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On Artincokernel of The Group ($Q_{2m}\times D_3$) Where m= 2p and p is prime number

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Abstract:

The main purpose of This paper is to find Artin's character table $\boldsymbol{Ar(Q_{2m}\times D_3)}$*when m is even number such that m =2p,and p is a prime number ;where* Q_{2m} *is denoted to Quaternion group of order 4m, time is said to have only one dimension and space to have three dimension ,the mathematical quaternion partakes of both these elements ; in technical language it may be said to be "time plus space" ,or "space plus time" and in this sense it has , or at least involves a reference to four dimensions ,and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols.,1882,1885,1889)) ,and* $\boldsymbol{D_3}$ *is Dihedral group of order 6.In 1962 ,C. W. Curits & I. Reiner studied Representation Theory of finite groups.*

In 1976, I.M.Isaacs studied Characters Theory of finite groups, In 1982, M. S. Kirdar studied The Factor Group of the Z-Valued class function modulo the group of the Generalized Characters, In1994, H. H. Abass studies The Factor Group of class function over the group of Generalized Characters of D_n *and found* $\equiv^*(D_n)$*. In 1995, N. R. Mahmood studies The Cyclic Decomposition of the factor Group*

cf(Q_{2m},Z)/ $\bar{R}(Q_{2m})$, In 2002, K-Sekiguchi studies Extensions and the Irreducibilies of the Induced Characters of Cyclic P-Group. In 2006, A. S. Abid studies Characters Table of Dihedral Group for Odd number.

Key words: Discrimination, Disability, Family, Society, Women.

Introduction:

Representation Theory is a branch of mathematics that studies abstract algebra structures by representing their elements as linear transformations of vector spaces. A representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication, in which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication. Moreover, representation and character theory provide applications, not only in other branches of mathematics but also in physics and chemistry.
Fore a finite group G ,The factor group **$\bar{R}$(G)/T(G)** is called the Artincokernel of **G** denoted **AC(G)**, **$\bar{R}$(G)** denoted the a belian group generated by Z-valued characters of G under the operation of pointwise addition**, T(G)** is a subgroup of **$\bar{R}$(G)** which is generated by Artin's characters.

3. Preliminars:[1]:(3,1)

The Generalized Quaternion Group Q_{2m}:

For each positive integer m≥2 ,Thegeneralized Quaternion Group **Q_{2m}** of order 4m with two generators x and y satisfies **Q_{2m}**=$\{x^h y^k, 0 \le h \le_{2m-} 1, k=0,1\}$
Which has the following properties $\{x^{2m}=y^4=I, yx^my^{-1}=x^{-m}\}$.

Let **G** be a finite group, all the characters of group **G** induced from a principal character of cyclic subgroup of **G**are called Artin characters of **G** .
Artin characters of the finite group can be displayed in a table called Artin characters table of **G** which is denoted by **Ar(G);** The first row is Γ-conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G(cl_a)|$ and other rows contains the values of Artin characters .

Theorem:[5]:(3,3)
The general form of Artin characters table of $\mathbf{Cp^s}$
When p is a prime number and s is a positive integer number is given by :-
$\mathbf{Ar(Cp^s)=}$

Γ-classes	[1]	$[x^{ps-1}]$	$[x^{ps-2}]$	…	[x]
$\|cl_a\|$	1	1	1	…	1
$\|cp^s(cl_a)\|$	p^s	ps	p^s	…	p^s
Φ_1	p^s	0	0	…	0
Φ_2	P^{s-1}	P^{s-1}	0	…	0
⋮	⋮	⋮	⋮	…	⋮
Φ_s	p	p	p	…	0
Φ_{s+1}	1	1	1	…	1

Figure (1): Artin charaters of $\mathbf{Ar(Cp^s)}$

Corollary:[5]:(3,4)
Let $n=P_1^{a1}.P_2^{a2}\ldots P_n^{an}$where g.c.d($P_i$,$P_j$)=1 if i≠j and P_i are a prime numbers, and a_i any positive integer for all 1≤i≤n Then :
$\mathbf{Ar(C_m)=Ar(Cp_1^{a1})\otimes Ar(Cp_2^{a2})\ldots\otimes Ar(Cp_n^{an})}$ such that

$\mathbf{Ar(Cp^1)=}$

Γ-class	[1]	[x]
$\|cl_a\|$	1	1
$\|cp(cl_a)\|$	p	p
Φ_1	p	0
Φ_2	1	1

Figure (2): Artin charaters of $\mathbf{Ar(Cp^1)}$

Where **$Ar(C_{2m})$,m=2.p is $Ar(C_{2.2p})=Ar(C_{2^2}p)=Ar(C_{2^2})\otimes Ar(p^1)$.**

