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Astrophysical Signatures of Scalar-Photon Interaction

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Abstract:

Dimensions-five photon (γ) scalar (φ) interaction term usually appear in the Lagrangians of bosonic sector of unified theories of electromagnetism and gravity. This interaction makes the medium dichoric and induces optical activity. Considering a toy model of a rotation powered, ultra-cold magnetized compact star [White Dwarf (WD) or Neutron Star (NS)], we have modeled the propagation of very low energy photons with such interaction, in the environment of these stars. Assuming synchro-curvature process as the dominant mechanism of emission in such environments, we have tried to understand the polarimetric implications of photon scalar coupling on the produced spectrum of the same. Furthermore assuming the emission energy vs emission –altitude relation, that is believed to hold in such (i.e. rotation powered cold magnetized WD or NS) environments. We have tried to point out the possible modifications to

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the radiation spectrum when the same is incorporated along with dimension-five photon-scalar mixing operator.

Key words: Dimension-five Scalar-photon interaction, stokes parameters, compact stars.

1 Introduction

Scalar photon interaction through dim-5 operators, originates in many theories beyond standard model of particle physics, usually in the unified theories of electromagnetism and gravity, with extra dimensions. The scalars involved can be moduli fields of string theory, KK particle from extra dimension, scalar component of the gravitational multiplet in extended supergravity models etc.,[(Schmutzer 1983), (Scherk 1979), (Bergmann 1968), (Gasperini 1984), (Roll et al. 1964), (Braginsky & Panov 1972), (Gibbons and Whiting1981) and (Fishbach et. al 1986)], to name a few. An extensive review on this topic can be found in (Duff, Nilsson & Pope 1986). The emphasis of such models had been unification of forces. Similar interaction has been invoked to solved the dark matter and dark energy [(Komatsu et al. 2009) and (Dunkley et al. 2009)] problems of the universe, in the chameleon models for dark matter [(Brax et al. 2004), (Khoury and Weltman 2004a), (Khoury and Weltman 2004b), (Brax, Van de Bruck & Davis 2007) and (Davis, Schelpe & Shaw 2009)].

Scalar photon γ-φ interaction, in theories with extra dimension, originate through compactification of the extra-dimension. Usually the physics of compactification of extra dimensions, introduces various kinds of model dependent interactions. The original Kaluza Klein (KK) model [(Kaluza 1921), (Klein 1926) & (Lee 1984)] constructed to unify gravity with electromagnetism, introduced dimension-five, γ-φ interaction, through compactification of the extra fifth dimension.

Interestingly enough, the dimension-five photon-scalar interaction term generated in the Lagrangian [(Kaluza 1921), (Klein 1926)] by compactification of the extra dimension, would eventually bear optical signature of the same. The purpose of this note is to study such effects coming from dimension-five scalar photon interaction in some detail and point out their possible astrophysical consequences.

Usually dimension-five interactions (be it scalar or pseudoscalar (axion) photon interactions), induce optical activity. The scalar or pseudoscalar photon interaction turns vacuum into birefringent and dichoric one [(Raffelt & Stodolosky 1988), (Miani, Petronzio & Zavattini 1986), (Gasperini 1987) and (Ganguly & Parthasarathy 2003)]. As a result, the plane of polarization of a plane polarized beam of light keeps on rotating as passes through vacuum. This particular aspect of the theory, for pseudoscalar photon interaction, has been extensively in the literature in cosmological and astrophysics contexts, in [(Giovannini & Kunze 2009), (Giovannini 2005), (Agarwal et al., 2008), (Das et al. 2005), (Hutsem kers & Lamy 2001),(Hutsem kers 2005),(Hutsem kers 1998), (Jain, Panda & Sarla 2002),(Jain, Narain & Sarala 2004),(Agarwal, Kamal & Jain 2011),(Agarwal et al. 2012) and (Payez et al. 2011.)].

Keeping this in view, in this note, initially, we have studied the optical signature of dimension-five, $\varphi F_{\mu\nu} F^{\mu\nu}$ interaction in an ambient magnetic field of strength of ~10^{13} Gauss. Following this, we have considered a toy model for the stellar environment of a strongly magnetized, rotation-powered, ultra cold, compact astrophysical object like White Dwarf (WD) or Neutron Star (NS) to understand some issue related to – emission energy and altitude (Gil & Kijak 1993), that is believed to affect the produced spectra of electromagnetic radiations (EM), originating there. Next, we have made an effort to explore the potential of the later, in modifying the usual spectral signatures of the tree-level dimension-five, $\varphi F_{\mu\nu} F^{\mu\nu}$ coupling from such astrophysical situations. We would like to mention here, that, this model of ours is a toy model. The purpose of this construction is, to motivate further investigations of the possible modifications to the emission spectra, from a realistic WD or NS environment, with $\varphi F_{\mu\nu}F^{\mu\nu}$ interaction.

While investigating the effect of $\varphi F_{\mu\nu}F^{\mu\nu}$ interaction in this note, we have been able to achieve three objectives: (i) verifying the earlier results (Ganguly & Parthasarathy 2003) through an independent approach, (ii) Showing the possibility of generation of circular and elliptic polarization from a plane polarized light beam–as the same passes through the stellar environment. (iii) Pointing out the possibility of some extra modifications to the usual polarimetric signatures of $\varphi F_{\mu\nu}F^{\mu\nu}$ coupling, due to emission altitude vs energy relation, that has been discussed in the literature (Gil & Kijak 1993).

It was realized for the first time in (Ganguly & Parthasarthy 2003) that existence of superluminal propagation modes for low frequency photons are possible, in a mode with dimension-5 $\varphi F_{\mu\nu}F^{\mu\nu}$ interaction term present in the Lagrangian. The analysis there was performed in terms of the gauge potentials, using Lorentz gauge, leaving a scope of *gauge ambiguities* as a source of the problem. In order to rule out any role of the same, for the afore mentioned problem, we have taken a different approach here. We have used the field strength tensors and Bianchi identity for deriving the same. Our new approach reproduces the results obtained earlier.

