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# Comparison of Wind Energy Potential using Different Mathematical Methods for Pasni, (Pakistan)

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## Abstract:

*This paper gives a detailed analysis of measured wind speed data in an attempt to estimate wind energy potential for Pasni, Baluchistan, Pakistan. The wind speed data over a period of 10 years (2002-2011) on daily basis measured at Midnight & Noon for Pasni and used Weibull probability distribution function for data fitting. In order to calculate two Weibull parameters, i.e., shape and scale parameters seven different statistical methods are used. These methods include (EM) Empirical Method, (MLM) Maximum Likelihood Method, (PDM) probability density method, (MMLM) Modified Maximum Likelihood Method, (MLE) Method of Least Squares, (MoM) Method of Moments, and (EPFM) Energy Pattern Factor Method. Their validity is tested using, statistical analysis for the goodness of fit is performed using (RMSE) Root mean Square Error and Coefficient of determination or R-square tests. Cumulative distribution function (CDF) and Weibull probability density function (PDF) are determined for the actual time-series wind speed data using the specified shape and scale parameters.*

**Key Words:** Wind Energy Potential, Wind Speed Data, Weibull Probability Distribution Function, Root mean Square Error, Coefficient of determination.

## Introduction:

Today, the main part of World's energy necessity fulfil from burning large amount of fossil fuels which is one of the cause of special weather condition observed in different locations around the World. Acid rains & snow falls, urban smog, climate change, regional haze, frequent tornados, etc., have become rampant around the World. Wind, biomass, solar, and geothermal energy i-e renewable energy resources are better than burning fossil fuels. Wind is one of the promising renewable energy source which can be harnessed in a commercial manner. Renewable energy sources effectively reduce environmental pollution and the burning up of fossil fuel. With effective planning and execution any kind of wind power engineering project leads to a reduction in the cost of generating electrical power.

Wind energy conversion systems design required considerable efforts for recognizing a suitable statistical model for wind speed frequency distribution. The widely used function to model wind speed data is Weibull distribution function [1]. More recently it has become a reference distribution function in commercially used wind energy software i-e Wind Atlas Analysis and Application Program [2]. We characterised Weibull distribution by two parameters, a scale parameter and a shape parameters.

For wind speed data, Weibull distribution graphical method and lognormal models were used by Garcia et al. (1998) [3]. The modified maximum likelihood method (MMLM) recommended by Seguro and Lambert (2000) [4] for the assessment of Weibull parameters using the time series wind data. This was based on a limited number of wind speed data of three days and he suggested that the true evaluation of the method requires many months/years of measured wind speed data. Sulaiman etal. (2002) [5] used the graphical method for determining the Weibull parameters Wind characteristics for Oman. Several authors have used various statistical methods to assess Weibull parameters, for example, the widely used empirical method (EM), maximum likelihood method (MLM), method of moment (MoM), modified maximum likelihood method (MMLM), and energy pattern factor method (EPFM) [6-15].

To analyze the wind power density at 10, 30, and 60 m heights in Kingdom of Bahrain, Jowder [16] used the graphical & empirical methods. Empirical method gives more precise prophecy of average wind speed and power density. Dorvlo [17] conclude that the Chi-square method provided better evaluation for Weibull parameters than graphical & moment method, based on the Kolmogorov–Smirnov statistic while analyzing the wind data from 04 stations in Oman. The wind data observed does not necessarily follow the Weibull distribution but above mentioned numerical methods indicates that wind speed data follows the Weibull probability distribution.

## Theoretical Background

The measured wind speed distribution is modelled to a theoretical distribution function for the calculation of wind energy potential. Weibull distribution is characterized by a velocity function of two parameters (k, c) [18]. It can be described by its probability density function *f(v)* and cumulative distribution function *F(v)* given as:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

$$F(v) = 1 - exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (2)$$

where k the dimensionless shape parameter, $v$ is the wind speed, and c the scale parameter having the same dimension as $v$. If its shape parameter k is 2, the distribution is named Rayleigh distribution. The Weibull mean wind speed $v_m$, using Gamma function $\Gamma$, is expressed by equation (3), the energy density $P_v$ is expressed by equation (4) ($\rho_a$: air density 1.225 kg/m$^3$) and available energy density for all wind speeds $E_d$ (Weibull energy density) is expressed by equation (5) [19, 20].

