the chosen math topic. They were analyzed and explained qualitatively primarily. Additionally, the level of analogy inference usage of the participants was assessed according to the scale above.

3. RESULTS AND DISCUSSION

Table 3. Statistics of experimental results

Topics	Groups	Analogy inference analysis in textbooks	Levels
The dot product of two vectors	1CH	Textbook authors use similar reasoning to teach the discovery of 2-vector scalar problems	3
	1SV	The textbook guide helps create a similar relationship between the knowledge of the known work and the new knowledge of the dot product of the two vectors.	3
	8SV	The presentation in textbooks helps students make similar connections between learned knowledge and new knowledge.	4

		It is only a guide to present a concept, not an activity for students to discover new knowledge by themselves.	
Vectors in space.	2CH	From the concept of vectors in the known plane, the concept of vectors in space is given by analogy.	4
	2SV	Textbooks remind students of similar sources. The lead word creates a link between the source and the destination. Emphasize there is a difference between vectors in plane and space. No activities have been given for students to discover new knowledge.	4
	9SV	The textbook guides vector relationships in the plane when introducing vectors in space to help create a connection between learned knowledge and new knowledge. Create a link between the source and the destination.	4
Parametric equation of a line in space	3CH	The textbook repeats the form of the parametric equation of the line in the plane, thereby posing the problem of what the equation of the line in space looks like.	3
	3SV	Remind the similar source, create the connection of the source and the target so that the students understand the similar relationship. No activities have been given for students to discover new knowledge by themselves.	3
	10SV	Reminiscent of similar sources. Ask questions when entering a new lesson to evoke learning motivation for students. The way that the textbook presented helps students understand the similar relationship between old and new knowledge; both lead sequentially and create activities for students to reason.	4
The formula for calculating the distance from a point to a plane	4CH	Textbooks evoke learning motivation for students. Inspire students to find similarities in the two formulas; students can easily remember the new formula.	4
	4SV	Remind students to remember similar formulas, creating knowledge connections. New formulas are given immediately without requiring students to set up based on knowledge.	3
	11SV	Arousing students' memories of similar situations. Establish correspondence between source and destination.	3
The formula for calculating the general	5CH	The quote in the textbook evokes a similar relationship between the public and the multiplier.	4

term of a geometric sequence		It is not clear how similar inference is used? How to deduce the general term formula of the geometric sequence.	
	5SV	Show the relationship between this theorem and the known theorem	0
	12SV	The textbook guides the additive relationship in finding the general terms of the geometric sequence. Make connections between old and new knowledge, develop new knowledge from the same background. Have not guided students to explore and discover new knowledge.	4
Way of solving the equation $\cos x = a$	6CH	Similar solutions between equations $\sin x = a$ and $\cos x = a$	3
	6SV	From how to solve the equations $\cos x = \frac{1}{2}$; $\cos x = \frac{3}{4}$, they generalize to general problems $\cos x = a$.	3
	13SV	When teaching to solve the trigonometric equation $2\sin x - 1 = 0$, the textbook considers finding *x*, leading to solving basic trigonometric equations.	0
Way of solving the equation $Az^2 + Bz + C = 0$ with A, B, C are complex numbers	7CH	Textbooks use the analogy with the case A, B, C are real numbers. Help students recognize the relationship between old and new knowledge, point out the difference when solving quadratic equations with real and complex coefficients. It is recommended to add an activity that repeats how to solve quadratic equations with real coefficients.	3
	7SV	Calculating the square root of complex numbers is similar to calculating the square root of real numbers. Textbooks recall similar sources, creating a link between source and destination. Has not yet promoted students' curiosity and discovery of new knowledge	3
	14SV	Solving the quadratic equation with complex coefficients is similar to solving the quadratic equation with real coefficients. Conclude those different ways to find the square root of real and complex numbers.	2

In question 1, most of the groups analyzed the same source that the textbook used. The participants commented that the purpose that the textbook used analogous reasoning was to evoke the motivation to start the lesson and connect ancient knowledge and new knowledge. This approach created a foundation for teachers to use analogous reasoning in the teaching process. Some groups suggested that some activities needed to be improved and enhanced to help students explore and discover new knowledge based on similar known knowledge. A few groups (groups 5SV, 6SV, 13SV) had not properly identified the same source that the textbook has used.

