

Mathematical Model for Estimating Income Tax Revenues in the Philippines through Regression Analysis Using Matrices

JACKIE D. URRUTIA
JOSEPH MERCADO
JUDY ANN QUITE

Polytechnic University of the Philippines, Parañaque Campus
The Philippines

Abstract:

The aim of the study is to develop a mathematical model for estimating Income Tax Revenues in the Philippines. The researchers considered the following factors: Real Gross Domestic Product Growth Rate (x_1), Employment Population (x_2), Unemployment Rate (x_3), Annual Domestic Crude Oil Prices (x_4) and Inflation Rate (x_5) as the explanatory variables of the fluctuations in the Income Tax Revenues in the Philippines. The researchers created a model through regression analysis using matrices and used $\alpha = 0.01$.

The study examined data for 34 years from 1980 to 2013. The data were obtained from National Statistical Coordination Board (NSCB), Department of Labor and Employment (DOLE) and Philippine Institute for Development Studies. After having satisfied the multiple linear regression assumptions, the mathematical model was obtained using Matlab and was written as:

$$\ln y = -39.812 + 0.138 \ln x_1 + 5.197 \ln x_2 - 0.024 \ln x_3 - 0.288 \ln x_4 + 0.147 \ln x_5$$

The study shows that there are three significant factors that actually affect the dependent variable –Income Tax Revenue in the Philippines (y). These are: Employment Population (x_2), Annual Domestic Crude Oil Prices (x_4) and Inflation Rate (x_5). The predicted values of the dependent variable (y) were obtained from the model with a coefficient of determination of 99%. The Paired T-Test examined that there is no significant difference between the actual and predicted values.

This study will be of significance in estimating future Income Tax Revenues in the Philippines. Tax revenues being the primary source of government funds, is a basic data needed in development planning activities and in preparing the national budget.

Key words: Income Tax Revenues, Regression Analysis, Matrices, Matlab, Log Transformation

1. Introduction

Fiscal policy is a means of the government to influence the economy. It deals with how the government uses its fund to sustain the need of the nation, and far as the government fund is concern, one of its primary sources of fund are the taxes. Taxes are mandatory payments, ruled by laws. Income tax revenue is collected from the whole society with differentiated intensity, inspired by considerations of justice, efficiency and effectiveness.

Income tax revenue calls for revenue generation and income distribution. Majority of developing countries are depending on income tax revenue for their economic development and Philippines is one of those [1].

There are several factors which affects the Income Tax Revenue. The researchers consider five (5) explanatory variables. These are: Real Gross Domestic Product Growth Rate

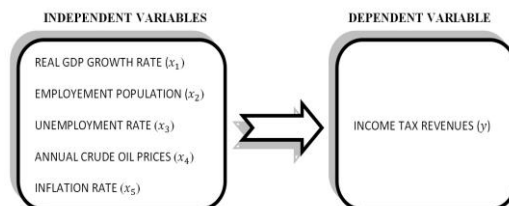
(x_1), Employment Population (x_2), Unemployment Rate (x_3), Annual Domestic Crude Oil Prices (x_4) and Inflation Rate (x_5). The effects of these factors will be tackled in Review of Related Literature.

This study will be of significance in estimating future Income Tax Revenues in the Philippines. Accuracy of estimated income tax revenue is a very much important for nation's budget planning. Both underestimation and overestimation of planed revenue could bring forth problems when the revenue is used in financing government activities. Tax revenues being the primary source of government funds, is a basic data needed in development planning activities and in preparing the national budget.

2. Objective of the Study

The Objective of the study was to formulate mathematical model through regression analysis using matrices to estimate the Income tax Revenue in the Philippines given the factors affecting it such as Real GDP Rate(x_1), Number of Employed(x_2), Unemployment Rate(x_3), Annual Domestic Crude Oil Prices(x_4), and Inflation Rate(x_5). This model will be of great value in estimating future Income Tax Revenues in the Philippines. Tax revenues being the primary source of government fund, is a basic data needed in development planning activities and in preparing the national budget.

Figure 1: Research Paradigm



The researchers used the variables shown in the figure 1 to make model that estimate Income Tax Revenue in the Philippines. Significant relationships among variables were obtained after some transformation. Multiple Regression using Matrices was then used after satisfying the assumptions to make model that will estimate Income Tax Revenue in the Philippines.

3. Statement of the Problem

This study is conducted to formulate mathematical model through regression analysis using matrices to estimate income tax revenue of the Philippines. In particular, the point of the study was to answer the following questions:

1. What is the behavior of graph of the following variables?
 - 1.1. Real GDP Growth Rate (x_1)
 - 1.2. Employment Population (x_2)
 - 1.3. Unemployment Rate (x_3)
 - 1.4. Annual Crude Oil Prices (x_4)
 - 1.5. Inflation Rate (x_5)
 - 1.6. Income Tax Revenue (y)
2. Is there a significant relationship between the Dependent from the Independent variable?
3. What mathematical model can be formulated through regression analysis using matrices that estimate the Income Tax Revenue?
4. What are the significant factor(s) that can actually predict the Income Tax Revenue (y)?
5. Is there a significant difference between the Actual and Predicted value?

4. Scope and Limitation

The researchers limit this study for 34 years. It considered years from 1980 up to 2013. The data were gathered from National Statistical Coordination Board, Department of Labor and Employment, inflationdata.com and World Bank. The researchers formulate regression model using matrices by considering independent variables such as Real GDP Rate (x_1), Employment Population (x_2), Unemployment Rate (x_3), Annual Domestic Crude Oil Prices (x_4) and Inflation Rate (x_5).

5. Review Related Literature

This area presents a review of related literatures that would be beneficial to the study summarized from previous writings and studies, revealing facts stated by people and pioneer in this field of study.