<u>Theorem:[3](3,5)</u>

The Artin characters table of Quaternion group **Q_{2m}** when m is an even number and m=2p; p is prim number is given as follows:

$Ar(C_{2^2}p)$=

Γ-classes	[I]	$[x^{2p}]$	$[x^p]$	$[x^4]$	$[x^2]$	[x]
$\|cl_a\|$	1	1	1	1	1	1
$\|c_{2^2}p(cl_a)\|$	p	p	p	p	p	p
Φ_1	p	0	0	0	0	0
Φ_2	p	p	0	0	0	0
Φ_3	p	p	p	0	0	0
Φ_4	1	0	0	1	0	0
Φ_5	1	1	0	1	1	0
Φ_6	1	1	1	1	1	1

Figure(3): Artin charaters of $Ar(C_{2^2}p)$

$Ar(Q_{2^2}p)$=

	Γ-classes of $(C_{2^2}p)$							
Γ-classes	[1]	$[x^{2p}]$	$[x^p]$	$[x^4]$	$[x^2]$	[x]	[y]	[xy]
$\|cl_a\|$	1	1	2	2	2	2	2p	2p
$\|C_{Q_{2^2}p}(cl_a)\|$	8p	8p	4p	4p	4p	4p	4	4
Φ_1	2p	0	0	0	0	0	0	0
Φ_2	2p	2p	0	0	0	0	0	0
Φ_3	2p	2p	2p	0	0	0	0	0
Φ_4	2	0	0	2	0	0	0	0
Φ_5	2	2	0	2	2	0	0	0
Φ_6	2	2	2	2	2	2	0	0
Φ_7	2p	0	0	0	0	0	2	0
Φ_8	2p	0	0	0	0	0	0	2

Figure(4): Artin charaters of group $Ar(Q_{2^2}p)$

[1] [r] [s]

$$Ar(D_3)=\begin{bmatrix} 6 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

Proposition:[4]:(3,5)

The number of all distinct Artin characters on group G is equal to the number of Γ-classes on **G** .Furthermore, Artin characters are constant on each Γ-classes .

4. The main results:

Theorem:(4,1)

The Artin's character table of the group **(Q_{4p}×D_3)** where m=2p, and p is prime number ; is given as follows:

Ar($Q_{2.2p}$×D_3)=

The Artin's character table of matrix from degree24×24 of group **(Q_{4p}×D_3)** Table(5)

Proof:

Let g_{ij}=(qi,dj) ;qi∈Q_{4p} ,dj∈D_3

Case (*I*):-

Consider the group **G=(Q_{4p}×D_3)** and if H is a cyclic subgroup of **(Q_{4p}×{I})** then H=<(q,1)> and Φ the principle character of H and Φ_jArtin's characters of **Q_{4p}** ,1≤j≤i+2 ,The cyclic subgroup of **Q_{4p}** which are {<I>},{<x^{2p}>},{<x^p>},{<x^4>},{<x^2>},{<x>},{<y>},{<xy>}, and cyclic subgroup of **D_3** which are {<I>},{<r>},{<s>}, by using theorem:

$$\Phi j(g)=\begin{cases}\frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & if\ hi\in H\cap cl(g)\\ 0 & if\ H\cap cl(g)=\emptyset\end{cases}$$

H=<q,1>:-

1:-H_{11}=<(I,1)> if g=(I,1) then $\emptyset_{11}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|CH(g)|}.1=\frac{6|CQ4p(1)|}{|C<x>(1)|}$ .1=6.$\emptyset$j(1)

Since H∩cl(I,1)=(I,1)

2:-H_{21}=<(x^{2p},1)> ; (a) ifg=(I,1)then$\emptyset_{21}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$==$\frac{24.2p}{|CH(g)|}$.1=$\frac{6|CQ4p(1)}{|C<x>(x2p)|}$ .1= 6$\emptyset j(I)$

(b)if g=(x^{2p},1) then $\emptyset_{11}$(g)=$\frac{|CG(g)|}{|CH(g)|}.\emptyset(g)=\frac{24.2p}{|CH(g)|}$.1=$\frac{24.2p}{|C<x>(x^{2p})|}$ .1 =$\frac{6|CQ4p(x^{2p})|}{|C<x>(x^{2p})|}$.1=6.$\emptyset j(x^{2p})$since H∩ $cl(x^{2p})$=(I,1),(x^{2p},1)otherwise=0

3:-H_{31}=< $(x^p,1)$ >;(a) if g=(I,1) then $\emptyset_{31}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|CH(g)|}$. 1= =$\frac{6|CQ4p(g)|}{|C<x>(x^p)}$.1=$\frac{6|CQ4p|}{|C<x>(x^p)|}$.1=6.$\emptyset$j(I)

(b) if g=(x^{2p},1) then $\emptyset_{31}$(g)=$(x^{2p},1)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C<x>(x^p)|}$.1 =6.$\emptyset j(x^{2p})$

(c)if g=(x^p,1) then

$\emptyset_{31}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})$=$\frac{12.2p}{|C<x>(x^p)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x^p)|}$.2=6.$\emptyset j(x^p)$ since H∩ $cl(x^p)=\{(I,1),(x^{2p},1),(x^p,1)\}$ otherwise=0

4:-H_{41}=< $(x^4,1)$ > ; (a) if g=(I,1) then

$\emptyset_{41}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C<x>(x^4)|}$.1=$\frac{6|CQ4p(q)|}{|C<x>(x^4)|}$ .1= 6.$\emptyset j(I)$.