In question 2, most of the master students and pre-service teachers had designed lessons using analogous reasoning to promote students' activeness. The average use of similarity inference was 3.1. This result proved that they had mastered the teaching process with analogous reasoning and could apply it well to teaching specific topics.

3.1. Topic 1: Dot product of two vectors

In question 1, groups of master students and pre-service teachers commented on the similar relationship between work done by a force and the dot product of two vectors. Specifically, the students of group 1SV wrote the following (see Figure 1):

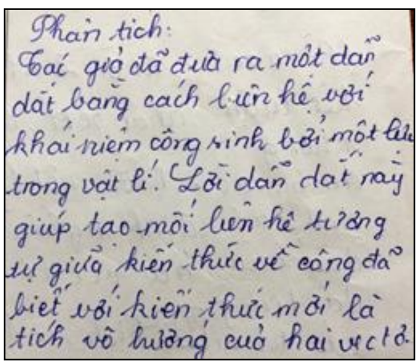


Figure 1. Analyzing the concept of the dot product of two vectors in textbooks of group 1SV

Additionally, it was noticed that the students in the group of 8 students also commented on the presentation in the textbook: "However, it is only a guide to present a concept, not an activity for students to do self-discovery of new knowledge" (see Figure 2).

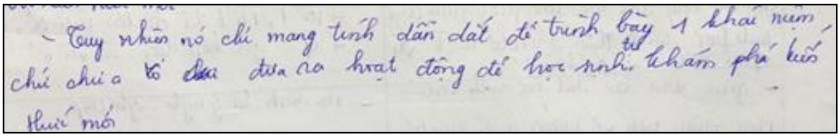


Figure 2. Analyzing the concept of the dot product of two vectors in textbooks of group 1SV

This comment used teaching concept discovery with analogous reasoning to teach the dot product of two vectors. The students in the group of 8 students designed the activities of teachers and students in 5 steps. They also used a similar source of work done by a force in physics. They designed more activities for students to analyze the characteristics of the source, such as "state the formula for the work of force?", "explain the symbols in the formula?". Then, they asked questions to help students establish the correspondence between the source and the destination; please rewrite the formula for work done by a force in 2 vectors ". Students could construct a new formula with similar characteristics to work done by a force from this question. The teacher introduced the dot product of two vectors and asked students to state the definition and give examples to apply.

3.2. Topic 2: Vectors in space

When analyzing how to use mathematical knowledge in the Geometry 11 textbook (see Figure 4.4), the group of 2 students commented on the purpose of using mathematical knowledge to remind students of similar sources, creating a link between source and destination. Also, the textbook author emphasized the difference between vectors in space, that was, pointed out the difference in both source and destination. Moreover, they also commented that this presentation was only "guided to present a new concept".

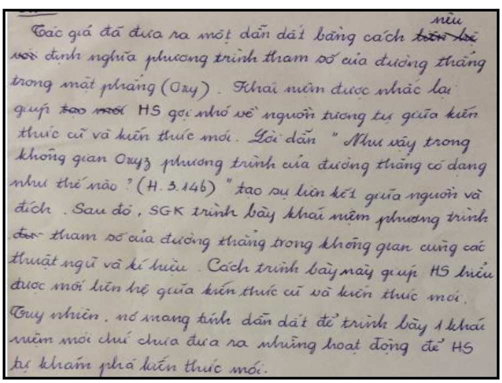
Khi viết "*Ở lớp 10 chúng ta đã được học về vector trong mặt phẳng.*", tác giả SGK đã gợi nhớ cho học sinh về nguồn tương tự. Lời dẫn dắt "*Do đó định nghĩa vector trong không gian cùng với một số nội dung có liên quan đến vector như độ dài của vector, sự cùng phương, cùng hướng của hai vector, giá của vector, sự bằng nhau của hai vector và các quy tắc thực hiện các phép toán về vector được xây dựng và xác định hoàn toàn tương tự như trong mặt phẳng.*" tạo ra sự liên kết giữa nguồn và đích. Tác giả còn nhấn mạnh những khác biệt nhất định giữa vector trong không gian và mặt phẳng "*trong không gian, chúng ta sẽ gặp những vấn đề mới về vector như việc xét sự đồng phẳng hoặc không đồng phẳng của ba vector hoặc việc phân tích một vector theo ba vector không đồng phẳng.*" Cuối cùng SGK trình bày định nghĩa vector trong không gian cùng các khái niệm có liên quan.