According to Christina Romer and David Romer (2010) *"Tax changes have very large effects: an exogenous tax increase of 1 percent of GDP lowers real GDP by roughly 2 to 3 percent."* Tax changes that are made to promote long-run growth, or to reduce an inherited budget deficit, in contrast, are undertaken for reasons essentially unrelated to other factors influencing output. Thus, examining the behavior of output following these relatively exogenous tax changes is likely to provide more reliable estimates of the output effects of tax changes.[2]

According to Bretschger (2010), he found negative impacts of corporate taxes on openness and total tax revenue to the economic growth in 12 Organization Economic Co-operation and Development (OECD) countries. He also mentioned on the tax competition theory that argues that, when tax rate of capital is reduced, it will cause the capital inflow to a country. This is because; the tax rate is one of the costs for capital holder (Bucovetsky, 1991 and Wilson, 1991). These two researches

were found that private return on investment is influenced by the changes in capital taxes. [3]

William McBride (2012) said “While there are a variety of methods and data sources, the results consistently point to significant negative effects of taxes on economic growth even after controlling for various other factors such as government spending, business cycle conditions, and monetary policy. In this review of the literature, I find twenty-six such studies going back to 1983, and all but three of those studies, and every study in the last fifteen years, find a negative effect of taxes on growth.”[4]

In Philippines, Rosario G. Manasan (2013) analyzed the effects of inflation on the individual income tax structure for the ten-year period starting 1964 to 1974 using hypothetical families with income levels ranging from P2,000 to P40,000 per annum and family sizes of 2, 4 and 6. The authors concluded that (1) taxable portion of the constant real income had consistently increased over the period implying that the value of exemptions and deductions in real terms had continually declined for the same period; (2) the real effective tax rates had risen steadily; (3) given the same real income, real disposable income had shrunk over the ten-year period as a consequence of larger tax obligations; and (4) families with more dependents and families in the lower income brackets were more adversely affected by inflation, e.g. , the taxable portion of their constant real income and the real effective tax rate had increased faster. The paper attributes the above mentioned findings to the fact that the taxable base increases with inflation in as much as the principal tax deductible items are expressed in nominal fixed amounts. [5]

Minea and Villieu (2009) have shown theoretically that inflation targeting provides an incentive for governments to improve institutional quality in order to enhance tax revenue performance. [6]

In New Delhi, India,(2011) The Revenue Secretary Sunil Mitra said “Inflation can affect domestic demand and thereby adversely affect GDP growth... and consequently our tax collection” [7]

Roger Salmons (2011) Oil and gas extraction plays a dominant role as a source of export earnings and, to a lesser extent, employment in many developing countries. But the most important benefit for a country from development of the oil and gas sector is likely to be its fiscal role in generating tax and other revenue for the government. [8]

Farzanegan & Markwardt (2013), Oil plays important strategic role in Iran’s exports. In fact, Iran is one of the largest oil exporting countries in the world. Since a major share of Iran’s budget revenue comes from oil revenue so the Iranian economy depends very much on oil exporting. [9]

Ilona V. Tregub (2011), Stimulating fiscal policy's aim is to reduce unemployment and to encourage economic activity at the period of a recession. To do so, government increases its spending (G), increases transfers as well (Tr) and reduces taxation (T_x). [10]

Milan(2011),When the labour market is imperfectly competitive the composition of the tax budget and the side that is legally taxed are relevant for assessing the economic effects of taxes. The degree of progressivity may affect both employment levels and human capital accumulation. The effects of single taxes cannot be correctly evaluated without taking into account how the government disposes of the tax receipts and the overall structure of the tax system. [11]

Joshua A. Cuevas (2012) states: A stronger economy will bring in more tax dollars because more people will be employed. Simple enough. Except that for the last three decades, politicians, think tanks, and special interest groups have been making the case that lower taxes will strengthen the economy because it frees up capital for job creators and those in the private sector to then spend, thus keeping businesses thriving

and people employed. This argument presupposes a cause and effect relationship. It suggests that low tax rates lead to more money circulating through the system, creating a stronger economy and lower unemployment. [12]

Henrik Jacobsen Kleven (2013): When the government levies income taxes to finance transfer programs and public goods, individuals respond by changing labor supply. By revealed preference, each individual prefers the new labor supply to the old one at the tax-inclusive prices. So, everyone is better off because of the labor supply adjustments? No, the revealed preference argument applies to each individual separately, not to the population as a whole. Behavioral responses affect government revenue and create a fiscal externality—the deadweight loss of taxation.[13]

6. Research Methodology

The methods of research that was used by the researchers are Jarque-Bera test for normality, Wald test for Linearity, Durbin Watson Test of Independence, Breusch-Pagan Test for Heteroscedasticity and Variance Inflation Factor for Multicollinearity.

6.1 Statistical Treatment

Since there were more than one independent variables involved, multiple regression using matrices was utilized to establish the mathematical model that would best assess the income tax revenue in the Philippines.

All calculations were made using normal equation. Matrix calculations were calculated using MATLAB and regression models were verified using Eviews.

In fitting a multiple linear regression model, knowledge of matrix theory can facilitate the mathematical manipulations considerably. Using matrix notation, we can write the equation

Where

$$y = X\beta + c$$

$$y = [y], X = [1 X_1 X_2 X_3 \dots X_k], \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

Then the least squares method for estimation of, it involves finding for b for which $SSE = (y - Xb)'(y - Xb)$ is minimized, then the X matrix is

$$A = X'X = \begin{bmatrix} n & \sum_{i=1}^n X_{1i} & \dots & \sum_{i=1}^n X_{2i} & \sum_{i=1}^n X_{ki} \\ \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{1i}^2 & \dots & \sum_{i=1}^n X_{1i} X_{2i} & \sum_{i=1}^n X_{1i} X_{ki} \\ \vdots & \vdots & \dots & \vdots & \vdots \\ \sum_{i=1}^n X_{ki} & \sum_{i=1}^n X_{ki} X_{1i} & \dots & \sum_{i=1}^n X_{ki} X_{2i} & \sum_{i=1}^n X_{ki}^2 \end{bmatrix}; g = X'y$$

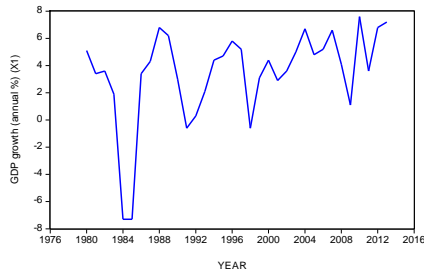
$$= \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n X_{1i} y_i \\ \vdots \\ \sum_{i=1}^n X_{ki} y_i \end{bmatrix}$$

Allows the normal equations to be put in the matrix from $Ab = g$. [14]

7. Presentation, Analysis and Interpretation of Data

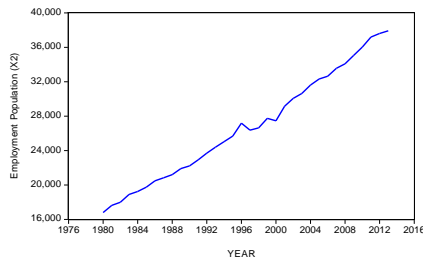
7.1. Behavior of the graph of the variables used