To shed light on the second issue, we have assumed the radiation beam, produced at the production point, to be plane polarized electromagnetic wave (with orthogonal planes of polarization, as is the case for synchro-curvature radiation). What we find is: *the same generates a significant amount of scalar component through $\varphi F_{\mu\nu}F^{\mu\nu}$ interaction, well before it is out of the stellar atmosphere:* this is something that needs to be considered when one is looking for signatures of dim-5 mixing operators from astrophysical polarimetric data. Also, though the initial beam of radiation is plane polarized, however during its passage, it picks up significant amount of elliptic/circular polarization through mixing. The amount of elliptic/circular polarization generated through mixing is energy (ω) dependent (having a complex dependence on ω), along with the strength of the magnetic field B, coupling constant $g_{\gamma\gamma\varphi}$ as well as the distance z traveled in the stellar atmosphere. During our analysis, we have also looked into the pattern of polarization angle ψ and ellipticity χ that the radiation beam generates at different energies (ω), after propagation through the same distance. What is very interesting, is the existence of identical polarization angle ψ and ellipticity angle χ for various values of energy (ω), when the traversing path remains same. The details of the same are discussed later.

The organization of the document is as follows, in section-II, we have derived the equations of motions. Section III is dedicated to the determination of the dispersion relations and the solutions of the equations of motions. In section IV we discuss about the possible observables and applications, including a brief introduction stokes parameters. Introduction to the physics of magnetized astrophysical compact objects and ideas behind energy vs emission altitude mapping [(Gil & Kijak 1993), (Gangadhara and Gupta 2001) & (D' Angelo & Rafikov 2007)] is presented in section V. Results are presented in section VI. Lastly we conclude, by pointing out the relevance of our analysis, in realistic astrophysical or cosmological contexts.

2 From the Action to the Equations of Motion

To bring out the essential features of $\varphi F_{\mu\nu} F^{\mu\nu}$ coupling term on the dynamics of the system, we would work in flat four dimensional space time. The action for this coupled scalar photon system, in flat four dimensional space time is given by

$$S = \int d^4x \left[\frac{1}{2}\left(\partial_\mu\varphi\, \partial^\mu\varphi\right) - \frac{g_{\varphi\gamma\gamma}}{4}\varphi F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]. \qquad \textbf{(1)}$$

Here $g_{\varphi\gamma\gamma}$ is is the effective coupling strength between scalar and electromagnetic field. The equations of motion can be obtained by varying the action with respect to φ and $F_{\mu\nu}$ and demanding invariance of the same under this arbitrary variation, the result is:

$$\partial_\mu\left[F^{\mu\nu} + g_{\varphi\gamma\gamma}\varphi F^{\mu\nu}\right] = 0, \qquad (2)$$
$$\partial_\mu\partial^\mu\varphi + \frac{1}{4}\, g_{\varphi\gamma\gamma}F_{\alpha\beta}F^{\alpha\beta} = 0. \qquad (3)$$

As a next step we decompose the EM field into two parts, the mean field ($F_{\mu\nu}$) and the infinitesimal fluctuation ($f_{\mu\nu}$), i.e.,

$$F_{\mu\nu} = F_{\mu\nu} + f_{\mu\nu}. \qquad (4)$$

Assuming the magnitude of the scalar field to be of the order of the fluctuating electromagnetic field $f_{\mu\nu}$, one can linearize the equations in [2]. The equations of motion for the scalar field and the electromagnetic fields turn out to be

$$\partial_\mu f^{\mu\nu} = -g_{\varphi\gamma\gamma}\partial_\mu\varphi\bar{F}^{\mu\nu}, \qquad (5)$$
$$\partial_\mu\partial^\mu\varphi = -\frac{1}{2}\, g_{\varphi\gamma\gamma}\bar{F}^{\mu\nu}f_{\mu\nu} = 0. \qquad (6)$$

The two equations [5] and [6] are the two equations of motion governing the dynamics of the system in an external magnetic field described by the tensor $\bar{F}^{\mu\nu}$. Here we consider $\bar{F}^{\mu\nu}$ to be a very slowly varying function of coordinates, so that the same can be considered to be effectively constant. We note here that these two equations carry the information about the three degrees of freedom of the system, two for the two polarization states of the photon and the third one for the scalar degree of freedom.

3 Dispersion Relations

In this section we would get the dispersion relation for the scalar photon coupled system of equations. Equation [5] in general would provide three equations corresponding to two transverse and one longitudinal states of polarization for the photon. However in vacuum photons has only two transverse degrees of freedom, so one of the three would be redundant. The last equation i.e. [6] would provide the dynamics of scalar degree of freedom, thus making the total number of degrees of freedom for the coupled system to be three.

We can get to the dynamics of the three degrees of freedom for the scalar photon system by two methods, (a) by using the gauge potentials and choosing a particular gauge or (b) by making use of Bianchi Identity, In this work we would chose the second method that is use of Bianchi Identity:

$$\partial_\mu f_{\nu\lambda} + \partial_\nu f_{\lambda\mu} + \partial_\lambda f_{\mu\nu} = 0. \qquad (7)$$

If we now multiply the Bianchi Identity by $\bar{F}^{\nu\lambda}$ and operate with ∂_μ, we arrive at the identity

$$\partial_\mu \partial^\mu \left(f_{\lambda\rho} \bar{F}^{\lambda\rho}\right) = -2\partial^\lambda \partial_\mu \left(f^{\mu\rho} \bar{F}_{\lambda\rho}\right). \qquad (8)$$

Now one can take, eqn.[5] and multiply by $F_{\mu\nu}$ and operate with ∂^λ, and subsequently use eqn.[8] to get to

$$\partial_\mu \partial^\mu \left(\frac{f\bar{F}}{2}\right) = g_{\varphi\gamma\gamma} \partial^\lambda \partial_\alpha \varphi\left(\bar{F}^{\alpha\nu} \bar{F}_{\nu\lambda}\right). \qquad (9)$$

Next one may introduce a new variable, $\psi = \frac{f^{\mu\nu}\bar{F}_{\mu\nu}}{2}$, use it in eqn. [9], and go to the momentum space. The resulting equation in momentum space is

$$k^2\psi = g_{\varphi\gamma\gamma}\left(k_\alpha \bar{F}^{\alpha\nu}\bar{F}_{\lambda\nu}k^\lambda\right)\varphi\,. \qquad (10)$$