Figure 3. Analysis of the concept of vectors in space in textbooks of group 2SV

Based on the analysis of textbooks, the students of group 2 students designed a lesson on the concept of vectors in space and their properties (see Figure 3). They based the five steps of the teaching process on analogous reasoning. The selected analog source was an in-plane vector. Some activities analyzed the points of the source, let students discuss to predict some vector operations in space, such as adding, subtracting vectors, multiplying vectors by a number. Moreover, they also helped students discover the box rule, a rule similar to the parallelogram rule. Finally, there were some examples for students to apply in the exercise.

3.3. Topic 3: Parametric equations of lines in space

In question 1, the 3SV group commented on the similar relationship between the parametric equation of the line in the plane and space (see Figure 4). There was agreement among the students that the purpose of the presentation in the textbook was to recall a similar source, thus creating a connection between the source and the target. That helped students understand the relationship between learned knowledge and new knowledge.



Tác giả đã đưa ra một dẫn dắt bằng cách nêu định nghĩa phương trình tham số của đường thẳng trong mặt phẳng (Oxy). Khái niệm được nhắc lại giúp ~~tạo mới~~ HS gợi nhớ về nguồn tương tự giữa kiến thức cũ và kiến thức mới. Lời dẫn "Như vậy trong không gian Oxyz phương trình của đường thẳng có dạng như thế nào? (H. 3.146)" tạo sự liên kết giữa nguồn và đích. Sau đó, SGK trình bày khái niệm phương trình tham số của đường thẳng trong không gian cùng các thuật ngữ và kí hiệu. Cách trình bày này giúp HS hiểu được mối liên hệ giữa kiến thức cũ và kiến thức mới. Tuy nhiên, nó mang tính dẫn dắt để trình bày 1 khái niệm mới chứ chưa đưa ra những hoạt động để HS tự khám phá kiến thức mới.

Figure 4. Conceptual analysis of parametric equations of straight lines in space in textbooks of group 3SV

When designing a lesson on the concept of parametric equations of straight lines in space (see Figure 4), the 3SV group organized for students to review their knowledge of parametric equations of lines in planes, from there, they asked students to predict the parametric equation form of the line in space and the corresponding signs between the parametric equation of the line in the plane and space. However, they only asked students to make predictions but did not instruct them on checking whether this prediction was correct or incorrect. In our opinion, it was necessary to additionally verify the formula given because the correctness of the conclusions when using the same inference had not been confirmed. This way helped students avoid making some mistakes when using analogous reasoning later.

3.4. Topic 4: Formula to calculate the distance from a point to a plane (Group 4CH, 4SV, 11SV)

For calculating the distance from a point to a plane, the researcher mentioned how to analyze textbooks and design lessons of the 4CH group. When analyzing textbooks (see Figure 5), the students pointed out the effects of using similar reasoning in textbooks, such as: motivating students to learn, prompting students to find similarities in two works and formulas, helping students easily remember new formulas.