7.1.1 GDP growth rate (X_1)



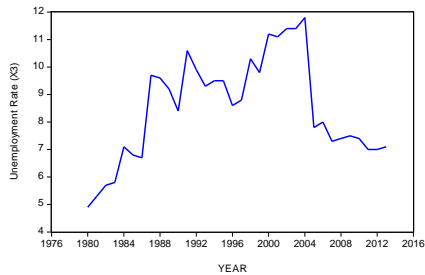
It can be seen from the graph that GDP growth is generally fluctuating for 34 years. From 1980 to 1983, it shows a decrease in rate up to approximately 9.2% in 1984 until 1985 where GDP is also at its lowest point. It also recorded 4 years of having a negative growth rate: In 1984, 1985, 1991 and 1998 as follows. Then it went up and down for the next few years and reached a peaked on 2010.

7.1.2. Employment Population (X_2)



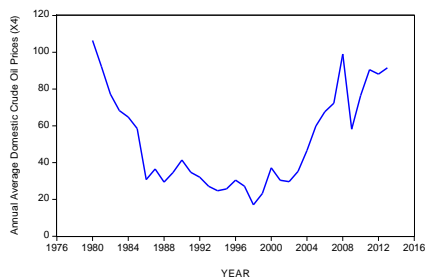
The graph refers to employment population. As an overall trend, it is clear that no. of employment increases. In 1997 it declined to around 3% then, continues to rise for the following two years but fell again on 2000. After that, it remained increasing.

7.1.3. Unemployment Rate (X_3)



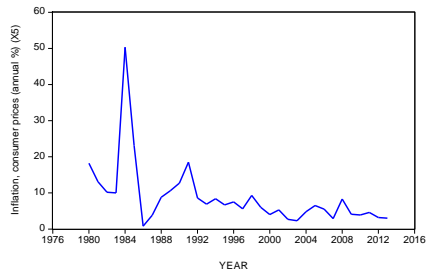
The graph illustrates that unemployment rate increased from 1980 to 1984 and slightly fell for the next two years. In 1987 it rose to around 3% then continuously decline until 1990. It goes up at a faster pace on 1991 until it hits the highest point in 2004 which was subsequently followed by gradual decline. The rate fluctuates constantly until year 2007 where it becomes relatively steady at around 7%.

7.1.4. Annual Crude Oil Prices (X_4)



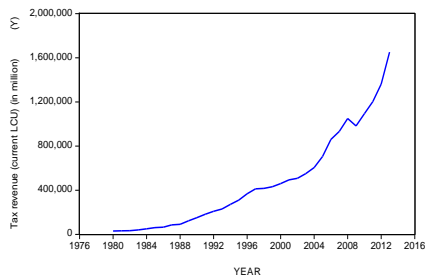
It can be seen from this graph that oil prices are much lower from 1984 to 2002. For the first five years, oil prices decline gradually then dropped sharply on 1986. From 1987 to 2004, it goes up and down and on 1998 it noted its lowest point at about 17. For the year 2005 up to 2013, it remained fluctuating wildly.

7.1.5. Inflation Rate (X_5)



The graph illustrates that inflation fluctuates widely for 34 years. It peaked at approximately 50 on 1984 and only after a year, a significant decrease in inflation was observed. That decrease results to the lowest point of inflation which is approximately 1. In between the years 1987 and 2013, it fluctuates continuously

7.1.6. Tax Revenue (Y)



It can be seen from the graph of Income Tax Revenue in the Philippines that collection mainly increased for 34 years. From 1980 to 1988, rate of increase is much lower than the following years. In 2009, collection fell but it was followed by a more rapid increase for the last five years.

7.2. Relationships of the Independent and Dependent Variable(s)

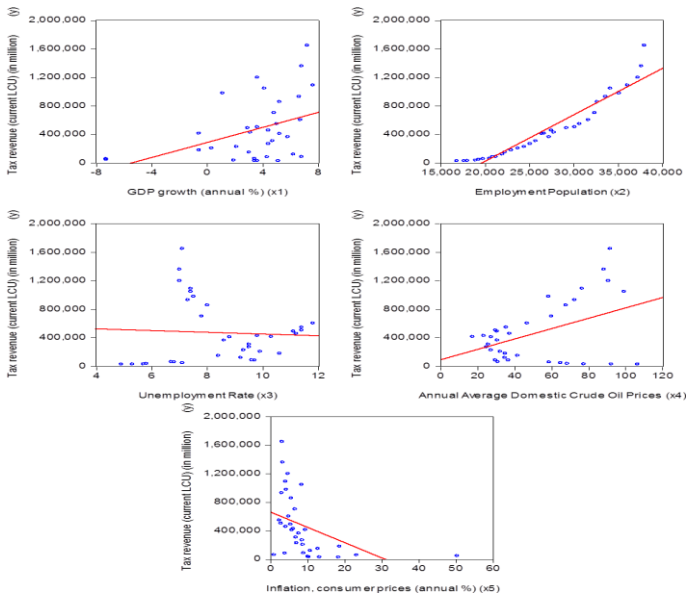
The relationships of the Independent to the Dependent Variables using original data (Table 7.2.1) were ascertained by Pearson's coefficient of correlation, as shown in the table below.

TABLE 7.2.1

	Real GDP growth rate (x_1)	Employment Population (x_2)	Unemployment Rate (x_3)	Annual Crude Oil Prices (x_4)	Inflation Rate (x_5)
Income Tax Revenue (y)	0.416650	0.955178	-0.052520	0.437586	-0.435113
p-value	0.0142	0.0000	0.7680	0.0097	0.0101

The table 7.2.1 shows that Real GDP Rate and Unemployment Rate were not significantly correlated with the Dependent variable at $\alpha = 0.01$. In contrary, the Employment Population, Annual Crude Oil Prices and Inflation Rate were significantly correlated with the Income tax Revenue.

SCATTER DIAGRAM 7.2.1



The scatter diagram 7.2.1 shows that there is significant Linear relationship between Income Tax Revenue (y) and Independent variables: Employment Population (x2), Annual Crude Oil Prices (x4) and Inflation Rate (x5) given that the resulted p-value of the Independent variables are less than $\alpha = .01$. It also shows that there is no significant Linear relationship between Income Tax Revenue (y) and Independent variables: Real GDP rate (x1) and Unemployment Rate (x3) provided that the resulted p-value of the Independent variables are greater than $\alpha = .01$.