$$v_m = c\Gamma\left(1 + \frac{1}{k}\right) \quad (3)$$

$$P_v = \frac{1}{2}\rho_a v^3 \quad (4)$$

$$E_d = \frac{\rho_a c}{2}\Gamma\left(1 + \frac{3}{k}\right) \quad (5)$$

$$\frac{P}{A} = \frac{\rho_a c^3}{2}\Gamma\left(1 + \frac{3}{k}\right) \quad (6)$$

$$\frac{E}{A} = \frac{\rho_a c^3}{2}\Gamma\left(1 + \frac{3}{k}\right)T \quad (7)$$

where $\rho_a$ is the air density 1.225 kg/m$^3$ and $\Gamma()$ is the Gamma function expressed by

$$\Gamma(x) = \int_0^\infty t^{x-1} exp(-t)dt \quad (8)$$

The survey of six statistical methods for estimating Weibull parameters are given in the following sections. i-e (MoM), (MLE), (MLM), (MMLM), (EM), (PDM).

**Statistical Error Analysis & Goodness of Fit**

In order to analyze the efficiency of the seven methods used in estimating Weibull parameters and the goodness of fit of the measured data to Weibull function, RMSE and $R^2$ tests are performed. These tests are as;

$$RMSE = \left[ \frac{1}{N} \sum_{i=1}^{N} (y_i - x_i)^2 \right]^{1/2} \quad (9)$$

$$\chi^2 = \frac{\sum_{i=1}^{N} (y_i - x_i)^2}{N - n} \quad (10)$$

$$R^2 = \frac{\sum_{i=1}^{N} (y_i - z_i)^2 - \sum_{i=1}^{N} (y_i - x_i)^2}{\sum_{i=1}^{N} (y_i - z_i)^2} \quad (11)$$

Where, N is the number of observations, $z_i$ is the mean of $y_i$, $y_i$ is observed frequency for the bin i, $x_i$ is expected frequency for bin i and is calculated using Weibull distribution.

Root Mean Square Error (RMSE) is measured for discrete data points and is commonly used to estimate error or uncertainty in locations. Test RMSE is the square root of the variance of residuals. This test gives the absolute measure of the fit of the model to the measured data. RMSE's lower values indicate a better fit. The $R^2$ test give relative measure of the fit of the model to the measured data in compare to RMSE. A value of $R^2$ closer to one shows that greater proportion of variation in data is being explained by the model [21].

In order to test the suitability of the theoretical probability density function a test, known as the Kolmogorov–Smirnov test, is performed. The test is defined as max-error between 02 cumulative distribution functions:

$$Q = \max \left| F(v) - O(v) \right| \quad (12)$$

where *F(v)* ≡ the cumulative distribution function for wind speeds calculated using specified Weibull parameters.

*O(v)* ≡ the cumulative distribution functions for observed or randomly generated wind speed data.

The critical value for the Kolmogorov–Smirnov test at 95% confident level is given by:

$$Q_{95} = \frac{1.36}{\sqrt{n}} \quad (13)$$

The wind speed data (measured) can be pictured by a histogram. It is mainly useful in comparing distribution of the wind speed variations with modelled Weibull distribution. The choice of bin size is critical as the shape of histogram depend on the bin size. The bin size (B) can be determined using the following empirical expression [22]:

$$B = \frac{v_{max}}{[3.3\ln(n)+1]} \quad (14)$$

where $v_{max}$ is maximum wind speed in data set and n is the data number.