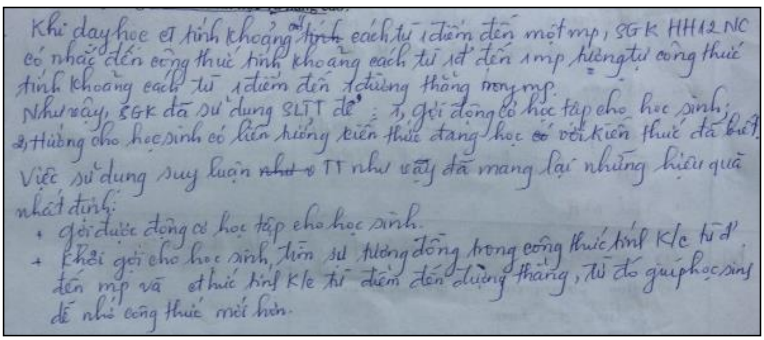

Khi dạy học ct tính khoảng cách từ 1 điểm đến một mp, SGK HH12 NC có nhắc đến công thức tính khoảng cách từ 1 đ đến 1 mp tương tự công thức tính khoảng cách từ 1 điểm đến 1 đường thẳng trong mp.
Như vậy, SGK đã sử dụng SLTT để: 1, gợi động cơ học tập cho học sinh; 2, Hướng cho học sinh có liên hướng kiến thức đang học với kiến thức đã biết.
Việc sử dụng suy luận TT như vậy đã mang lại những hiệu quả nhất định:
+ gợi được động cơ học tập cho học sinh.
+ Khơi gợi cho học sinh thấy sự tương đồng trong công thức tính K/c từ đ đến mp và cthức tính K/c từ điểm đến đường thẳng, từ đó giúp học sinh dễ nhớ công thức mới hơn.

Figure 5. Analysis of the formula for calculating the distance from a point to a plane in the textbooks of group 4CH

Based on these analyses, the master students designed a lesson plan for this formula (see Figure 5). After asking the question whether the formula for calculating the distance from a point to a plane was similar to the formula for calculating the distance from a point to a line in grade 10, the teacher asked students to repeat learned knowledge, such as formulas, ways building a formula for calculating the distance from a point to a line, how to determine the distance from a point to a plane in class 11. From analyzing the characteristics of the source, the teacher gave the problem of building a formula for calculating the distance from point M to the plane *(P)* and instructions, suggestions for students, then, ask students to verify and state the formula. Finally, the teacher stated the theorem and gave examples to apply. However, here the group had not given any specific examples.

3.5. Topic 5: Formula to calculate the general term of the exponent

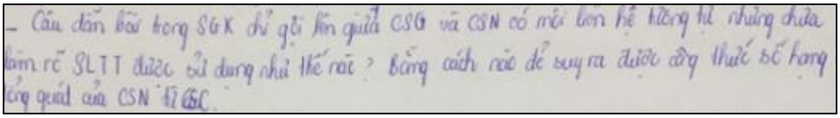
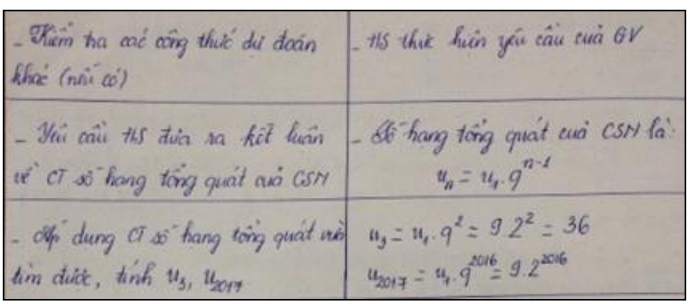

- Câu dẫn bài trong SGK chỉ gợi lên giữa CSC và CSN có mối liên hệ tương tự nhưng chưa làm rõ SLTT được sử dụng như thế nào? Bằng cách nào để suy ra được công thức số hạng tổng quát của CSN từ CSC.