The assumptions of multiple linear regression are the researchers' basis in formulating mathematical model that estimate the Income tax Revenue of the Philippines. The Jarque-Bera enables them to know if the data are normally distributed. Since such assumption of multiple regression did not meet, the researchers decided to use natural log transformation to transform both the dependent and independent variable(s). Given that the data for Real GDP has a negative value, the researchers add a constant of 10 for the values of GDP. [15]

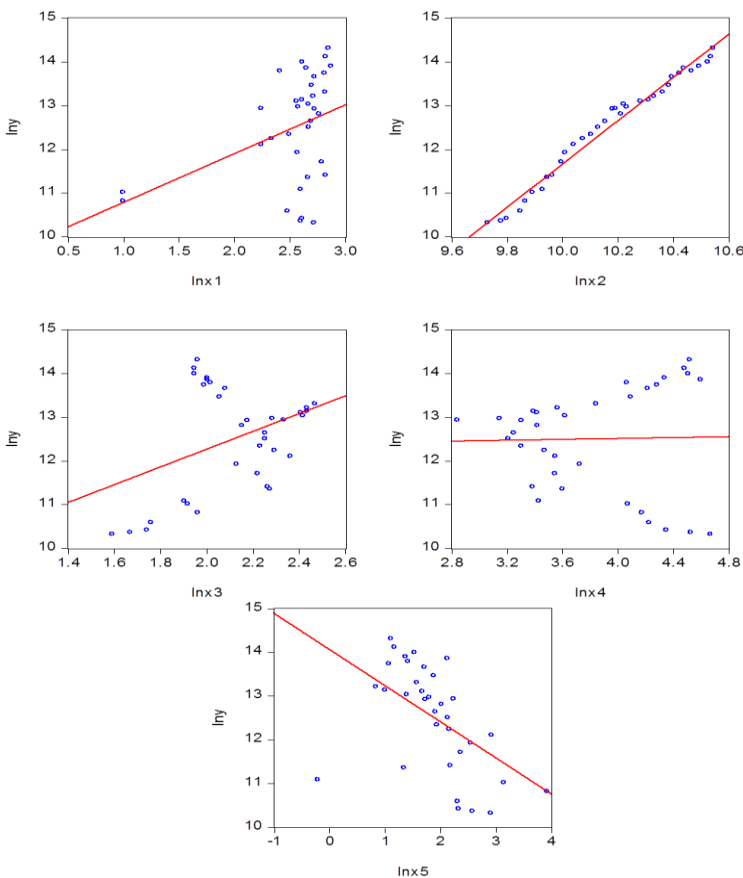
The relationships of the Independent to the Dependent Variables using transformed data (Table 7.2.2) were ascertained by Pearson's coefficient of correlation, as shown in the table below.

TABLE 7.2.2

	Real GDP growth rate ($\ln x_1$)	Employment Population ($\ln x_2$)	Unemployment Rate ($\ln x_3$)	Annual Crude Oil Prices ($\ln x_4$)	Inflation Rate ($\ln x_5$)
Income Tax Revenue ($\ln y$)	0.389182	0.987814	0.389073	0.021117	-0.520883
p-value	0.0229	0.0000	0.230	0.9056	0.0016

The table 7.2.2 shows that the transformed variables Real GDP Rate, Unemployment Rate, and Annual Crude Oil Prices were not significantly correlated with the dependent variable. In contrary, the transformed variables Employment population and Inflation Rate were significantly correlated with Dependent variable a $t \alpha = 0.01$ level.

SCATTER DIAGRAM 7.2.2



The scatter diagram 7.2.2 shows that there is significant Linear relationship between Income Tax Revenue (ln y) and Independent variables: Employment Population (ln x2) and Inflation Rate (ln x5) given that the resulted p-value of the

Independent variables are less than $\alpha = .01$. It also shows that there is no significant Linear relationship between Income Tax Revenue ($\ln y$) and Independent variables : Real GDP rate ($\ln x_1$), Unemployment Rate ($\ln x_3$) and Annual Crude Oil Prices ($\ln x_4$) provided that the resulted p-value of the Independent variables are greater than $\alpha = .01$.

7.3. Proposed Mathematical Model

Matrix theory is used in facilitating mathematical manipulations since the researchers have more than two variables in fitting a Multiple linear regression.

The least squares estimating equations $(X'X) b = X'y$

$$y = \begin{bmatrix} 34 & 86.306363 & 345.745277 & 71.892146 & 130.055632 & 63.902825 \\ 86.306363 & 224.979946 & 878.937252 & 182.897451 & 330.071967 & 155.736242 \\ 345.745277 & 878.937252 & 3517.80263 & 731.633518 & 1323.03578 & 646.471494 \\ 71.892146 & 182.897451 & 731.633518 & 153.788292 & 272.039045 & 133.814298 \\ 130.055632 & 330.071967 & 1323.03578 & 272.039045 & 506.038416 & 245.9443 \\ 63.902825 & 155.736242 & 646.471494 & 133.814298 & 245.9443 & 139.366686 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

$$= \begin{bmatrix} 425.28566 \\ 1086.12969 \\ 4334.25817 \\ 902.860982 \\ 1627.21774 \\ 783.419097 \end{bmatrix}$$

and then, using the relation $b = (X'X)^{-1}X'y$, the estimated regression coefficients are obtained as:

$$b_0 = -39.812; \quad b_1 = 0.318; \quad b_2 = 5.197; \quad b_3 = -0.024; \quad b_4 = -0.288; \quad b_5 = 0.147$$

The coefficients were obtained using MATLAB. Therefore, the Income Tax Revenue in the Philippines can be computed using the regression equation:

$$\ln \hat{y} = -39.812 + 0.138 \ln x_1 + 5.197 \ln x_2 - 0.024 \ln x_3 - 0.288 \ln x_4 + 0.147 \ln x_5$$