Similarly, defining $\tilde{\psi} = \frac{f^{\mu\nu}\tilde{\bar{F}}_{\mu\nu}}{2}$, and the same procedure, one arrives at the equation for $\tilde{\psi}$. The same turns out to be

$$k^2\tilde{\psi} = 0. \qquad (11)$$

Finally the equation of motion for the scalar field in momentum space is given by

$$k^2\varphi = g_{\varphi\gamma\gamma}\psi. \qquad (12)$$

Assuming, $\bar{F}^{12} \neq 0$ therefore $\tilde{\bar{F}}^{03} \neq 0$ and denoting $\bar{F}^{12} = B$, we may use the following compact representations for various Lorentz scalars e.g., $\left(\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}\right) = 2B^2$, $\left(k_\alpha \bar{F}^{\alpha\nu}\bar{F}_{\lambda\nu}k^\lambda\right) = k_\perp^2 B^2$, $\bar{F}^{\mu\nu}\tilde{\bar{F}}_{\mu\nu} = 0$ and $\left(k_\alpha \tilde{\bar{F}}^{\alpha\nu}\tilde{\bar{F}}_{\nu\lambda}k^\lambda\right) = (k^2 + k_\perp^2)B^2$, appearing in the equations of motion. In these expressions $k_\perp$ is the component of $\vec{K}$ that is orthogonal to B and Θ is the angle between the magnetic field B and the propagation direction $\vec{K}$. In terms of these we can rewrite the following expressions as

$$\begin{aligned} k_\perp^2 B^2 &= K^2 Sin^2(\theta)B^2 \approx \omega^2 Sin^2(\theta)B^2, \\ (k^2 + k_\perp^2)B^2 &= \left(\omega^2 - K^2 Cos^2(\theta)\right)B^2, \\ &\approx \omega^2 Sin^2(\theta)B^2. \end{aligned} \qquad (13)$$

While deriving the expression in eqn. [13], we have assume that to order $g_{\varphi\gamma\gamma}$, $\omega \approx K$. Using these relations (i.e. those in eqn.[13]), the equations of motion for the combined photon and scalar system can be written, in matrix form, as:

$$\begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & -g_{\gamma\gamma\varphi}(\omega B Sin\Theta)^2 \\ 0 & -g_{\gamma\gamma\varphi} & k^2 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \psi \\ \varphi \end{bmatrix} = 0 \qquad (14)$$

The matrix eqn. [14] does not does not look symmetric because the dimension of φ and the dimensions of ψ or $\tilde{\psi}$ are different. To bring the same in symmetric form, we multiply the φ equation in eqn.[12] by *ωSin(θ)B,* and redefine φ by Φ = *ωSin(θ)Bφ*, to arrive at

$$\begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & -g_{\gamma\gamma\varphi}(\omega B Sin\Theta) \\ 0 & -g_{\gamma\gamma\varphi}(\omega B Sin\Theta) & k^2 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \psi \\ \Phi \end{bmatrix} = 0 \quad (15)$$

3.1 Decoupling of the Dispersion Relations.

As can be seen from eqn.[15], that because of the presence of off diagonal elements, only two dynamical degrees of freedom, out of the three (ψ, $\tilde{\psi}$ and Φ), during their propagation mix with each other. The matrix in eqn. [15] is real symmetric, so we can go to a diagonal basis, by an orthogonal transformation to diagonalize the same. The orthogonal transformation matrix is given by

$$O = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{pmatrix}, \qquad (16)$$

where $\theta = \frac{\pi}{4}$, for the case in hand.

On diagonalizing eqn.[15], we arrive at:

$$\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 - g_{\gamma\gamma\varphi}(\omega B Sin\Theta) & 0 \\ 0 & 0 & k^2 - g_{\gamma\gamma\varphi}(\omega B Sin\Theta) \end{pmatrix}$$

$$\times \begin{pmatrix} \tilde{\psi} \\ \frac{\Phi-\Psi}{\sqrt{2}} \\ \frac{\Phi+\psi}{\sqrt{2}} \end{pmatrix} = 0 \qquad (17)$$

It is easy to see from the equation above that, $\tilde{\psi}, \frac{\Phi+\Psi}{\sqrt{2}}$ and $\frac{\Phi-\Psi}{\sqrt{2}}$ satisfies the following dispersion relations:

$$\omega = K \qquad (18)$$

$$\omega_+ = \pm \sqrt{K^2 + g_{\gamma\gamma\varphi}(\omega B Sin\Theta)} \qquad (19)$$

$$\omega_- = \pm \sqrt{K^2 - g_{\gamma\gamma\varphi}(\omega B Sin\Theta)} \qquad (20)$$

Where, the quantity $g_{\varphi\gamma\gamma} B Sin\theta\omega$ depends on the strength of the external electromagnetic field, scalar photon coupling constant $g_{\varphi\gamma\gamma}$, ω and the sine of the angle Θ between the direction of propagation $\vec{K}$ and the magnetic field B.

It should be noted that equation [15] or [17], incorporates all the dispersive features of a photon propagating in a magnetized vacuum with dimension-5 scalar photon interaction. One can verify that the dispersion relations obtained from [15] or [17] are identical to those in [(Miani, Petronzio & Zavattini 1986), (Gasperini 1987) and (Ganguly & Parthasarathy 2003)], provided appropriate limits are taken.

It's worth noting that eqn. [15] or [17], actually shows that, in the case of scalar photon interaction, photons with polarization state ⊥ to the magnetic field B remains unaffected and propagates with speed of light and the same with polarization state ∥ to the magnetic field couples to the scalar and undergoes modulation It can be demonstrated, that there exists a critical energy (ω_c), below which the ∥ mode have imaginary ω, hence they would be non- propagating. However the ⊥ mode doesn't suffer from this pathological problem hence (for energy below ω_c) they would propagate freely. Therefore light coming from distant sources with $\omega < \omega_c$ would appear to be linearly polarized provided this kind of interaction does exist in nature.

We emphasize here, that, much of our analysis performed in this paper would have remained the same even, if we had dim-5 pseudoscalar photon interaction; thus making it difficult to identify the type of interaction responsible for polarimetric observation, that's being invoked in this note.

However, the way out is, to note that the parallel and perpendicularly polarized components of the photon with these two different kind of interactions, (*scalar or pseudoscalar*) interchange their role in presence of an external magnetic field B. Hence the polarization state of the linearly polarized light for scalar photon interaction (with $\omega < \omega_c$) would be orthogonal to the same with Axion-photon system. Therefore in principle, one can look for this signature in polarimetric observations, to point out the kind of interaction responsible, for the observed signal.