Figure 6. Analysis of the formula for calculating the general term of the geometric sequence in the textbooks of group 5CH

The 5CH group analyzed using analogous reasoning in the Advanced Algebra and Calculus 11 textbook (see Figure 6). The group commented: "The quote in the textbook that evokes the work and multiplier has a similar relationship but has not clarified how the analogy is used? How to deduce the general term formula of the exponential. This outcome showed that the textbook authors had used analogous reasoning but only mentioned the source but had not established the correspondence between the source and the target to deduce a new formula. The 5CH group improved this in the lesson design with the following reasoning (see Figure 7):



- Kiểm tra các công thức dự đoán khác (nếu có)	- HS thực hiện yêu cầu của GV
- Yêu cầu HS đưa ra kết luận về CT số hạng tổng quát của CSN	- Số hạng tổng quát của CSN là: $u_n = u_1.q^{n-1}$
- Áp dụng CT số hạng tổng quát vừa tìm được, tính u_3, u_{2017}	$u_3 = u_1.q^2 = 9.2^2 = 36$ $u_{2017} = u_1.q^{2016} = 9.2^{2016}$

Figure 7. Design lesson to teach the formula to calculate the general term of the geometric sequence of group 5CH

Because in the problem that when *n* was too large, the search would take time and complexity, leading teachers to find a general formula. Next, the teacher asked the students to restate the formula for calculating the general term of the geometric sequence and predict the formula for the general term of the exponent. The requirement to check the correctness and prove the formulas just predicted helps students avoid making errors when using analogous reasoning.

3.6. Topic 6: The way of solving the equation cos*x* = a

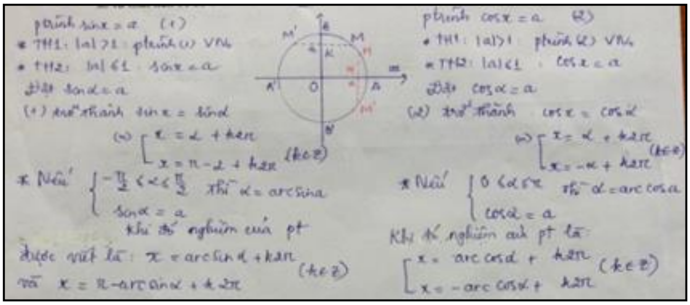
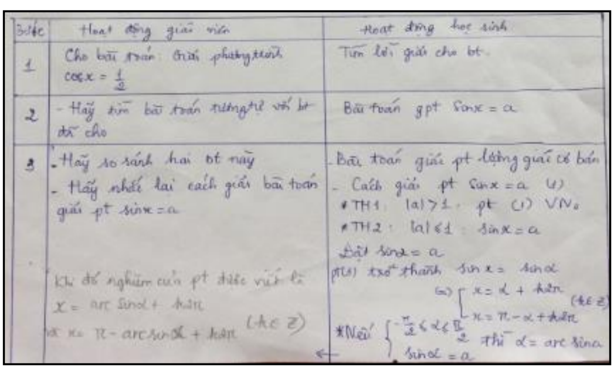


Figure 8. Analyzing how to solve PT cosx=a in textbooks of group 6CH

When analyzing how to solve the equation cosx=a (see Figure 8), the 6CH group listed the similarities and differences between the equations sinx=a and cosx=a. This comparison created favorable conditions for designing lessons with analogous reasoning. However, the 6CH group had not made any comments or comments on the presentation of analogous reasoning in textbooks, such as: How did the textbook use similar reasoning and used for what purpose? What are the advantages and disadvantages of the presentation?



Bước	Hoạt động giáo viên	Hoạt động học sinh
1	Cho bài toán: Giải phương trình cosx = 1/2	Tìm lời giải cho bt.
2	- Hãy tìm bài toán tương tự với bt đã cho	Bài toán gpt sinx = a
3	- Hãy so sánh hai bt này - Hãy nhắc lại cách giải bài toán giải pt sinx = a Khi đó nghiệm của pt được viết là x = arc sinα + k2π và x = π - arcsinα + k2π (k ∈ Z)	- Bài toán giải pt lượng giác cơ bản - Cách giải pt sinx = a (1) * TH1: |a| > 1: pt (1) VN. * TH2: |a| ≤ 1: sinx = a Đặt sinα = a pt(1) trở thành sinx = sinα ⇔ x = α + k2π; x = π - α + k2π (k ∈ Z) *Nếu -π/2 ≤ α ≤ π/2, sinα = a thì α = arc sina