(b) if g=(x^4,1) then

$\emptyset_{41}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{|C<x>(x4)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x^4)|}$.2= 6 .$\emptyset j(x^4)$

Since H∩ $cl(x^4)$={$(I,1),(x^4,1)$}. Otherwise=0 and $\emptyset(g)=\emptyset(g^{-1})$=1.

5:-H_{51}=< $(x^2,1)$ > ;(a) if g=(I,1) then $\emptyset_{51}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C<x>(x^2)|}$ .1=$\frac{6|CQ4p(q)|}{|Cx>(x^2)|}$.1=6$\emptyset j(I)$

(b) if g=$(x^{2p},1)$ then $\emptyset_{51}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C<x>(x^2)|}$.1=$\frac{6|CQ4p(q)|}{|C<x>(x^2)|}$ .1=6 .$\emptyset j(x^{2p})$.

(c) if g=$(x^4,1)$then

$\emptyset_{51}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{|C<x>(x^2)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x^2)|}$.2=6 $\emptyset j(x^4)$.

(d) if g=(x^2,1) then

$\emptyset_{51}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+(g^{-1}))$=$\frac{12.2p}{|C<x>(x^2)|}$=$\frac{3|CQ4p(q)|}{|C<x>(x^2)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x^2)|}$.2= 6

$\emptyset j(x^2)$. Since H$\cap cl(x^2)$={$(I,1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1)$}.
Otherwise=0.and $\emptyset(g)=\emptyset(g^{-1})$

6:-H_{61}=$<(x,1)>$; (a)ifg=(I,1) then
$\emptyset_{61}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C<x>(x)|}$.1=$\frac{6|CQ4p|}{|C<x>(x)|}$.1=6 $\emptyset j(I)$
(b) if g=(x^{2p},1) then
$\emptyset_{61}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C<x>(x)|}$.1=$\frac{6|CQ4p(q)|}{|C<x>(x)|}$.1=6.$\emptyset j(x^{2p})$.
(c) if g=(x^p,1) then
$\emptyset_{61}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2p}{|C<x>(x)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x)|}$.2=6.$\emptyset(x^p)$.
(d) if g=(x^4,1) then
$\emptyset_{61}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{|C<x>(x)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x)|}$.2=6.$\emptyset j(x^4)$.
(e) if g=(x^2,1) then
$\emptyset_{61}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{|C<x>(x)|}$(1+1)=$\frac{3|CQ4P(q)|}{|c<X>(X)|}$.2=6 $\emptyset J(x^2)$.
(F) IF g=(X,1) then
$\emptyset_{61}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{|C<x>(x)|}$(1+1)=$\frac{3|CQ4p(q)|}{|C<x>(x)|}$.2=6 $\emptyset j(x)$.
Since
H$\cap cl(x)=\{(I.1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1),(x,1)\}\emptyset(g)=\emptyset(g^{-1})=1$
otherwise =0.

7:-H_{71}=$<(y,1)>$;(a) ifg=(I,1) then$\emptyset_{71}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{4}$.1=6
$\emptyset_{i+1}$(I).
(b) if g=(y^2,1) $=(x^{2p},1)then\emptyset_{71}(g)$=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{4}$.1=6 $\emptyset_{i+1}(x^{2p})$.
(c) if g=(y,1) or (y^3,1) then
$\emptyset_{71}$(g)=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})$=$\frac{24}{4}$(1+1)=6.2=12.
Since H$\cap cl(g)=\{(I,1),(y^2,1),(y,1)\}$ and $\emptyset(g)=\emptyset(g^{-1})$otherwise =0.
8:- H_{81}=$<(xy,1)>$; (a) if g=(I,1) then $\emptyset_{81}$(g)=$\frac{|CG(G)|}{|cH(g)|}\emptyset(g)$=$\frac{24.2p}{4}$.1=6
$\emptyset_{i+2}$(I).

(b) if g=$((xy)^2,1)$=(x^{2p},1) then $\emptyset_{81(g)}=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{4}.1=6\ \emptyset_{i+2}(x^{2p})$.