**Table-1: Monthly (mean) wind speed ( m/sec ) for Pasni at Midnight**

| Month | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|---|
| January | 1.7923 | 0.6638 | 0.8297 | 1.0289 | 1.3276 | 1.5267 | 2.6884 | 1.2280 | 0.7634 | 0.8629 |
| February | 1.1616 | 0.3651 | 0.4978 | 1.2612 | 0.7966 | 0.5310 | 1.7591 | 0.5642 | 0.9957 | 0.4315 |
| March | 0.0996 | 0.5642 | 1.8586 | 0.6306 | 0.8961 | 0.3983 | 0.0664 | 0.7966 | 1.3940 | 1.0289 |
| April | 0.4647 | 0.6638 | 1.6595 | 0.4813 | 1.4935 | 0.6970 | 0.4647 | 0.1991 | 1.6595 | 0.4647 |
| May | 0.3983 | 1.1948 | 1.0953 | 1.2944 | 1.8586 | 0.9625 | 0.7302 | 0.0332 | 1.4935 | 0.7966 |
| June | 0.4978 | 1.0621 | 1.4604 | 0.7302 | 1.5267 | 0.4978 | 0.3651 | 0.3651 | 1.3940 | 0.8961 |
| July | 3.1530 | 1.2280 | 1.2612 | 1.1616 | 1.3608 | 1.0289 | 0.5974 | 0.0000 | 1.4272 | 0.9957 |
| August | 2.6884 | 0.5310 | 1.6595 | 0.3651 | 1.0953 | 0.8961 | 0.5974 | 0.0000 | 1.2612 | 0.7966 |
| September | 1.5599 | 0.9625 | 1.2612 | 0.0664 | 0.2987 | 0.4647 | 0.3319 | 0.3983 | 0.7634 | 0.5974 |
| October | 0.3651 | 0.7966 | 0.6638 | 0.3651 | 0.8961 | 0.8297 | 0.2987 | 0.5974 | 0.0996 | 0.3983 |
| November | 0.9293 | 1.2612 | 0.1328 | 0.5974 | 0.8629 | 0.3319 | 0.7966 | 0.5974 | 0.0332 | 0.1991 |
| December | 1.6595 | 1.4604 | 5.6755 | 0.9625 | 1.5599 | 1.0953 | 0.1328 | 0.4978 | 0.3983 | 1.1948 |
| Annual Mean | 1.2308 | 0.8961 | 1.5046 | 0.7454 | 1.1644 | 0.7717 | 0.7357 | 0.4398 | 0.9736 | 0.7219 |

**Table-2: Monthly (mean) wind speed (m/sec) for Pasni at Noon**

| Month | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|---|---|---|---|
| January | 6.1567 | 4.1819 | 3.1199 | 4.3811 | 3.5181 | 3.9828 | 3.5513 | 3.1862 | 2.9207 | 3.7505 |
| February | 5.8746 | 3.3190 | 3.2526 | 4.3811 | 3.5845 | 4.5802 | 3.9164 | 3.8500 | 2.8211 | 3.5513 |
| March | 7.4345 | 4.9453 | 7.4843 | 5.4763 | 5.4100 | 5.2440 | 5.3104 | 4.7794 | 6.0074 | 4.3479 |
| April | 6.7376 | 4.5470 | 5.9908 | 7.0363 | 6.7542 | 5.3436 | 4.7462 | 4.4806 | 6.9035 | 4.9453 |
| May | 6.4057 | 4.7462 | 5.9078 | 7.5009 | 7.2354 | 6.7707 | 5.7750 | 4.9785 | 7.5341 | 7.9324 |
| June | 7.1026 | 5.1776 | 6.0738 | 7.4345 | 6.8039 | 5.3104 | 3.9496 | 4.6798 | 5.5759 | 5.4431 |
| July | 7.0695 | 4.1487 | 6.2397 | 5.4763 | 5.3104 | 5.1776 | 4.3147 | 4.4475 | 5.2772 | 5.9410 |
| August | 4.1156 | 4.9951 | 6.4388 | 5.4100 | 5.5095 | 5.7750 | 3.5513 | 4.6134 | 6.5052 | 3.0867 |
| September | 6.0074 | 4.9453 | 6.7376 | 5.6257 | 5.1776 | 5.2274 | 5.4100 | 5.1776 | 6.1733 | 5.1444 |
| October | 4.5636 | 4.0160 | 5.1776 | 4.8457 | 5.4100 | 4.1487 | 3.7837 | 3.4518 | 3.8168 | 3.5181 |
| November | 4.5138 | 3.2858 | 3.7837 | 3.0535 | 3.7837 | 4.0160 | 3.6675 | 2.8543 | 2.6552 | 2.3565 |
| December | 5.9078 | 5.0449 | 4.6466 | 3.2194 | 3.6841 | 3.4849 | 2.4561 | 3.0203 | 2.8875 | 3.1530 |
| Annual Mean | 5.9908 | 4.4461 | 5.4044 | 5.3201 | 5.1818 | 4.9218 | 4.2027 | 4.1266 | 4.9232 | 4.4309 |