7.4. Significant factors that can actually predict the Dependent Variable (y)

TABLE 7.4.1

Values of the Coefficients

Dependent Variable: $\ln y$

Method: Least Squares

Date: 03/24/14 Time: 14:42

Sample: 1980 2013

Included observations: 34

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-39.81238	1.211607	-32.85914	0.0000
$\ln x_1$	0.138294	0.067374	2.052624	0.0496
$\ln x_2$	5.196966	0.146490	35.47658	0.0000
$\ln x_3$	-0.024130	0.195314	-0.123543	0.9026
$\ln x_4$	-0.288436	0.087520	-3.295665	0.0027
$\ln x_5$	0.146969	0.043273	3.396350	0.0021
R-squared	0.990340	Mean dependent var		12.50840
Adjusted R-squared	0.988615	S.D. dependent var		1.210984
S.E. of regression	0.129211	Akaike info criterion		-1.095949
Sum squared resid	0.467476	Schwarz criterion		-0.826591
Log likelihood	24.63113	Hannan-Quinn criter.		-1.004090
F-statistic	574.1213	Durbin-Watson stat		1.505563
Prob(F-statistic)	0.000000			

As shown in Table 7.4.1, Real GDP ($\ln x_1$) has a p-value of 0.0496, Employment Population ($\ln x_2$) with p-value of 0.0000, p-value of 0.9026 for Unemployment Rate ($\ln x_3$), 0.0027 for Oil Prices ($\ln x_4$) and 0.0021 for Inflation Rate ($\ln x_5$). Thus, there are three variables which are significant, Employment Population ($\ln x_2$), Oil Prices ($\ln x_4$) and Inflation Rate ($\ln x_5$).

7.5 Difference between Actual and Predicted Value

TABLE 7.5.1

Actual Value	Predicted Value	Difference of Actual and Predicted Value
30461	25919.1	4541.903
31812	32545.81	-733.808
33630	36699.09	-3069.09
39848	48008.72	-8160.72
50118	55008.19	-4890.2
61190	57945.69	3244.311
65491	64286.42	1204.582
85923	83900.24	2022.762
90352	113281.6	-22929.6
122462	131148.2	-8686.16
151700	133527.8	18172.2
182275	166003	16272.04
208706	183215	25491.01
230170	221152.6	9017.428
271305	274515.5	-3210.55
310517	300440.7	10076.26
367895	396112.7	-28217.7
412165	332589.1	79575.91
416585	402366.2	14218.83
431686	447724.9	-16038.9
460034	351932.6	108101.4
493608	523782.4	-30174.4
507637	563932.7	-56295.7
550468	585907.7	-35439.7
604964	718884	-113920
705615	778128.8	-72513.8
859857	774273.3	85583.69
932937	810777.1	122159.9
1049189	915977.8	133211.2
981631	1077928	-96296.5
1093643	1215878	-122235
1202066	1350724	-148658
1361081	1406288	-45207.4
1651256	1443352	207904.1

Table 7.5.2

Paired T-Test Result

Hypothesis Testing for DIFF
 Date: 04/04/14 Time: 22:20
 Sample: 1980 2013
 Included observations: 34

Test of Hypothesis: Mean = 0.000000

Sample Mean = -1184.998

Sample Std. Dev. = 73278.69

<u>Method</u>	<u>Value</u>	<u>Probability</u>
t-statistic	-0.094293	0.9254

The above Table 7.5.2 divulges that p-value of the Paired T-Test results to 0.9254, which means that: There is no significant difference between the Actual and Predicted Value and that the model can actually predict the Income Tax Revenue in the Philippines.

8. Summary of Findings

From the results of data analysis, the following are the findings of the study:

8.1 Behavior of the graph of the variables used

8.1.1. The GDP growth rate recorded 4 years of having a negative growth rate: In 1984, 1985, 1991 and 1998 as follows. Having in year 1984 and 1985 a decrease of 9.2% where GDP is at its lowest point. It continues to fluctuate until it reached its peaked in 2010.

8.1.2. Employment population on the other hand, has a continuous increase in its number except for the years 1997 and 2000.

8.1.3. For the 34 years, the Unemployment rate continuous to fluctuate and recorded first year as its lowest point and hit the highest point at 2004.

8.1.4. Unlike in Unemployment rate, Oil Prices noted its peak in 1980 and its lowest point in year 1998.

8.1.5. The inflation rate graph fluctuates widely for 34 years. It strikes its peaked in 1984 and has its lowest point in year 1986.

8.1.6. The graph of Income Tax Revenue collection mainly increased for 34 years. In 1980 to 1988 there are lower rate of increase than others years and it fell in year 2009 but followed by a prompt increased in the last five (5) years.

8.2 Relationship of Independent Variables to Dependent Variable

Using the Original Data, based on the result of Pearson's coefficient of correlation, Employment Population and Annual Crude Oil Prices were significantly correlated with Income Tax Revenue in the Philippines at $\alpha = 0.01$. On the other hand, using the Transformed Data, Employment Population and Inflation Rate were significantly correlated with Income Tax Revenue in the Philippines.

8.3. Proposed mathematical model

$$\ln \hat{y} = -39.812 + 0.138 \ln x_1 + 5.197 \ln x_2 - 0.024 \ln x_3 - 0.288 \ln x_4 + 0.147 \ln x_5$$

The model estimates the income tax revenue in the Philippines and was significant with p-value of 0.000 and coefficient of determination $R^2 = 0.990340$.

8.4. Significant factors that can actually predict the dependent variable (y)

Out of the five (5) Independent Variables, there are only three (3) factors that are significant. These are: Employment Population ($\ln x_2$) with a p-value of 0.000, Oil Prices ($\ln x_4$) has 0.002 and 0.002 for Inflation Rate ($\ln x_5$).

8.5 Difference between actual and predicted value

The Paired T-Test results to 0.9254 which is higher than the level of significance 0.01. Therefore, there is no significant difference between the Actual and Predicted Value.

9. Conclusion

Based on the findings of the study, the conclusions were drawn:

The assumptions of Multiple Linear Regression were all satisfied. The formulated mathematical model shows that there are three significant factors that can actually predict the Income Tax Revenue (y). These are: Employment Population(X_2), Annual Crude Oil Prices(X_4), and Inflation Rate(X_5). The model also shows that it can actually estimate the Income Tax Revenue of the Philippines since there is no significant difference between the Actual and Predicted Value.

10. Recommendation

The researchers propose looking for more independent variables such as: tax efforts, poverty incidence, Gini coefficient and income distribution. It also suggests adding more series of data to assess Income Tax Revenue in the Philippines more accurately.