3.2 Inhomegeneous Wave Equations

The solutions for the dynamical degrees of freedom in coordinate space can be written as

$$\begin{pmatrix} \tilde{\psi} \\ \cos\theta\ \psi + \sin\theta\ \Phi \\ -\sin\theta\ \psi + \cos\theta\ \Phi \end{pmatrix} = \begin{pmatrix} A_0 e^{i(\omega t - k.x)} \\ A_1 e^{i(\omega_+ t - k.x)} \\ A_2 e^{i(\omega_- t - k.x)} \end{pmatrix} \quad (21)$$

The constants A_0, A_1 and A_2 has to be defined from the boundary conditions one imposes on the dynamical degrees of freedom. It can be verified from eqn. [21], that the solution for the dynamical variables turn out to be

$$\tilde{\psi}\,(x,t) = A_0 e^{i(\omega t - k.x)}$$
$$\psi\,(x,\,t) = \cos\theta e^{i(\omega_+ t - k.x)} - A_2 \sin\theta e^{i(\omega_- t - k.x)} \quad (22)$$

$$\Phi(x,t) = A_1 sin\theta e^{i(\omega_+ t - kx)} + A_2 cos\theta e^{i(\omega_- t - kx)}$$

In the following we consider the following boundary conditions, $\Phi(0,0) = 0$ and $\psi(0,0) = 1$. With this boundary condition we have, $\frac{A_2}{sin\theta}$ = -1. Where the angle is $\theta = \frac{\pi}{4}$ as has already been stated before. With these conditions the solution for ψ turns out to be

$$\Psi(x,t) = [cos^2\theta\, e^{i(\omega_+ t - kx)} + sin^2\theta\, e^{i(\omega_- t - kx)}] \quad (23)$$

Defining, $a_x^2(t) = (Re[\psi(t,0)])^2 + (Im[\psi(t,0)])^2$, we get the following form for $\Psi(x, t)$,

$$\psi(t,x) = a_x(t) e^{i\left(tan^{-1}\left[\frac{cos^2\theta sin\omega_+ t + sin^2\theta sin\omega_- t}{cos^2\theta cos\omega_+ t + sin^2\theta cos\omega_- t}\right] - kx\right)} \quad (24)$$

A wave equation of this type is usually called inhomogeneous wave equation. The phase velocity for such system, where the solution is represented by $a(t)e^{i\varphi(t) - kx}$ is defined by (Born & Wolf 1980),

$$v_p = \frac{1}{K}\frac{\partial\varphi(t)}{\partial t} \quad (25)$$

For the case under consideration, since $\theta = \frac{\pi}{4}$, the expression for the phase velocity can be evaluated exactly. It should be noted however, that, the same with nonzero scalar mass and/or other interactions present in the Lagrangian, may lead to a more complex situation and an exact analytical result may not be possible. We won't be elaborating on this issue any further (here), the same would be dealt with in a separate publication.

4 Application

The predictions for the class of theories under consideration, in this note, can be tested through optics based experiments, set up for laboratory or astrophysical environments. For instance, through the observations of the index of the power spectrum of the radiation beam checking the differential dispersion measure or through the measurements of polarization angle, ellipticity at different energies.

The analysis in this paper, are seemingly more suitable for dispersive and/or polarimetric measurements for verifying the predictions of these theories. Moreover, since we are more interested to find out the astrophysical implications of the theory being studied and optics based experiments are more suitable for the same; therefore, we will concentrate on dispersive or polarimetric analysis here.

4.1 Observables from non-thermal radiation

In this section we would apply our results for astrophysical situations. They are of interest because the ambient magnetic field available there, are many orders of magnitude more, than the same available in laboratory conditions; also the length of the path the light beam traverses is enormous. These are the main motivations to consider astrophysical situations. In astrophysical situations, most of the interesting emission mechanisms are non-thermal in nature. As the charged particles in these situations accelerate in the ambient electromagnetic field, they radiate Electro Magnetic (EM) radiations.

4.2 Polarized spectrum

The Electro Magnetic (EM) radiation, that the charged particles in such environments emit, are plane polarized in nature where the planes of polarization are mutually orthogonal to each other. The amplitudes of the polarized radiations coming from synchrotron or curvature ture radiations had been discussed in [(Ginzburg & Syrovatsky 1965), (Rybicki & Lightman 1979) and (Mena, Razzaque, S. & Villaescusa-Navarro 2011)]. It had been shown in these papers, that, the amplitudes of the emitted radiation (synchrotron or curvature) corresponding to $A_{\|}$ or $A_{\perp}$ states (w.r.t the $\hat{k}$-$\hat{B}$ plane), are given by,:

$$A_{\perp} \propto K_{\frac{1}{3}}\left(\frac{\omega}{2\omega_{sc}}\right)$$
$$A_{\|} \propto K_{\frac{2}{3}}\left(\frac{\omega}{2\omega_{sc}}\right) \qquad (26)$$

In eqn. [26], $\omega_{sc} = \frac{3}{2}\frac{\Gamma^3}{\rho}$ is the peak energy for radiation spectrum with, Γ, the Lorentz boost factor for the emitting particles and ρ the radius of curvature of the particle trajectory. The differential intensity spectrum, given by

$$\frac{d^2 I}{d\omega d\Omega} = \frac{(e\omega)^2}{4\pi^2}\left(\left|A_{\|}\right|^2 + \left|A_{\perp}\right|^2\right), \qquad (27)$$

grows as $\omega^{\frac{2}{3}}$ for $\omega \ll \omega_{sc}$, and falls off exponentially for $\omega \gg \omega_{sc}$. Equation [26] indicates the behavior of the amplitudes of the two orthogonally polarized states as available from the spectrum of synchrotron or curvature radiations.

4.3 Dispersive Measures

In astrophysical situations, dispersive and polarimetric measures are very effective to extract information about a system. For an object at a distance D we can measure the time taken by the signal to reach us by measuring $\frac{D}{v}$. That is, by measuring $t_m = \frac{D}{v}$ (carroll, Field & Jackiw 1990).

In cosmological situations red shift z can be, converted to proper distance using (for Ω =1), $D = \frac{2}{3H_0}\left(1 - \frac{1}{(1+z)^{\frac{3}{2}}}\right)$ with H_0 the Hubble constant being related to h, by $h = \frac{H_0}{100\ Sec\ M_{pc}}$ km and $h = 0.72 \pm 0.05$ obtained from the WMAP data (Spergel et. al.