## Results and Discussions:

A bi-daily wind speed data measured at Pasni Meteorological Office of ten years ( 2002-to-2011) is uses for estimating wind energy potentials for Pasni. The measured data is fitted to Weibull distribution function; Weibull parameters were estimated using seven statistical methods, i.e., PDM, MMLM, MLM, MoM, MLE, EPFM and EM. *Tables 1 & 2* provide mean monthly wind speeds for Pasni at Midnight and Noon. *Table 3 & 4* list skewness, Kurtosis, the standard deviation and calculated mean wind speed at Midnight and Noon. The statistical results of the seven numerical methods of estimation of Weibull shape and scale parameters lists in *Table 5.* Columns 2-5 listing calculated Weibull parameters, i.e., scale parameter c, shape parameter k, Weibull energy density, Weibull mean wind speed. Columns 6 & 7 list values of two statistical tests for each of the seven estimation methods.

Figure 1 shows histogram of hourly wind speed data overlaped by various Weibull fuctions with k & c values estimated usind seven methods. Figure 2a-2g: Histrogram of daily wind speed observed at Pasni with various computed Webull fuctions. The histograms are overlap by cumulative probability distribution function (cdf) and Weibull probability distribution function (pdf) calculated with specific shape and scale parameters for the wind speed data (actual) also for generated data. Figures show very fine agreement between the histograms, cdf and pdf of the actual time series data & of generated data where Weibull shape & scale parameters calculated using seven statistical methods.

From *table 5*, the $R^2$-test gives a value of 0.995 for all estimation methods, indicates a very fine fit among measured wind speed distribution & calculated Weibull distribution. In particular the fit explains on the average 99.5 % of the total variation in the measured data.

Measured wind speed data for Pasni is fitted to the two parameter Weibull distribution function i.e., shape parameter and scale parameter, are estimated using seven statistical method. For testing goodness of fit reliability of estimation methods, statistical techniques are performed such as Root mean Square Error (RMSE) and Coefficient of determination or $R^2$ tests. Weibull probability density function (PDF) and cumulative distribution function (CDF) are also determined for the actual time-series wind speed data using the specified shape and scale parameters.

*Figure 1* shows the analysis of real wind speed data which is histogram of wind speed data (actual) overlapped by calculated Weibull probability density function (PDF) all with particular shape and scale parameters estimated by using all seven statistical methods. Good agreement is seen between measured wind speed data and various Weibull functions whose k and c parameters are estimated using different methods of estimation.

**Figure 1: Histogram of hourly wind speed data overlaped by various Weibull fuctions with k & c values estimated usind seven methods.**