(c) if g=(xy,1) or($(xy)^3$,1) then

$\emptyset_{81}$(g)= | $\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})).\frac{24}{4}(1+1)=6.2=12.$

Since H $\cap\, cl(xy)=\{(I,1),(xy^2,1),(xy,1)\}$ and $\emptyset(g)=\emptyset(g^{-1})=1$

Case (II):-

Consider the group **G=(Q_{4p}×D_3)** and if H is a cyclic subgroup of **(Q_{4p}×{r})** then H=<(q,r)> and $\emptyset$ the principle character of H and $\emptyset j$ Artin's character of $\mathbf{Q_{4p}}$, 1≤j≤i+2 ,by using theorem:-

$$\Phi j(g)=\begin{Bmatrix}\frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & if\ hi\in H\cap cl(g)\\ 0 & if\ H\cap cl(g)=\emptyset\end{Bmatrix}$$

H=<(q,r)>:-

1:-H_{12}=<(I,r)> (a) if g=(I,1) then $\emptyset_{12}=\frac{|CG(g)|}{|CH(g)|}$

$$\emptyset(g)=\frac{24.2p}{|CH(g)|}(1)=\frac{24.2p}{|c<x>(1)|}.1=\frac{6|CQ4p(1)|}{3|C<x>(1)|}.1=2.\emptyset j(1)$$

If g=(I,r)or=(I,r^2) then (b)

$$\emptyset_{12}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{|C<x>(1)|}(1+1)=\frac{3|CQ4p(1)|}{3|C<x>(1)|}.2=2.\emptyset j(q)$$

Since H∩cl(g)={$(I,1),(I,r),(I,r^2)$} and $\emptyset(g)=\emptyset(g^{-1})=1$
othrewise=0.

2:-H_{22}=<(x^{2p},r)> (a) if g=(I,1) then

$\emptyset_{22}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C<x>(x^{2p})|}(1)=\frac{6|CQ4p|}{3|C<x>(x^{2p})|}(1)=2.\emptyset j(I)$

(b) if g=(x^{2p},1)

then$\emptyset_{22}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C<x>(x^{2p})|}(1)=\frac{6|CQ4p(x^{2p})|}{3|c<x>(x^{2p})|}(1)=2.\emptyset j(x^{2p})$.

(c) if g=(I,r)

then$\emptyset_{22}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(q)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(q)|}.2=2.\emptyset j(q)$.

(d) if g=(x^{2p},r)

then$\emptyset_{22}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(q)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(q)|}.2=2.\emptyset j(q)$.

Since H∩cl(g)={$(I,1),(x^{2p},1),(I,r),(x^{2p},r)$}.and
$\emptyset(g)=\emptyset(g^{-1})=1$.otherwise=0.

3:-H_{32}=<(x^p,r)> (a) if g=(I,1) then
$\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C\langle x\rangle(x^p)|}$.1=$\frac{6|CQ4p|}{3|C\langle x\rangle(x^p)|}$.1=2.$\emptyset j(I)$.
(b) if g=(x^{2p},1)
then$\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C\langle x\rangle(x^p)|}$.1=$\frac{6|CQ4p|}{3|C\langle x\rangle(x^p)|}$.1=2$\emptyset j(x^{2p})$.
(c) if g=(x^p,1) then
$\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C\langle x\rangle(x^p)|}$(1+1)=$\frac{3|CQ4p|}{3|C\langle x\rangle(x^p)|}$.2=2.$\emptyset j(x^p)$.
(d) if g=(I,r)
then$\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{3|C\langle x\rangle(x^p)|}$(1+1)=$\frac{3|CQ4p|}{3|C\langle x\rangle(x^p)|}$.2=2.
$\emptyset j(q)$.
(e) if g=(x^{2p},r)
then$\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{|C\langle x\rangle(x^p)|}$(1+1)=$\frac{3|CQ4p|}{3|C\langle x\rangle(x^p)|}$.2=2.
$\emptyset j(q)$.
(f) if g=(x^p,r) then
$\emptyset_{32}(g)\ =\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p}{3|C\langle x\rangle(x^p)|}$(1+1+1+1)=$\frac{6.4(2p)}{3|C\langle x\rangle(x^p)|}$=
$\frac{6|CQ4p|}{3|C\langle x\rangle(x^p)|}$=

2.$\emptyset j(q)$since H∩cl(g)={$(I,1),(x^{2p},1),(x^p,1),(I,r),(x^{2p},r),(x^p,r)$}.and
∅(g)=∅((g^{-1}) = 1.othrewise=0.