2003). Since we are using natural units, we can take t = D. This in principle can be performed for gamma-ray bursters or pulsar observations. The dispersion measure of time, estimated with high energy data samples, are expected to probe new physics induced modifications to the dispersive effects, more accurately than the low energy ones. Chief reason behind this being the fact that, propagation medium (intestellar or intergalactic) induced dispersion, affects the signals, in the low energy domain most.

It has already been mentioned before, that, for the kind of interaction under consideration, here, the vacuume turns out to be birefringent and dichoric (for the photons). So the two polarized modes of photon, would propagate with different speed. So, in principle, the orthogonally polarized signals from an astrophysical object, that is originating from the source at the same space time point, would reach the observer at two different epochs. A similar argument taking into account the effect of stellar magnetized plasma was initially put forward in [(Barnard and Arons 1986), (Mckinnon 1997), (Gallant 1998) and (Hoenbroech Lesch & Kunzl 1998)]. However, this argument has been discussed unfavorably in (Kramer 1999), and probably needs more refinement e.g., taking into account the kind of effect being discussed here and the effect of magnetized inter-galactic domains, etc. Detailed analysis of this effect is beyond the scope of this paper and would be performed elsewhere, using the techniques of [(Ganguly jain & Mandal 2009), (Agarwal et al., 2008) and (Das et.al 2005)].

4.4 Polarimetric Measure

Most of the astrophysical environments are associated with magnetic field, with strengths varying from 10^{-9} to 10^{13} Gauss. Since synchrotron or curvature radiation are some of the most efficient non-thermal radiative mechanisms, the astrophysical objects mostly radiate non-thermally via this processes. A characteristic signature of the radiation coming through this process is, the radiation is polarized along and perpendicular to the $\hat{k}$-$\hat{B}$ plane, where k is the direction vector from the source to the observer and $\hat{B}$ is the ambient magnetic field direction. The behavior of the amplitude and the spectrum of the radiation with respect to have already been discussed in subsection [4.2].

The intrinsically polarized nature of the produced radiations from compact astrophysical sources, turns out to be more useful to perform polarimetric analysis of the data from there. The observables for polarimetric analysis are degrees of polarization linear (Π_L) or circular (Π_c) and total polarization (Π_T) (Born & Wolf 1980). In view of this we would take a digression to the essentials of stokes parameters before estimating the polarimetric observables.

4.4 Digression of stokes parameters

In order to evaluate the polarimetric variables (Stokes parameters), one can construct the coherency matrix by taking different correlations of the vector potentials or the fields (Born & Wolf 1980). Various optical parameters of interest like polarization, ellipticity and degree of polarization of a given light beam, can be found from the components of the coherency matrix constructed from the correlation functions stated above (Ganguly 2012).

For a little digression, the coherency matrix, for a system with two degree of freedom is defined as an ensemble average (where the averaging is done over many energy bands) of direct product of two vectors:

$$\rho(z) = \langle \begin{pmatrix} \tilde{\psi}(z) \\ \psi(z) \end{pmatrix} \otimes \left(\tilde{\psi}(z)\psi(z)\right)^{*} \rangle$$
$$= \begin{pmatrix} \langle\tilde{\psi}(z)\tilde{\psi}^{*}(z)\rangle & \langle\tilde{\psi}(z)\psi^{*}(z)\rangle \\ \langle\psi(z)\tilde{\psi}^{*}(z)\rangle & \langle\psi(z)\psi^{*}(z)\rangle \end{pmatrix} \quad (28)$$

One important thing to note here, is, under any anti-clockwise rotation by an angle α about an axis i.e., perpendicular to the vectors $\tilde{\psi}$ and ψ, the coherency matrix would transform as

$$\rho(z) \rightarrow \rho'(z) = \langle R(\alpha) \begin{pmatrix} \tilde{\psi}(z) \\ \psi(z) \end{pmatrix} \otimes \left(\tilde{\psi}(z)\psi(z)\right)^{*} R^{-1}(\alpha) \rangle, \quad (29)$$

where R(α) is the rotation matrix. Now from the relations between the components of the coherency matrix and the stokes parameters:

$$\begin{aligned} I &= \langle \tilde{\psi}(z)\tilde{\psi}^{*}(z)\rangle + \langle \psi^{*}(z)\psi(z)\rangle, \\ Q &= \langle \tilde{\psi}(z)\tilde{\psi}^{*}(z)\rangle - \langle \psi^{*}(z)\psi(z)\rangle, \\ U &= 2\mathrm{Re}\, \langle \tilde{\psi}^{*}(z)\psi(z)\rangle, \\ V &= 2\mathrm{Im}\, \langle \tilde{\psi}^{*}(z)\psi(z)\rangle. \end{aligned} \quad (30)$$

It is easy to establish that:

$$\rho(z) = \frac{1}{2}\begin{pmatrix} I(z)+Q(z) & U(z)-iV(z) \\ U(z)+iV(z) & I(z)-Q(z) \end{pmatrix} \quad (31)$$

Therefore, under an anti-clockwise rotation by an angle α, about an axis perpendicular to the plane containing $\tilde{\psi}(z)$ and $\psi(z)$, the density matrix transforms as: ρ(z) → ρ' (z) ; hence the coherency matrix in the rotated frame would be given by

$$\rho'(z) = \frac{1}{2} R(\alpha) \begin{pmatrix} I(z)+Q(z) & U(z)-iV(z) \\ U(z)+iV(z) & I(z)-Q(z) \end{pmatrix} R^{-1}(\alpha). \quad (32)$$

Since for a rotation by an angle α–in the anticlock wise direction (about the axis that is perpendicular to the plane having $\tilde{\psi}$ and ψ on it) the rotation matrix R(α) is given by

$$R(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}. \quad (33)$$

As a consequence the two stokes parameters Q'(z) and U'(z), in the rotated frame of reference, would get related to the same in the unrotated frame, by the relation:

$$\begin{pmatrix} Q'(z) \\ U'(z) \end{pmatrix} = \begin{pmatrix} cos2\alpha & sin2\alpha \\ sin2\alpha & cos2\alpha \end{pmatrix} \begin{pmatrix} Q(z) \\ U(z) \end{pmatrix} \quad (34)$$

The other two parameters, i.e., I and V remain unaltered. It is for this reason that sometimes I and V are termed invariants under rotation.