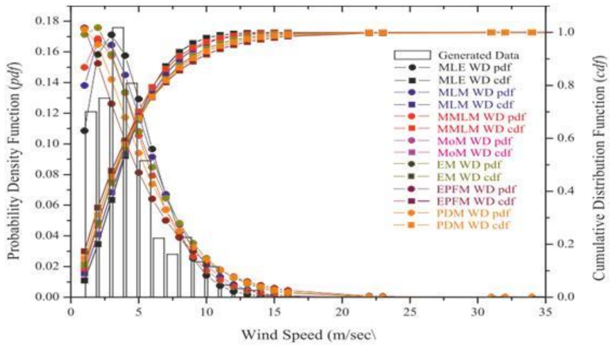


**Table-3: Descriptive Statistics of observed Wind Speed data at 0000 hours in Pasni**

| Months | Data Points | Speed Range ( m/sec) | Mean Wind Speed ( m/sec) | Standard Deviation (m/sec) | Kurtosis | Skewne ss | Correlation of Variation (CV) % | Power Density (W/m²) | $Q_{95}$ |
|---|---|---|---|---|---|---|---|---|---|
| January | 310 | 14.404 | 1.271 | 2.457 | 6.553 | 2.496 | 193.269 | 1.257 | 0.275 |
| Februar y | 310 | 12.347 | 0.836 | 1.835 | 12.993 | 3.356 | 219.428 | 0.358 | 0.205 |
| March | 310 | 31.896 | 0.773 | 2.285 | 113.124 | 9.025 | 295.462 | 0.283 | 0.255 |
| April | 310 | 6.688 | 0.825 | 1.229 | 2.397 | 1.601 | 149.039 | 0.344 | 0.137 |
| May | 310 | 7.202 | 0.986 | 1.429 | 3.940 | 1.875 | 145.016 | 0.587 | 0.160 |
| June | 310 | 6.173 | 0.880 | 1.322 | 2.080 | 1.599 | 150.356 | 0.417 | 0.148 |
| July | 310 | 7.202 | 1.221 | 1.442 | 0.775 | 1.118 | 118.082 | 1.114 | 0.161 |
| August | 310 | 6.173 | 0.989 | 1.255 | 1.465 | 1.260 | 126.860 | 0.592 | 0.140 |
| Septemb er | 310 | 4.116 | 0.670 | 1.089 | 1.907 | 1.655 | 162.481 | 0.184 | 0.122 |
| October | 310 | 6.173 | 0.531 | 0.887 | 6.947 | 2.241 | 166.947 | 0.092 | 0.099 |
| Novemb er | 310 | 11.318 | 0.574 | 1.240 | 22.165 | 3.864 | 215.872 | 0.116 | 0.139 |
| Decembe r | 310 | 59.676 | 1.464 | 4.889 | 77.654 | 7.859 | 334.040 | 1.920 | 0.546 |

**Table-4: Descriptive Statistics of Observed Wind Speed data at 1200 hours in Pasni**

| Months | Data Points | Speed Range m / sec | Mean Wind Speed m / sec | Standard Deviation m / sec | Kurtosis | Skew | Correlation of variation (CV) % | Power Density (W/m²) | $Q_{95}$ |
|---|---|---|---|---|---|---|---|---|---|
| January | 310 | 30.867 | 3.875 | 2.568 | 39.786 | 4.515 | 66.279 | 35.610 | 0.287 |
| Februar y | 310 | 13.376 | 3.913 | 2.491 | 2.078 | 1.020 | 63.654 | 36.667 | 0.278 |
| March | 310 | 85.912 | 5.644 | 5.314 | 177.865 | 11.735 | 94.157 | 110.030 | 0.594 |
| April | 310 | 31.381 | 5.748 | 3.284 | 11.657 | 2.130 | 57.134 | 116.226 | 0.367 |
| May | 310 | 14.404 | 6.479 | 2.885 | -0.112 | 0.740 | 44.523 | 166.447 | 0.322 |
| June | 310 | 15.433 | 5.755 | 2.912 | 0.696 | 0.736 | 50.594 | 116.651 | 0.325 |
| July | 310 | 13.376 | 5.340 | 2.332 | -0.237 | 0.682 | 43.674 | 93.191 | 0.261 |
| August | 310 | 11.832 | 5.000 | 2.197 | 0.269 | 0.825 | 43.935 | 76.500 | 0.246 |
| Septemb er | 310 | 14.404 | 5.563 | 2.778 | 0.022 | 0.430 | 49.948 | 105.361 | 0.311 |
| October | 310 | 14.404 | 4.273 | 2.011 | 4.405 | 1.854 | 47.071 | 47.747 | 0.225 |
| Novemb er | 310 | 22.121 | 3.397 | 1.868 | 32.654 | 3.523 | 55.004 | 23.990 | 0.209 |
| Decemb er | 310 | 45.271 | 3.750 | 3.656 | 69.982 | 7.203 | 97.474 | 32.273 | 0.409 |

*Table 5* lists the value of shape and scale parameters along with other Weibull estimates. The values of Weibull parameters, i.e., scale parameter $c$, shape parameter $k$, Weibull energy density, Weibull mean wind speed listed in columns 2-5. Columns 6 and 7 show results of statistical tests carried out for testing goodness of fit model to the experimental data.