4:-H_{42}=<(x^4,r)> (a) if g=(I,1)
then$\emptyset_{42(g)}=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{|C\langle x\rangle(x^p)|}$.1=$\frac{6|CQ4p|}{3|C\langle x\rangle(x^4)|}$.1=2.$\emptyset j(I)$.
(b)if g=(x^4,1)
then$\emptyset_{42(g)}=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{3|C\langle x\rangle(x^4)|}$(1+1)=$\frac{3|CQ4p|}{3|C\langle x\rangle(x^4)|}$.2=2.
$\emptyset j(x^4)$.
(c) if g=(I,r) then
$\emptyset_{42(g)}=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\ \frac{12.2p}{3|C\langle x\rangle(x^4)|}$(1+1)=$\frac{3|CQ4p|}{3|C\langle x\rangle(x^4)|}$.2=2.$\emptyset j(q)$.

(d) if

g=(x^4,r) then$n\emptyset_{42(g)} = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})) = \frac{6.2p}{3|C{<}x{>}(x^4)|}$(1+1+1+1)=

$\frac{6.4(2p)}{3|C{<}x{>}(x^4)|}=\frac{6|CQ4p|}{3|C{<}x{>}(x^4)|}$=2 .$\emptyset j(q)$.

since H∩ cl(g)=$\{(I,1),(x^4,1),(I,r),(x^4,r)\}$and

∅(g)=∅((g^{-1}) = 1.otherwise=0.

5:-H_{52}=<(x^2,r)> (a) if g=(I,1) then

$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C{<}x{>}(x^2)|}.1=\frac{6|CQ4p|}{3|C{<}x{>}(x^2)|}.1=2.\emptyset j(I)$.

(b) if g=(x^{2p},1) then

$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C{<}x{>}(x^2)|}.1=\frac{6|CQ4p|}{3|C{<}x{>}(x^2)|}.1=2.\emptyset j(x^{2p})$.

(c) if g=(x^4,1) then

$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}) = \frac{12.2p}{3|C{<}x{>}(x^2)|}(1+1)=\frac{3|CQ4p|}{3|C{<}x{>}(x^2)|}.2=2. .\emptyset j(x^4)$.

(d) if g=(x^2,1) then

$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})) = \frac{12.2p}{3|C{<}x{>}(x^2)|}(1+1)=\frac{3|CQ4p|}{3|C{<}x{>}(x^2)|}.2=2.\emptyset j(x^2)$.

(e) if g=(I,r) then

$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})) = \frac{12.2p}{3|C{<}x{>}(x^2)|}(1+1)=\frac{3|CQ4p|}{3|C{<}x{>}(x^2)|}.2=2. .\emptyset j(q)$.

(f) if g=(x^{2p},r)

the$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})) = \frac{12.2p}{3|C{<}x{>}(x^2)|}(1+1)=\frac{3|CQ4p|}{3|C{<}x{>}(x^2)|}.2=2.$

.$\emptyset j(q)$.

(g) if g=(x^4,r) then

$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})) = \frac{6.2p}{3|C{<}x{>}(x^2)|}(1+1+1+1)=\frac{6.4(2p)}{3|C{<}x{>}(x^2)|}=$

$\frac{6|CQ4p|}{3|C{<}x{>}(x^2)|}$ =2 .$\emptyset j(q)$.

(h) if g=(x^2,r)

then$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})) = \frac{6.2p}{3|C{<}x{>}(x^2)|}(1+1+1+1)=\frac{6.4(2p)}{3|C{<}x{>}(x^2)|}=$

$\frac{6|CQ4p|}{3|C{<}x{>}(x^2)|}=2.\emptyset j(q)$.since H∩

cl(g)=$\{(I,1),(x^{2p},1),(x^4,1),(x^2,1),(I,r),(x^{2p},r),(x^4,r),(x^2,r)\}$ and

∅(g)=∅((g^{-1}) = 1.othrewise=0.

6:-H_{62}=<(x,r)> (a) if g=(I,1) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C<x>(x)|}.1=\frac{6|CQ4p|}{3|C<x>(x)|}=2\ .\emptyset j(I)$

(b) if g=$(x^{2p},1)$then $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{|C<x>(x)|}.1=\frac{6|CQ4p|}{3|C<x>(x)|}=2\ .\emptyset j(x^{2p})$.

(c) if g=$(x^{p},1)$ then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x)|}.2=2\ .\emptyset j(x^{p})$

(d) if g=$(x^{4},1)$ then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x)|}.2=2\ .\emptyset j(x^{4})$.

(e) if g=$(x^{2},1)$ then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x)|}.2=2\ .\emptyset j(x^{2})$.

(f) if g=(x,1) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x)|}.2=2\ .\emptyset j(x)$.

(g) if g=(I,r) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x)|}.2=2\ .\emptyset j(q)$.

(h) if g=(x^{2p},r) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{3|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x)|}.2=2\ .\emptyset j(q)$.

(i) if g=(x^{p},r) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p)}{3|C<x>(x)|}=\frac{6|CQ4p|}{3|C<x>(x)|}$

$=2\ .\emptyset j(q)$.