We would like to point out here that, in any frame the Stokes parameters are expressed in terms of two angular variables χ and Ψ usually called the ellipticity parameter and polarization angle, defined as

$$\begin{aligned} I &= I_P, \\ Q &= I_P \cos 2\psi \cos 2\chi, \\ U &= I_P \sin 2\psi \cos 2\chi, \\ V &= I_P \sin 2\chi. \end{aligned} \quad (35)$$

The ellipticity angle χ, following [35], can be shown to be equal to

$$\tan 2\chi = \frac{V}{\sqrt{Q^2+U^2}}, \quad (36)$$

and the polarization angle can be shown to be equal to,

$$\tan 2\Psi = \frac{U}{Q}. \quad (37)$$

From the relations given above, it is easy to see that, under the frame rotation

$$R(\alpha) = \begin{pmatrix} cos2\alpha & sin2\alpha \\ -sin2\alpha & cos2\alpha \end{pmatrix}, \quad (38)$$

the tangent of χ, i.e., tanχ remains invariant, however the tangent of the polarization angle gets additional increment by twice the rotation angle, i.e,:

$$\tan(2\chi) \to \tan(2\chi)$$
$$\tan(2\Psi) \to \tan(2\alpha + 2\Psi). \quad (39)$$

It is worth noting that the two angles are not quite independent of each other, in fact they are related to each other. Finally we end the discussion of use of stokes parameters by noting that, the degree of polarization is usually expressed by

$$p = \frac{\sqrt{Q^2+U^2+V^2}}{I_{pT}} \quad (40)$$

Where I_{PT} is the total intensity of the light beam.

Since we already have the expressions for the stokes parameters in terms of the solutions of the field equations [23] one can substitute the solutions of the field equations in [30] to arrive at the expressions for I, Q, U and V. The expressions for the same are given by

$$I(\omega,z) = \frac{1}{2}\,[3 + \cos[(\omega_- - \omega_+)z]], \quad (41)$$

$$Q(\omega,z) = \frac{1}{2}\,[\cos[(\omega_- - \omega_+)z] - 1], \quad (42)$$

$$U(\omega,z) = 2.0\,\left[\cos\left[\left(\frac{\omega_+ - \omega_-}{2}\right)z\right]\right] \times \left[\cos\left[\left(\frac{\omega_+ - \omega_-}{2} - \omega\right)z\right]\right], \quad (43)$$

$$V(\omega,z) = 2.0\,\left[\cos\left[\left(\frac{\omega_+ - \omega_-}{2}\right)z\right]\right] \times \left[\sin\left[\left(\frac{\omega_+ - \omega_-}{2} + \omega\right)z\right]\right]. \quad (44)$$

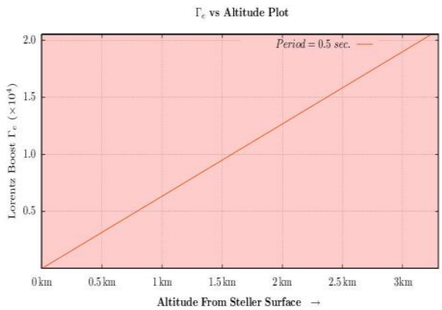

Fig. 1 Plot of Lorentz boost of charged fermions (emitting synchrotron-curvature radiation) versus altitude, for a cold compact star (white dwarf), with dipolar magnetic field strength ~ 10^{13} Gauss and period 0.5 sec.

5 Astrophysical Accelerators

Standard astrophysical accelerators of charged particles in our universe are, white dwarf (WD) pulsar, neutron star (NS) pulsars, supernova remnants (SNRs), to name a few. Their contribution to observed high energy cosmic ray flux has been discussed in, [(Kashiyama K., Ioka, K. and Kawanaka 2011), (Kawanaka,Ioka & Nojiri 2010), (Ho oper, Blasi& Serpico 2009), (Malyshev, Cholis & Gelfand 2009), (Grasso 2009), (Hu et al. 2009),), (Blasi 2009), (Blasi & Serpico 2009), (Mertsch & Sarkar 2009), (Biermann et al. 2009), (Ahlers, Mertsch & Sarkar 2009), (Heinz & Sunyaev 2002), and (Asano et al.2007)] to name a few. As the charged particles accelerate along the dipole magnetic field lines (Radhakrishanan & Cook 1969) of these astrophysical accelerators, they emit synchro-curvature radiation [(Goldreich & Julian 1969), (Ruderman & Sutherland 1975) and (Cheng & Ruderman 1986)], whose spectrum (in familiar notations), peaks at $\omega_{sc} = \frac{3\Gamma^3}{2\rho}$.

For our analysis we would dealing with the emission spectra and polarization studies of compact stars e.g white dwarf pulsar or neutron star pulsars. We will be following the analysis of [(Goldreich & Julian 1969) & (Cheng & Ruderman

1986)] and [(Usov 1993)] in this paper. According to the rotating dipole model (Radhakrishanan & Cooke 1969) of compact stars, a strong electric field of magnitude, $E_{||} = \frac{\vec{E}.\vec{B}}{|B|}$ is generated along the magnetic field in the co-rotating magnetosphere of the pulsar. Charged particles are accelerated to high energies in this field. However, it was noted in (Shabad & Usov 2011), that in similar situations, emitted rays, instead of decaying into $e^{+} - e^{-}$ pairs , may from a bound state, leaving the $E_{||}$ unscreened. This unscreened $E_{||}$ may further accelerate the charged particles to higher energies.