*Figures 2a – 2g* show the plots of the calculated Weibull functions overlaid on the observed wind speed histogram for Pasni. The plots contain histogram of the observed wind speed distribution, probability density function (*pdf*) and cumulative distribution function (*cdf*) for Weibull function calculated using the observed time series wind speed data and Weibull functions. A good agreement is observed between measured and generated data.

**Figure 2a-2g: Histrogram of daily wind speed observed at Pasni with various computed Webull fuctions.**

**Figure 2a**

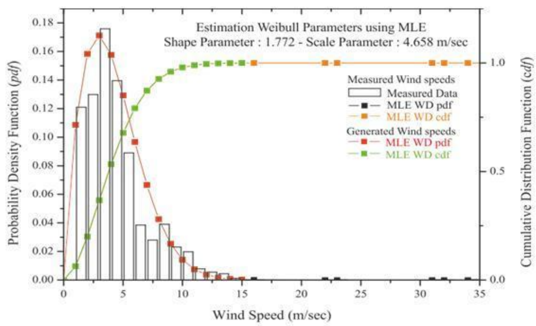


**Figure2b**

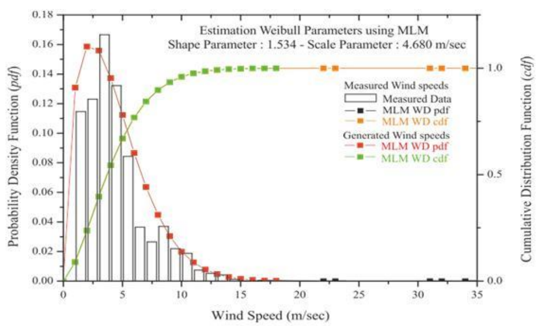


**Figure 2c**

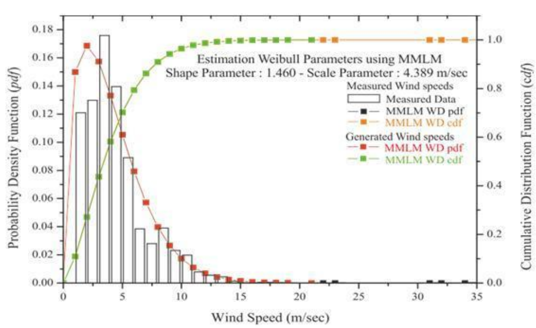


**Figure2d**

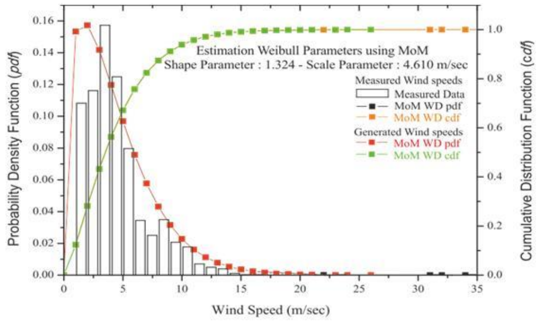


**Figure 2e**

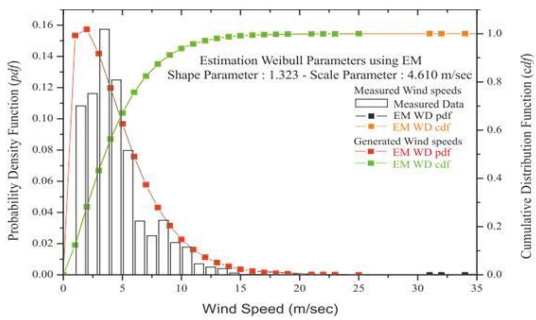


**Figure 2f**

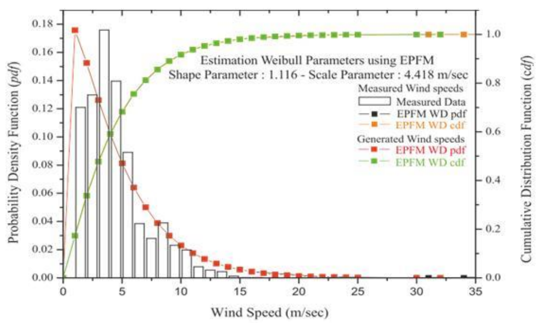


**Figure 2g**

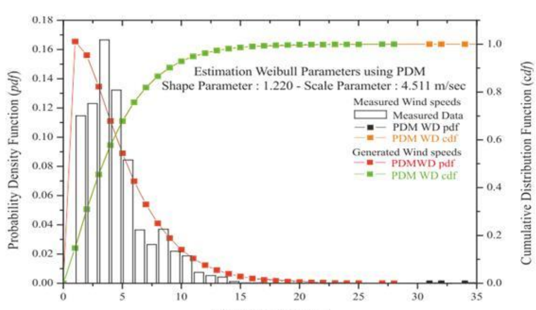


*Table 5* reveals that from all methods of estimation identical values of the parameters *k* and *c* (m/s) are obtained. Furthermore, statistical tests reveal

that the $R^2$-test gives a value of 0.995 for all methods, indicates a fine fit among the measured wind speed distribution and the fitted Weibull distribution function. More particularly the fit explain on the average 99.5% of the total deviation in the measured data. A similar trend for all estimation methods is seen in RMSE test whose values are of the order of $10^{-3}$ indicating good fit of the model to the experimental data. EPFM gave the highest RMSE value among seven methods, whereas value for MLM is lowest. The power density obtained from the seven methods of estimation is in the range 94 W/m$^2$ to 215 W/m$^2$. In general all seven methods are applicable and give good measure of the Weibull parameters.

**Table5: Estimates of Weibull parameters k and c using seven methods and results of statistical tests for Pasni.**

| Numerical Method | Weibull Parameter | | | | Statistical Test | |
|---|---|---|---|---|---|---|
| | ***k*** | ***c*** (m / s) | $v_m$ (m / s) | ***P/A*** (W / m$^2$) | **RMSE** x $10^{-3}$ | **$R^2$** |
| MLE | 1.772 | 4.658 | 4.105 | 94.209 | 1.257 | 0.995 |