(j) if g=(x^{4},r) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p)}{3|C<x>(x)|}=\frac{6|CQ4p|}{3|C<x>(x)|}$

$=2\ .\emptyset j(q)$.

(k) if g=(x^{2},r) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p)}{3|C<x>(x)|}=\frac{6|CQ4p|}{3|C<x>(x)|}$

$=2.\emptyset j(q)$.

(l) if g=(x,r) then

$\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p)}{3|C<x>(x)|}=\frac{6|CQ4p|}{3|C<x>(x)|}$

$=2.\ \emptyset j(q)$.

Since

H $\cap cl(g) = \{(I,1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1),(x,1),(I,r),(x^{2p},r),(x^p,r),(x^4,r),(x^2,r),(x,r)\}$

And Ø(g)=Ø(g^{-1}) = 1.otherwise=0.

7:-H_{72}=<(y,r)> (a) if g=(I,1) then $\emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{12}$.1=2.(2p) =2.$\emptyset_{i+1}$(I).

(b) if g=(y^2,1) or (x^{2p},1) then $\emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{12}$.1=2.(2p) =2.$\emptyset_{i+1}$($x^{2p}$).

(c) if g=(y,1) or (y^3,1) then $\emptyset_{72}(g)$=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{24}{12}$(1+1)=4.

(d) if g=(I,r) then$\emptyset_{72}(g)$=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{12}$(1+1)=2.(2p)=2.$\emptyset_{i+1}$(q).

(e) if g=(y^2,r) or (x^{2p},r) then $\emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12.2p}{12}(1+1)$=2.(2p)= 2.$\emptyset_{i+1}$(q).

(f) if g=((y,r) or (y^3,r) then $\emptyset_{72}(g)$=$\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{12}{12}$(1+1+1+1)=4.

Since H∩cl(g)=$\{(I,1),(y^2,1),(y,1),(I,r),(y^2,r),(y,r)\}$ And Ø(g)=Ø((g^{-1}) = 1.othrewise=0.

8:-H_{82}=<(xy,r)> (a) if g=(I,1) then$\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{12}$.1=2.(2p) =2.$\emptyset_{i+2}$(I).

(b) if g=(($xy)^2$,1)=(x^{2p},1) then$\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p}{12}$.1=2.(2p) =2.$\emptyset_{i+2}$($x^{2p}$).

(c) if g=(xy,1) or (($xy)^3$,1) then $\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$=$\frac{24}{12}(1+1)$=4.

(d) if g=(I,r) then $\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$ =$\frac{12.2p}{12}(1+1)$=2.(2p)= 2.$\emptyset_{i+2}$(q).

(e) if g=((xy)^2,r)=(x^{2p},r) then $\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))$

$=\frac{12.2p}{12}(1 + 1)$=2.(2p)= 2.$\emptyset_{i+2}$(q).

(f) if g=(xy,r) or((xy)^3,r) then

$\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12}{12}$(1+1+1+1)=4.

Since

H∩cl(g)=

$\{(I,1),((xy)^2,1)(xy,1),(I,r),((xy)^2,r),(xy,r)\} And\ \emptyset(g) = \emptyset((g^{-1}) = 1. othrewise = 0.$

Case (III):-

Consider the group **G=(Q_{4p}×D_3)** and if H is a cyclic subgroup of **(Q_{4p}×{s })** then H=<(q,s)> and ∅ the principle character of H and $\emptyset j$ Artin's character of $\mathbf{Q_{4p}}$, 1≤j≤i+2 ,by using theorem:-

$$\Phi j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & if\ hi \in H\cap cl(g) \\ 0 & if\ H\cap cl(g) = \emptyset \end{cases}$$

H=<(q,s)>

1:-H_{13}=<(I,s)> (a) if g=(I,1) then

$\emptyset_{13}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p}{2|C<x>(I)|}.1=\frac{6|CQ4p|}{2|C<x>(I)|}.1$=3.∅j(I).

(b) if g=(I,s) then $\emptyset_{13}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(I)|}.1=\frac{2|CQ4p|}{2|C<x>(I)|}.1=\emptyset j(q)$.

Since H∩cl(g)=$\{(I,1),(I,s)\}\ othrewise = 0$

2:-H_{23}=<(x^{2p},s)> (a) if g=(I,1) then

$\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^{2p})|}.1=\frac{6|CQ4p|}{2|C<x>(x^{2p})|}.1$

=3.∅j(I).

(b) if g=(x^{2p},1) then $\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^{2p})|}.1=\frac{6|CQ4p|}{2|C<x>(x^{2p})|}.1$

==3.∅j(x^{2p}).

(c) if g=(I,s) then

$\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^{2p})|}.1=\frac{2|CQ4p|}{2|C<x>(x^{2p})|}.1=\emptyset j(q)$.