REFERENCES

- [1] The Impact of Income Tax Rates (ITR) on the Economic Development of Botswana ;Bonu N. S.* and Pedro Motau P.; Faculty of Business, University of Botswana, Gaborone, Botswana.; Accepted 26 March, 2009
http://www.academicjournals.org/article/article1379341228_Bonu%20and%20Pedro.pdf
- [2] <http://www.nber.org/digest/mar08/w13264.html> Christina D. Romer & David H. Romer, 2010. "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks," American Economic Review, American Economic Association, vol. 100(3), pages 763-801, June.

[3]<http://www.intechopen.com/books/international-trade-from-economic-and-policy-perspective/the-impact-and-consequences-of-tax-revenues-components-on-economic-indicators-evidence-from-panel-gr> L. Bretschger, 2010 Taxes, mobile capital, and economic dynamics in a globalizing world. *Journal of Macroeconomics* 32594605

[4]http://taxfoundation.org/article/what-evidence-taxes-and-growth#_ftnref26 William McBride, *The Great Recession and Volatility in the Sources of Personal Income*, Tax Foundation Fiscal Fact No. 316 (June 13, 2012)

[5]<http://dirp4.pids.gov.ph/ris/wp/pidswp8103.pdf> PHILIPPINE ECONOMIC UPDATEACCELERATING REFORMS TO MEET THE JOBS CHALLENGE May 2013(Philippine Institute for Development Studies ,Working Paper 81-03 “PUBLIC FINANCE IN THE PHILIPPINES: A REVIEW OF THE LITERATURE By ROSARIO G. MANASAN)

[6]http://www.univ-orleans.fr/leo/images/espace_commun/seminaires/semmar2010/WP_29.pdf Adoption of Inflation Targeting and Tax Revenue Performance in Emerging Market Economies: An Empirical Investigation

[7]<http://www.thehindu.com/business/Economy/inflation-slowing-growth-to-impact-revenue-collection-finance-ministry/article2044876.ece> Inflation, slowing growth to impact revenue collection: Finance Ministry, NEW DELHI, May 24, 2011

[8]http://www.psi.org.uk/pdf/2011/fuel_duty_psi_green_alliance.pdf Road transport fuel prices, demand and tax revenues: impact of fuel duty escalator and price stabilizer

[9] The Relationship between Inflation Rate, Oil Revenue and Taxation in IRAN: Based OLS Methodology *Journal of Economics and Sustainable Development* www.iiste.org ISSN 2222-1700 (Paper) ISSN 2222-2855 (Online) Vol.4, No.19, 2013

- [10] Mathematical Model of Income Tax Revenue on the UK example: Svetlana Ivanitskaya, Ilona V. Tregub : Financial University under the Government of Russian Federation
<http://www.freit.org/WorkingPapers/Papers/Development/FREIT550.pdf>
- [11] <http://ec.europa.eu/social/BlobServlet?docId=7404&langId=en> UNIVERSITA' BOCCONI - Econpubblica - Centre for Research on the Public Sector - Milan, 10 May 2011
- [12] <http://www.counterpunch.org/2012/10/09/unemployment-and-marginal-tax-rates/> Unemployment And Marginal Tax Rates
- [13] http://darp.lse.ac.uk/pdf/EC426/EC426_11.pdf LABOR SUPPLY RESPONSES TO TAXES AND TRANSFERS: PART I (BASIC APPROACHES) Lecture Notes for MSc Public Finance (EC426): Lent 2013
- [14] Probability and Statistics for Engineers and Scientist; Eight Edition; Ronald Walpole; pg. 446-447
- [15] Practical Assessment Research and Evaluation; A peer-reviewed electronic journal. ISSN 1531-7714
<http://pareonline.net/getvn.asp?v=8&n=6>

Data Sources

National Statistical Coordination Board
Department of Labor and Employment
Philippine Institute for Development Studies
WorldBank

Appendices

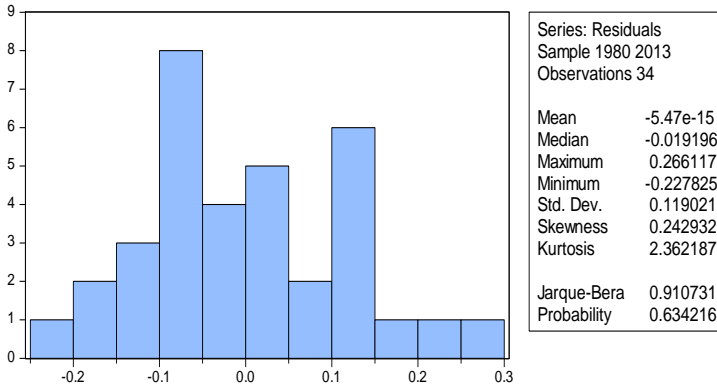
Appendix 1: Original Data

Year	GDP growth (annual %) (X_1)	Employment Population (X_2)	Unemployment Rate (X_3)	Annual Average Domestic Crude Oil Prices (in \$/Barrel) (Inflation Adjusted Price) (X_4)	Inflation, consumer prices (annual %) (X_5)	Tax revenue (current LCU) (in million) (Y)
1980	5.1	16794	4.9	106.36	18.2	30461
1981	3.4	17631	5.3	92.1	13.1	31812
1982	3.6	17993	5.7	77.21	10.2	33630
1983	1.9	18898	5.8	68.32	10	39,848
1984	-7.3	19238	7.1	64.75	50.3	50,118
1985	-7.3	19749	6.8	58.54	23.1	61,190
1986	3.4	20489	6.7	30.8	0.8	65,491
1987	4.3	20833	9.7	36.54	3.8	85,923
1988	6.8	21205	9.6	29.45	8.8	90,352
1989	6.2	21908	9.2	34.58	10.6	122,462
1990	3	22212	8.4	41.4	12.7	151,700
1991	-0.6	22914	10.6	34.7	18.5	182,275
1992	0.3	23696	9.9	32.09	8.6	208,706
1993	2.1	24382	9.3	27.13	6.9	230,170
1994	4.4	25032	9.5	24.71	8.4	271,305
1995	4.7	25677	9.5	25.72	6.7	310,517
1996	5.8	27186	8.6	30.5	7.5	367,895
1997	5.2	26365	8.8	27.17	5.6	412,165
1998	-0.6	26631	10.3	17.1	9.3	416,585
1999	3.1	27742	9.8	23.2	6	431,686
2000	4.4	27452	11.2	37.19	4	460,034
2001	2.9	29156	11.1	30.4	5.3	493,608
2002	3.6	30062	11.4	29.64	2.7	507,637
2003	5	30635	11.4	35.22	2.3	550,468
2004	6.7	31613	11.8	46.6	4.8	604,964
2005	4.8	32313	7.8	59.88	6.5	705,615
2006	5.2	32636	8	67.63	5.5	859,857
2007	6.6	33560	7.3	72.3	2.9	932,937
2008	4.1	34089	7.4	99.06	8.3	1,049,189
2009	1.1	35061	7.5	58.2	4.1	981,631
2010	7.6	36035	7.4	76.38	3.9	1,093,643
2011	3.6	37192	7	90.52	4.6	1,202,066
2012	6.8	37600	7	88.11	3.2	1,361,081
2013	7.2	37917	7.1	91.54	3	1,651,256