| MLM | 1.534 | 4.680 | 4.174 | 119.457 | 1.125 | 0.995 |
| MMLM | 1.460 | 4.389 | 3.938 | 107.788 | 1.140 | 0.995 |
| MoM | 1.324 | 4.610 | 4.203 | 153.989 | 1.372 | 0.995 |
| EM | 1.323 | 4.610 | 4.202 | 153.998 | 1.344 | 0.995 |
| EPFM | 1.116 | 4.418 | 4.203 | 214.842 | 1.733 | 0.995 |
| PDM | 1.220 | 4.511 | 4.186 | 176.771 | 1.443 | 0.995 |

## Conclusions:

Measured wind speed data for Pasni, is analysed using Weibull Distribution Function. The characteristic shape and scale parameters for Weibull distribution are determined using seven methods of estimation. Comparison between seven methods of estimation is performed and for all methods satisfactory results are obtained. Lower values of RMSE, i.e., $10^{-3}$ are obtained for seven methods of estimation of Weibull parameters. This indicates a good response of the Weibull distribution function to the measured wind speed data. Comparison of the seven methods of estimation i.e. MLE, MLM, MMLM, MoM, EM, EPFM and PDM show consistency of the estimated shape and scale parameters in terms smaller values of RMSE. A slight difference in estimated values of *k* and *c* is observed for EPFM and PDM. This suggests that Weibull distribution function is a good match for the measured wind speed distribution.

For all methods of estimation, RMSE and $R^2$ tests are performed giving values in the recommended range suggesting reliability of the methods used for estimating Weibull parameters and consequently a better approximation of measured wind speed data distribution. The statistical tests reveal that Weibull distribution function adequately explain the measured wind speed distribution.

The analysis of the measured wind speed data reveals that among the seven methods of estimation of the Weibull parameters, Maximum likelihood Method and Modified Maximum likelihood Methods are more accurate as compared to other methods.
Weibull distribution function is fitted to measured wind speed distribution. The agreement between the observed wind speed distribution & Weibull distribution was analyzed by performing statistical tests, such as RMSE and $R^2$ tests. The tests indicated good agreement between the observed and fitted distribution function.

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