(d) if g=(x^{2p},s) then

$\emptyset_{23}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^{2p})|}.1=\frac{2|CQ4p|}{2|C<x>(x^{2p})|}.1=\emptyset j(q)$.

Since

H⌒cl(g)=$\{(I,1),(x^{2p},1),(I,s),(x^{2p},s)\}$ $And\ \emptyset(g)=\emptyset((g^{-1})$ =1

$othrewise = 0.$

3:-H_{33}=<(x^p,s)> (a) if g=(I,1) then

$\emptyset_{33}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^p)|}.1=\frac{6|CQ4p|}{2|C<x>(x^p)|}.1=3.\emptyset j(I).$

(b) if g=(x^{2p},1) then

$\emptyset_{33}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^p)|}.1=\frac{6|CQ4p|}{2|C<x>(x^p)|}.1=3.\emptyset j(x^{2p}).$

(c)if g=(x^p,1) then

$\emptyset_{33}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x^p)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x^p)|}.2=3.\emptyset j(x^p).$

(d) if g=(I,s) then $\emptyset_{33}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^p)|}.1=\frac{2|CQ4p|}{2|C<x>(x^p)|}.1=\emptyset j(q).$

(e) if g=(x^{2p},s) then

$\emptyset_{33}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^p)|}.1=\frac{2|CQ4p|}{2|C<x>(x^p)|}.1=\emptyset j(q).$

(f) if g=(x^p,s) then

$\emptyset_{33}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x^p)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x^p)|}.2=\emptyset j(q).$

Since

H⌒cl(g)=

$\{(I,1),(x^{2p},1),(x^p,1),(I,s),(x^{2p},s),(x^p,s)\}$ $And\ \emptyset(g)=\emptyset((g^{-1})$ =1

$othrewise = 0.$

4:-H_{43}=<(x^4,s)> (a) if g=(I,1) then

$\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^4)|}.1=\frac{6|CQ4p|}{2|C<x>(x^4)|}.1=3.\emptyset j(I).$

(b)if g=(x^4,1) then

$\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x^4)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x^4)|}.2=3.\emptyset j(x^4).$

(c) if g=(I,s) then $\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^4)|}.1=\frac{2|CQ4p|}{2|C<x>(x^4)|}.1=\emptyset j(q).$

(d) if g=(x^4,s) then

$\emptyset_{43}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x^4)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x^4)|}.2=\emptyset j(q).$

Since

H⌒cl(g)=$\{(I,1),(x^4,1),(I,s),(x^4,s)\}$ $And\ \emptyset(g)=\emptyset((g^{-1})$ =1

$othrewise = 0.$

5:-H_{53}=<(x^2,s)> (a) if g=(I,1) then
$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^2)|}.1=\frac{6|CQ4p|}{2|C<x>(x^2)|}.1=3.\emptyset j(I).$

(b) if g=(x^{2p},1) then
$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x^2)|}.1=\frac{6|CQ4p|}{2|C<x>(x^2)|}.1=3.\emptyset j(x^{2p}).$

(c) if g=(x^4,1) then
$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x^4)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x^4)|}.2=3.\emptyset j(x^4).$

(d) if g=(x^2,1) then
$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x^2)|}.2=3.\emptyset j(x^2).$

(e) if g=(I,s) then $\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^2)|}.1=\frac{2|CQ4p|}{2|C<x>(x^2)|}.1=\emptyset j(q).$

(f) if g=(x^{2p},s) then$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x^2)|}.1=\frac{2|CQ4p|}{2|C<x>(x^2)|}.1=\emptyset j(q).$

(g) if g=(x^4,s) then
$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x^2)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x^2)|}.2=\emptyset j(q).$

(h) if g=(x^2,s) then
$\emptyset_{53}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x^2)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x^2)|}.2=\emptyset j(q).$

Since
H∩cl(g)=
$\{(I,1),(x^{2p},1),(x^4,1),(x^2,1),(I,s),(x^{2p},s),(x^4,s),(x^2,s)\} And\ \emptyset(g)=\emptyset((g^{-1})=$
$1\ othrewise=0.$

6:-H_{63}=<(x,s)> (a) if g=(I,1) then
$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x)|}.1=\frac{6|CQ4p|}{2|C<x>(x)|}.1=3.\emptyset j(I).$

(b) if g=(x^{2p},1) then
$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{2|C<x>(x)|}.1=\frac{6|CQ4p|}{2|C<x>(x)|}.1=3.\emptyset j(x^{2p}).$

(c) if g=(x^p,1) then
$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x)|}.2=3.\emptyset j(x^p).$

(d) if g=(x^4,1) then
$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x)|}.2=3.\emptyset j(x^4).$

(e) if g=(x^2,1) then
$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x)|}.2=3.\emptyset j(x^2).$