The relation between Γ (Lorentz Boost) and the scalar potential *V* was obtained in (Fawley, Arons & Scharlman 1977), along the lines of the argument presented below. A relativistic charged particle of mass m, if moves through a distance ds, in an electric field ($E_{||}$), then its change of energy (in natural units, h = c = 1), can be written as;

$$d\,(\Gamma m) = \overrightarrow{E_{||}}.\overrightarrow{ds} = -\vec{\nabla}\,(eV).\,\overrightarrow{ds}. \qquad (45)$$

The solution of eqn. [45], provides the expression of Γ as a function of position. In one dimension, it turns out to be:

$$\Gamma(s) = -\frac{eV(s)}{m} + \text{constant}, \qquad (46)$$

where the distance s, is measured from the center of the star. The value of the constant is fixed by assuming that on the surface of the compact object, velocity of the charged particle is zero and on the surface of the star parallel component of the electric field vanishes, hence, one arrives finally at:

$$\Gamma(s) = 1 - \frac{e\,V(s)}{m}. \qquad (47)$$

The unity in eqn. (47) is usually neglected for large Lorentz boost (Usov 1993), however for consistency it has to be retained. Using the relation V (r) = - $\int_0^{\omega} E_{\|}$.ds (when R is the radius of the star), one can find out the value of Γ(s) for a particular position *s*. According to polar cap model, a compact star with surface magnetic field B_S, angular velocity $\Omega = \frac{2\pi}{P}$ (when P is the period), would have electric field $E_{\|}$, close to the surface, given by $E_{\|} \approx \frac{B_S}{192}\left(\frac{\Omega R}{c}\right)^{\frac{5}{2}}\frac{s}{\Delta R_p}$ for $0 \leq s \leq \Delta R_p$ [(Usov 1988), (Usov 1993) and (Arons & Scharlmann 1979)]. The Polar cap radius is denoted by $\Delta R_p = \sqrt{\frac{\Omega R}{c}}$, with c as the velocity of light and is equal to unity according to our system of units. This relation makes it is easy to observe that, the value of the Lorentz boost at a height $r = \alpha\Delta R_p (0 \leq \alpha \leq 1)$ from the surface of the star is

$$\Gamma(r) = \frac{1}{16\sqrt{3}}\Omega R^3\left(\frac{eB}{m}\right)R\alpha^2 \qquad (48)$$

When the distance r = α(ΔRP), i.e., some fraction of the polar cap radius from the surface of the star. One can combine this result with the expression for ω_{sc} to get a relation between emission energy vs hight.

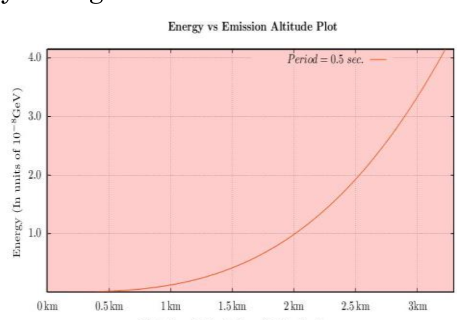

Fig. 2 Plot of energy of emitted synchrotron radiation versus emission-altitude, for a relatively cold compact star (white dwarf) with dipolar magnetic field strength ~ 10^{13} Gauss and period 0.5 sec.

Normally astrophysical objects like, WD or NS appear with surface magnetic field strength varying between 10^9 - 10^{13} Gauss. The surface dipolar magnetic field strength for young pulsars are more (< 10^9 Gauss) than the older ones (> 10^9 Gauss). Assuming the surface dipolar magnetic field strength to be around critical magnetic field, i.e. 4.4×10^{13} Gauss and a period of 0.5 sec, we have plotted the Lorentz boost vs altitude, as well as synchrotron emission energy ω_{sc} vs emission-altitude, in Fig. [1] and Fig. [2] respectively. As can be seen form the plots, that, for a compact star of radius 10 km or 10^6 cm, the synchro-curvature radiation reaches the value of 10^{-8} GeV, within 2 km hight from the surface of the star. The same keeps on increasing as one moves further away from the surface of the star.

There are two relevant points worth mentioning here, (i) with increase in the time period P of the compact object, energy of the emitted radiation at a fixed altitude would tend to increase (ii) we would be assuming that the photon emission process, takes place close to the last open field line and it's quasi tangential to the surface of the star. Although the magnetic field strength is expected to vary as $B(r) = \frac{B_S(R)R^2}{r^2}$ (a condition that follows from the flux conservation), but because of the special emission geometry we have assumed,[2] the observables, like Ψ, χ etc., would not undergo significant variation because of the variation of the ambient magnetic field.

6 Result and Analysis

Earlier we have shown that, the curvature radiation amplitudes for the plane polarized photons in the magnetized stellar environment follows from eqn. [26]. Since our objective in this work to bring out salient features of such emission, therefore we have assumed the initial amplitudes of the two orthogonal polarized modes to be of same magnitude. Though this is a simplified assumption, to be true for modelling of realistic emission processes taking place in compact astrophysical objects (WD or NS), however, as it will be clear below, that this is sufficient to bring out the new physics issues those we wish to focus on.

The physics of optical activity, in this scenario, is the following, as the produced electromagnetic beam propagates in the magnetized stellar environment, the photons with polarization orthogonal to the magnetic field, keep mixing with the scalars resulting in a change of phase for the same; and the photons with polarization along the magnetic field propagates freely. The superposition of the two decides the net polarization of the system. Since the change of phase is dependent on (a) the path traversed by the radiation beam, (b) the strength of the ambient magnetic field and (c) the frequency of the photons the final magnitude of the net polarization of the radiation beam depends on all the three.

Since we are interested in the wave propagation in an ambient (magnetic) field of strength~10^{13} Gauss, believed to exist close to surface of the star, we need know the critical synchro-curvature energy ω_{sc}, of the emitted photons there in.

The same has been obtained, as already mentioned using the relations of section IV. The critical energy of the emitted photons, as a function of altitude from a ultra cold, (WD or NS) pulsar (with period P = 0.5 sec.) was evaluated and plotted in Fig. [2]. The numerical data showed that, at altitudes very close to the surface of the star ($\leq$ 0.1 km), the critical energy, turns out to be of the order of O (10^{-16}) GeV, and it reaches the value of O (10^{-18}) GeV, at an altitude of 3.5 km or so.

Therefore assuming the initial amplitudes for both the polarized modes to be the same, we have estimated the stokes parameters numerically for a frequency range of 4.0×10^{-16} GeV to 2.0×10^{-15} GeV, for $g_{\gamma\gamma\varphi} = (10^{11} GeV)^{-1}$ [from PVLAS data (Zavattini2008)] and magnetic field strength B = 4.4×10^{13}

Gauss. The propagation length is taken to be about 5 $\times$ 10^5 cm (about $\frac{R_S}{10}$, when R_S = 10^6 cm, is the stellar radius which is close to that of the astrophysical compact objects). The result is plotted in the Fig. [3]. As can be verified from the initial conditions that, at z = 0 the stokes parameter U is non-zero but Q and V are both zero (i.e. because we have started with linearly polarized light at the source). However the variations of the same, after propagation through, a distance $\frac{R_S}{10}$ as a function of energy ω can be seen from Fig. [3]. As is clear from the plots, that at low energy, elliptic polarization, defined by Stokes parameter V, though is small in magnitude but the same undergoes modulation with increasing . Similar behavior is also observed to be taking place with U. However, the Stokes parameter Q doesn't undergo similar modulation in magnitude at high energies.