Figure 9. Design lesson on how to solve the equation cosx=a of group 5CH

In the lesson design teaching how to solve the equation cosx=a, group 6CH gave the opening problem of solving the equation $\cos x = \frac{1}{2}$ that evoked the motivation (see Figure 9). From there, the teacher asked the students to find a similar problem that was sinx=a. The teacher motivated students by analyzing the similarities of the two problems and how to solve the source problem. Next, the teacher suggested that students inferred a similar solution for the target problem and presented a specific solution for solving the equation $\cos x = \frac{1}{2}$. Here, in our opinion, the 6CH group needed more specific instructions on how to deduce the solution of the cosx=a equation from the solution of the sinx=a equation analyzed above.

3.7. Topic 7: The way of solving the quadratic equations with complex coefficients

With the topic of solving quadratic equations with complex coefficients, the 7CH group confirmed that the same source that the textbook used was a quadratic equation with real coefficients (see Figure 10). The purpose of using an analogy here was to help students recognize the relationship between known and new knowledge and point out the difference when solving quadratic equations with real and complex coefficients. Furthermore, the 7CH group also proposed an improvement: they should add an activity that repeats how to solve quadratic equations with real coefficients.

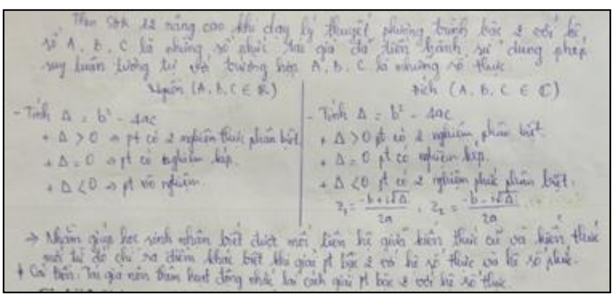


Figure 10. Analyzing how to solve quadratic equations of complex coefficients in textbooks of group 7CH

Based on the above analysis, the 7CH group designed this lesson with the following reasoning (see Figure 10): Ask students to repeat how to solve quadratic equations with real coefficients and solve exercises with three specific quadratic equations (real coefficients). Teachers guided students to predict the process of solving quadratic equations in complex numbers: in case $\Delta < 0$ quadratic equations had two complex solutions. However, this had not been clearly explained but only confirmed by teachers. In our opinion, this was the limitation of this design.

4. CONCLUSION

Thus, from the consideration of analogous reasoning used in textbooks, master students and pre-service math teachers clarified the similar relationship between objects, activities that could be used and limitations that needed improvement. From here, they designed lessons using analogous reasoning in the direction of promoting the activeness of students. Besides textbook analysis and lesson plan design, researchers can guide teachers in predicting and designing scenarios to correct some mistakes made by students when using analogy inference. Moreover, teachers can also make similar deductions to help students systematize knowledge.

From the results of this study, several recommendations are made. In order to contribute to promoting students' activeness in learning mathematics, teachers can analyze and improve activities using analogous reasoning in textbooks, thereby applying them to teaching to explore concepts, define, explain math problems and help students correct mistakes using analogous reasoning. Specifically, teachers can introduce new knowledge as an extension of source knowledge. Also, they introduce similar concepts, formulas and properties. Additionally, teachers can also generalize from known knowledge, help present knowledge more concisely, design problem situations to help students make predictions about new knowledge.

On the student's part, they need to realize that using an analogy in learning mathematics is helping them infer new knowledge based on what they already know. Thereby, students can review old knowledge, understand the relationship between knowledge.

This study deploys the theory and method of using analogous reasoning in teaching in training pre-service teachers majoring in mathematics pedagogy and master students majoring in theory and teaching mathematics methods. From theoretical knowledge about analogous inference, students and students can apply analogous reasoning in teaching practice to promote students' activeness and conduct scientific research to apply analogies in teaching. Also, the study can put the theory of analogy inference in teaching and implementing it at high schools, training high school math teachers and training students in mathematics education at high schools and other pedagogical schools.

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