(f) if g=(x,1) then

$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p}{2|C<x>(x)|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x)|}.2=3.\emptyset j(x).$

(g) if g=(I,s) then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x)|}.1=\frac{2|CQ4p|}{2|C<x>(x)|}.1=\emptyset j(q).$

(h) if g=(x^{2p},s) then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{2|C<x>(x)|}.1=\frac{2|CQ4p|}{2|C<x>(x)|}.1=\emptyset j(q).$

(i) if g=(x^p ,s) then

$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x)|}.2=\emptyset j(q).$

(j) if g=(x^4,s) then

$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x)|}.2=\emptyset j(q).$

(k) if g=(x^2,s) then

$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x)|}.2=\emptyset j(q).$

(l) if g=(x,s) then

$\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p}{2|C<x>(x)|}(1+1)=\frac{|CQ4p|}{2|C<x>(x)|}.2=\emptyset j(q).$

Since H

∩cl(g)=

$\{(I,1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1),(x,1),(I,s),(x^{2p},s),(x^p,s),(x^4,s),(x^2,s),(x,s)\}$

$And\ \emptyset(g)=\emptyset((g^{-1})=1\ othrewise=0.$

7:-H_{73}=<(y,s)> (a) if g=(I,1) then

$\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{8}.1=3.\emptyset_{i+1}(I).$

(b) if g=(y^2,1) =(x^{2p},1) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{8}.1=3.\emptyset_{i+1}(x^{2p}).$

(c) if g=(y,1) or (y^3,1) then

$\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{24}{8}(1+1)=6.$

(d) if g=(I,s) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{8}.1=\emptyset_{i+1}(q).$

(e) if g=(y^2,s)=(x^{2p},s) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{8}.1=\emptyset_{i+1}(q).$

(f) if g=(y,s) or (y^3,s) then$\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{8}{8}(1+1)=2.$

Since

H∩cl(g)=$\{(I,1),(y^2,1),(y,1),(I,s),(y^2,s),(y,s)\}$ $And\ \emptyset(g)=\emptyset((g^{-1})=1$ $othrewise=0.$

8:-H_{83}=<(xy,s)> (a) if g=(I,1) then
$\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{8}.1=3.\emptyset_{i+2}(I)$.

(b) if g=$((xy)^2,1)$ =$(x^{2p},1)$ then
$\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p}{8}.1=3.\emptyset_{i+2}(x^{2p})$.

(c) if g=(xy,1) or $((xy)^3,1)$ then
$\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{24}{8}(1+1)=6$.

(d) if g=(I,s) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{8}.1=\emptyset_{i+2}(q)$.

(e) if g=$((xy)^2,s)$ =(x^{2p},s) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p}{8}.1=\emptyset_{i+2}(q)$.

(f) if g=(xy,s) or $((xy)^3,s)$ then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))$
$=\frac{8}{8}(1+1)=2$.

Since H∩cl(g)
$=\{(I,1),((xy)^2,1),(xy,1),(I,s),((xy)^2,s),(xy,s)\}$ *And* $\emptyset(g)=\emptyset((g^{-1})=1$
othrewise $=0$.

Note: $(xy)^2=y^2$ since $(xy)^2$=xyxy=
$xyxy.y^2y^2=xyxy^3y^2=x(yxy^3)y^2=xx^{-1}y^2=y^2$.

Example:-(4,2)

Let **p=3;m=2.p=2.3=6** ;**Q_{2m}=Q_{12}**;To find Artin's character of the group **(Q_{12}×D_3)** the cyclic subgroup of **Q_{12}**which are {<I>},{<x^6>},{<x^3>},{<x^4>},{<x^2>},{<x>},{<y>},{<xy>} and cyclic subgroup of **D_3** which are {<1>},{<r>},{<s>}
∴Thecyclic subgroup of **(Q_{12}× D_3)** are
{<I,1>,<x^6,1>,<x^3,1>,<x^4,1>,<x^2,1>,<x,1>,<y,1>,<xy,1>,<I,r>,
<x^6,r>,<x^3,r >,<x^4,r >,<x^2,r >,<x,r>,<y,r>,<xy,r>,<I,s>,<x^6,s>,<x^3,s>,<x^4,s>,<x^2,s>,<x,s>,<y,s>,
<xy,s>.
by using theorem:-

$$\Phi j(g)=\begin{Bmatrix}\frac{|CG(g)|}{|CH(g)|}\sum_{i=1}^{n}\emptyset(hi) & if\ hi\in H\cap cl(g)\\ 0 & if\ H\cap cl(g)=\emptyset\end{Bmatrix}$$

Then $\mathbf{Ar(Q_{12}\times D_3)= Ar(Q_{2^2.3}\times D_3)= Ar(Q_{2^2.3}) \otimes Ar(D_3) =}$

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