Appendix 2: Transformed Data

Year	LNX1	LNX2	LNX3	LNX4	LNX5	LNy
1980	2.714695	9.728777	1.589235	4.66683	2.901422	10.3242
1981	2.595255	9.777414	1.667707	4.522875	2.572612	10.3676
1982	2.61007	9.797738	1.740466	4.346529	2.322388	10.42317
1983	2.476538	9.846811	1.757858	4.224203	2.302585	10.59283
1984	0.993252	9.864643	1.960095	4.170534	3.918005	10.82214
1985	0.993252	9.890858	1.916923	4.06971	3.139833	11.02174
1986	2.595255	9.927643	1.902108	3.427515	-0.223144	11.08967
1987	2.66026	9.944294	2.272126	3.598408	1.335001	11.36121
1988	2.821379	9.961992	2.261763	3.382694	2.174752	11.41147
1989	2.785011	9.994607	2.219203	3.543275	2.360854	11.71556
1990	2.564949	10.00839	2.128232	3.723281	2.541602	11.92966
1991	2.24071	10.0395	2.360854	3.54674	2.917771	12.11327
1992	2.332144	10.07306	2.292535	3.468544	2.151762	12.24868
1993	2.493205	10.1016	2.230014	3.30064	1.931521	12.34657
1994	2.667228	10.12791	2.251292	3.207208	2.128232	12.511
1995	2.687847	10.15335	2.251292	3.247269	1.902108	12.64599
1996	2.76001	10.21046	2.151762	3.417727	2.014903	12.81555
1997	2.721295	10.17979	2.174752	3.302113	1.722767	12.92918
1998	2.24071	10.18983	2.332144	2.839078	2.230014	12.93985
1999	2.572612	10.2307	2.282382	3.144152	1.791759	12.97545
2000	2.667228	10.22019	2.415914	3.61604	1.386294	13.03906
2001	2.557227	10.28042	2.406945	3.414443	1.667707	13.1095
2002	2.61007	10.31102	2.433613	3.389125	0.993252	13.13752
2003	2.70805	10.3299	2.433613	3.561614	0.832909	13.21852
2004	2.815409	10.36132	2.4681	3.841601	1.568616	13.31292
2005	2.694627	10.38322	2.054124	4.092343	1.871802	13.46683
2006	2.721295	10.39317	2.079442	4.214052	1.704748	13.66452
2007	2.809403	10.42109	1.987874	4.280824	1.064711	13.74609
2008	2.646175	10.43673	2.00148	4.595726	2.116256	13.86353
2009	2.406945	10.46484	2.014903	4.063885	1.410987	13.79697
2010	2.867899	10.49225	2.00148	4.335721	1.360977	13.90502
2011	2.61007	10.52385	1.94591	4.505571	1.526056	13.99955
2012	2.821379	10.53476	1.94591	4.478586	1.163151	14.12379
2013	2.844909	10.54315	1.960095	4.516776	1.098612	14.31705

Appendix 3: Test for Normality Results



Ho: The residuals follow a normal distribution
 Ha: The residuals do not follow a normal distribution

Rejection Rule: If p-value < $\alpha = 0.01$, reject the null hypothesis

Conclusion

Since p-value 0.634216 is greater than 0.01 then, FAIL TO REJECT the null hypothesis for the Jarque-Bera Test. Therefore, the residuals follow a normal distribution.

Appendix 4: Test for Linearity Results

Dependent Variable: *lny*
 Method: Least Squares
 Date: 03/28/14 Time: 11:36
 Sample: 1980 2013
 Included observations: 34

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-39.81242	1.211645	-32.85815	0.0000
<i>lnx₁</i>	0.138292	0.067376	2.052532	0.0496
<i>lnx₂</i>	5.196970	0.146495	35.47550	0.0000
<i>lnx₃</i>	-0.024127	0.195320	-0.123523	0.9026
<i>lnx₄</i>	-0.288436	0.087522	-3.295564	0.0027

lnx_5	0.146969	0.043274	3.396264	0.0021
R-squared	0.990340	Mean dependent var	12.50840	
Adjusted R-squared	0.988615	S.D. dependent var	1.210984	
S.E. of regression	0.129215	Akaike info criterion	-1.095890	
Sum squared resid	0.467504	Schwarz criterion	-0.826532	
Log likelihood	24.63013	Hannan-Quinn criter.	-1.004031	
F-statistic	574.0865	Durbin-Watson stat	1.505555	
Prob(F-statistic)	0.000000			

Ho: No explanatory variable has an effect on the dependent variable.

Ha: At least one explanatory variable has an effect on the dependent variable.

Rejection Rule: If $p\text{-value} < \alpha = 0.01$, reject the null hypothesis

Conclusion

Since $p\text{-value}$ 0.000000 is less than 0.01 then, REJECT the null hypothesis for the Wald Test. Therefore, at least one explanatory variable has an effect on the dependent variable.