It can be checked from the plots that there are situations when both Q and V are simultaneously zero except U, signaling linear polarization and vice-verse. As the frequency changes, the degree of linear polarization decreases and that of circular polarization increases. This is due to $\gamma - \varphi$ mixing effect. We would like to emphasize here that, degree of linear and circular polarization due to $\varphi F_{\mu\nu} F^{\mu\nu}$ coupling need to be of very close order at all energies.

The other important observation that we had mentioned already is, that the ellipticity χ and polarization angles Ψ are generally multi-valued functions of the energy ω. There can be various values of energies ω at which the Ψ and χ could turn out to be the same, but mostly they are not, as can be seen from Fig. [4].

It should however be noted that, the effects we have discussed so far are purely due to mixing, where the polarized multi wavelength beams are supposed to have travelled the same distance. This may be achievable in laboratory conditions, however,the same may not hold for all class of astrophysical objects and all kinds of emissions mechanisms.

Compact astrophysical objects (stars) usually radiates through synchro-curvature radiation, in many energy bands. Energy of an emitted photon depends on the emission altitude– measured from the surface of the star.

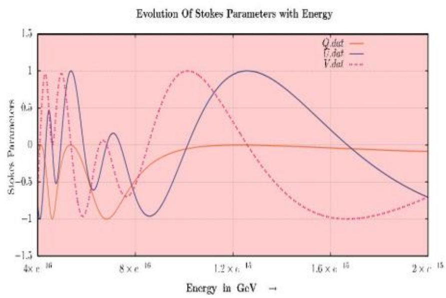

Fig. 3 Plots for Stokes parameters, Q (orange line), U (blue line) and V (dotted orange line), for coupling constant $g_{\gamma\gamma\varphi}$ = 10^{-11} Gev^{-1} and B = 4.4 $\times$ 10^{13} Gauss vs Energy.

For compact stars, this relationship, is referred at times as altitude energy mapping. The nature of this mapping depends on the details of the model of the compact object. In our illustrative model the low energy photons originate close to and higher energy photons originate far away from the surface of the compact star. Therefore the low energy modes would pass through larger distance in the magnetized media than the high energy ones.

So the states of polarizations that two light beams (say at two different energies ω_1 and ω_2, with $\omega_1 < \omega_2$), are going to acquire, would be different, once they are out of the stellar environment. This is because of the two different path lengths, covered by the the two beams in the magnetized stellar media, due to emission geometry. Since for the kind of physical picture for we have in mind (WD, NS or Quasars), a radiation beam, due to altitude energy effect (following dipole emission models), would be having multiple energies where each of these individual components would be traveling different path-lengths in the magnetized stellar atmosphere, once out of the stellar environment. Therefore, in addition to the $\gamma\varphi$ mixing induced polarization effects, an additional path dependent effect would also show up, at various energy bands of the synchro-curvature radiations coming from WD or NS. This is due to emission-altitude vs energy relation that these radiations follow. Therefore, for a consistent interpretation of the observations, the same should be taken into consideration.

6.1 Modified Boundary Conditions for Scalars

Another important point that we would like to point out here, is, although the number of scalar particles produced in the synchro-curvature emission model of WD or NS are usually zero (from kinematical or other considerations), however, by the time the emitted radiation is out of the stellar environment a significant amount of scalars may be generated because of the mixing effect.

Therefore, while analyzing synchro-curvature spectra of radiations from far away WD or NS, one would need to take this in to consideration for fixing the boundary conditions. However one may take the effect of multiple magnetized intergalactic domains into consideration for analyzing data from very far away objects (similar model was considered in Tiwari & Jain (2012), for different class of sources).

7. Discussion and Outlook

In this section we briefly summarize the findings of this work. We have tried to point out, here the possible combined effect of φ-γ interaction and emission-altitude-energy relation, in the synchro- curvature radiation spectra of compact objects (WD or NS), near or far away from us.

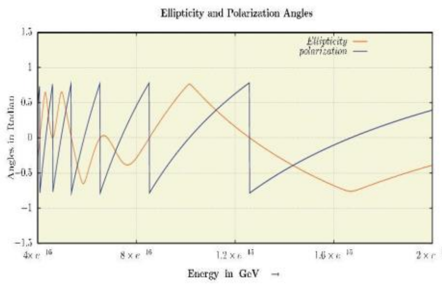

Fig. 4 Plots for polarization angle (blue line) and ellipticity (orange line), for coupling constant $g_{\gamma\gamma\varphi}$ = 10^{-11} Gev^{-1} and B = 4.4 $\times$ 10^{13} Gauss vs Energy.

Our findings are the following: the mixing effect itself is capable of producing (a) Elliptic or Circular polarization from initially Plane Polarized beams, (b) the amount of or plane, circular or elliptic polarization generated at different energies need not be of same magnitude at all energy bands, for the beams traveling through the same distance and same fields strength B, (c) Polarization and Ellipticity angles are multivalued functions of energy and there may be several energy bands where the angles repeat themselves, (d) significant amount of scalars may be produced in the radiation beam once they are out of the stellar environment (e) the physics of emission of synchro-curvature radiation for these sources, makes the monochromatic beams at different energy bands, travel different path lengths in the stellar environment (altitude vs energy relation). Hence a phase difference, coming from the difference of path travelled by the beams of light at different, will contribute to their polarimetric signatures. Therefore even with running the risk of repetition, we would like to say, that, the degree of linear and circular polarization may not be close to each other, with dim-5 photon scalar coupling. Inclusion of emission-altitude-energy would make the situation more complex.

Once the electromagnetic signal is out of the stellar environment, it would propagate through the ambient magnetized interstellar, galactic and inter-galactic space before reaching the observer. Signals from far away objects may also travel through multiple magnetized domains in the inter-galactic space. Since EM waves undergo Faraday Rotation (Ganguly, Konar & Pal 1999) in magnetized environment, therefore to find out the contribution of scalar photon mixing term to polarimetric data, one needs to estimate further, the Faraday rotation induced contribution to the same, following (Ganguly, jain & Mandal 2009). However, this analysis is outside the scope of current study and will be undertaken else-where.

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