Appendix 5: Test for Independence Results

Dependent Variable: lny

Method: Least Squares

Date: 03/27/14 Time: 14:31

Sample: 1980 2013

Included observations: 34

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-39.81238	1.211607	-32.85914	0.0000
lnx_1	0.138294	0.067374	2.052624	0.0496
lnx_2	5.196966	0.146490	35.47658	0.0000
lnx_3	-0.024130	0.195314	-0.123543	0.9026
lnx_4	-0.288436	0.087520	-3.295665	0.0027
lnx_5	0.146969	0.043273	3.396350	0.0021

Jackie D. Urrutia, Joseph Mercado and Judy Ann Quite- **Mathematical Model for Estimating Income Tax Revenues in the Philippines through Regression Analysis Using Matrices**

R-squared	0.990340	Mean dependent var	12.50840
Adjusted R-squared	0.988615	S.D. dependent var	1.210984
S.E. of regression	0.129211	Akaike info criterion	-1.095949
Sum squared resid	0.467476	Schwarz criterion	-0.826591
Log likelihood	24.63113	Hannan-Quinn criter.	-1.004090
F-statistic	574.1213	Durbin-Watson stat	1.505563
Prob(F-statistic)	0.000000		

General Rule: The p-value should range from 0 to 4 and the residuals are not correlated if the Durbin-Watson statistics is approximately 2, and an acceptable range is 1.50 to 2.50.

Conclusion

Since the result of Durbin Watson test resulted to 1.505563 which is in the range of the rule of thumb (1.5-2.5) therefore, it satisfies the Assumption that residuals are independent.

Appendix 6: Test for Homoscedasticity Results

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	0.939987	Prob. F(5,28)	0.4706
\[Obs*R-squared	4.886792	Prob. Chi-Square(5)	0.4299
Scaled explained SS	2.257298	Prob. Chi-Square(5)	0.8125

Test Equation:

Dependent Variable: RESID²

Method: Least Squares

Date: 03/27/14 Time: 14:34

Sample: 1980 2013

Included observations: 34

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.057414	0.153436	0.374189	0.7111
LNX1	0.010882	0.008532	1.275456	0.2126
LNX2	-0.022208	0.018551	-1.197138	0.2413
LNX3	0.040700	0.024734	1.645487	0.1111
LNX4	0.017352	0.011083	1.565585	0.1287
LNX5	0.001125	0.005480	0.205363	0.8388

Jackie D. Urrutia, Joseph Mercado and Judy Ann Quite- **Mathematical Model for Estimating Income Tax Revenues in the Philippines through Regression Analysis Using Matrices**

R-squared	0.143729	Mean dependent var	0.013749
Adjusted R-squared	-0.009176	S.D. dependent var	0.016289
S.E. of regression	0.016363	Akaike info criterion	-5.228793
Sum squared resid	0.007497	Schwarz criterion	-4.959435
Log likelihood	94.88948	Hannan-Quinn criter.	-5.136934
F-statistic	0.939987	Durbin-Watson stat	2.549888
Prob(F-statistic)	0.470591		

Ho: Errors are Homocedastic

Ha: Errors are Heterocedastic

Rejection Rule: If p-value < $\alpha = 0.01$, reject the null hypothesis

Conclusion

Since p-value 0.470591 is greater than 0.01 then, FAIL TO REJECT the null hypothesis for the Breusch-Pagan Heterocedasticity Test. Therefore, errors are homoscedastic.

Appendix 7: Test for Multicollinearity Results

Variance Inflation Factors

Date: 03/27/14 Time: 14:40

Sample: 1980 2013

Included observations: 34

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	1.467992	2989.518	NA
LNX1	0.004539	61.16893	1.603568
LNX2	0.021459	4521.540	2.475883
LNX3	0.038148	351.3891	4.053748
LNX4	0.007660	232.1644	3.924509
LNX5	0.001873	15.63086	2.160317

General Rule: The Variance Inflation Factor should be less than 10 to satisfy Multicollinearity.

Conclusion

Since the result Multicollinearity test resulted to VIF of all less than 10 therefore, it satisfies the Assumption that there is no multicollinearity among the variables.

Appendix 8: Values of Actual and Predicted Y

Year	ACTUAL VALUE	PREDICTED VALUE
1980	30461	25919.1
1981	31812	32545.81
1982	33630	36699.09
1983	39848	48008.72
1984	50118	55008.19
1985	61190	57945.69
1986	65491	64286.42
1987	85923	83900.24
1988	90352	113281.6
1989	122462	131148.2
1990	151700	133527.8
1991	182275	166003
1992	208706	183215
1993	230170	221152.6
1994	271305	274515.5
1995	310517	300440.7
1996	367895	396112.7
1997	412165	332589.1
1998	416585	402366.2
1999	431686	447724.9
2000	460034	351932.6
2001	493608	523782.4
2002	507637	563932.7
2003	550468	585907.7
2004	604964	718884
2005	705615	778128.8
2006	859857	774273.3
2007	932937	810777.1
2008	1049189	915977.8
2009	981631	1077928
2010	1093643	1215878
2011	1202066	1350724
2012	1361081	1406288
2013	1651256	1443352

Appendix 9: Difference of the Actual and Predicted Value

Difference Between the Actual and Predicted Value	
1980	4541.903
1981	-733.8082
1982	-3069.093
1983	-8160.718
1984	-4890.195
1985	3244.311
1986	1204.582
1987	2022.762
1988	-22929.58
1989	-8686.157
1990	18172.2
1991	16272.04
1992	25491.01
1993	9017.428
1994	-3210.55
1995	10076.26
1996	-28217.68
1997	79575.91
1998	14218.83
1999	-16038.94
2000	108101.4
2001	-30174.42
2002	-56295.71
2003	-35439.67
2004	-113920
2005	-72513.79
2006	85583.69
2007	122159.9
2008	133211.2
2009	-96296.5
2010	-122235
2011	-148657.8
2012	-45207.44
2013	207904.1

Appendix 10: Paired T-Test Result

Hypothesis Testing for DIFFERENCE

Date: 03/27/14 Time: 14:07

Jackie D. Urrutia, Joseph Mercado and Judy Ann Quite- **Mathematical Model for Estimating Income Tax Revenues in the Philippines through Regression Analysis Using Matrices**

Sample: 1980 2013

Included observations: 34

Test of Hypothesis: Mean = 0.000000

Sample Mean = 709.4236

Sample Std. Dev. = 73149.71

<u>Method</u>	<u>Value</u>	<u>Probability</u>
t-statistic	0.056550	0.9552

Ho: There is no significant difference between the Actual and Predicted Value.

Ha: There is a significant difference between the Actual and Predicted Value.

Rejection Rule: If p-value < $\alpha = 0.01$, reject the null hypothesis

Conclusion

Since p-value 0.9552 is greater than 0.01 then, FAIL TO REJECT the null hypothesis for the Paired T-Test. Therefore, there is no significant difference between the Actual and Predicted Value