EUROPEAN ACADEMIC RESEARCH
Vol. III, Issue 4/ July 2015

Impact Factor: 3.4546 (UIF)
DRJI Value: 5.9 (B+)

ISSN 2286-4822
www.euacademic.org

The Influence of the Effective Moment of Inertia on the Deflection of FRP Reinforced Concrete Members

ENIO DENEKO
Polytenchnic University of Tirana
Faculty of Civil Engineering, Tirana, Albania
ANJEZA GJINI
Polytenchnic University of Tirana
Faculty of Civil Engineering, Tirana, Albania

Abstract:

Fiber reinforced polymer (FRP) bars, beacause of their non-corrosive properties, emerged as an alternative to steel bars in RC structures in agressive environments. Mechanical characteristics of FRP materials, such as lower elastic modulus, lower ratio between Young's modulus and the tensile strength, lower bond strength of FRP bars and concrete, compared to steel reinforcement, can lead to large deflections and crack width, so this make that serviceability design determine the design of FRP reinforced concrete, based on the serviceability requirements. Different parameters can influence on the stresses in materials, maximum crack width and the allowed deflections, but one of the most important especially for the deflection design, is the effective moment of inertia. In this paper, reference is made ACI, EC2 and other models, regarding deflections of FRP reinforcement concrete beams comparing with each other and also with steel reinforcement concrete formulas. Concrete beams reinforced with glass-fiber (GFRP) bars, exhibit large deflections compared to steel reinforced concrete beams, because of low GFRP bars elasticity modulus. For this purpose we have used equations to estimate the effective moment of inertia of FRP-reinforced concrete beams, based on the genetic algorithm, known as the Branson's equation. In the last two decades, a number of researchers adjusted the Branson's equation *based on a lot of test results. In the paper, is elaborated a numerical example to check the deflection of a FRP-RC based on these equations of different code provisions and models, used for predicting the deflection of FRP-reinforced concrete beams.*

Key words: FRP bars, reinforced concrete beams, serviceability, deflection, effective moment of inertia, modulus of elasticity.

1 Introduction

FRP bars can be effectively used in corrosive environments because of their corrosion resistant property. The problems seem similar to steel RC, but solutions, limits and analytic models are different, because of the very large band of FRP bars on the market, with a large variety of mechanical characteristics. There are many types of fibers including glass (GFRP), carbon (CFRP) and aramid (AFRP), with different grades of tensile strength and modulus of elasticity. The behavior of FRP-RC beams differs from steel RC beams, because FRP bars display a linear elastic behavior up to the point of failure and do not demonstrate ductility. Also the bond strength of FRP bars and concrete is lower than that of steel bars, leading to an increase in the depth of cracking, a decrease of stiffening effect, and so an increase of the deflection of FRP-RC beams for an equivalent cross-section of reinforcement of steel reinforced concrete beams.

GFRP-RC beams have lower elastic modulus quite 20-25% of that of steel bars. Because of it, the deflection criterion may control the design of long FRP-RC beams. Consequently a method is needed in order to know the expected service load deflections with a high degree of accuracy. Only some countries have developed a code in FRP-RC design that is still in preparation phase, so is very difficult to operate in this new field.

2 Methodology

FRP are anisotropic materials and are characterized by high tensile strength with no yielding in the direction of the reinforcing fibers. An FRP-RC member is designed on its required strength and then checked for serviceability and ultimate state criteria (e.g., deflection, crack width, fatigue and creep rapture). In most instances, serviceability criteria will control the design so safety checks for FRP-RC are more important than that for steel-RC.

A well-known deflection model for steel-RC is proposed by ACI and EC2, later modified for FRP-RC. This deflection model simulates the real behavior of the structure by taking into account cracking, but not the tension-stiffening effect of concrete.

ACI 318-95 is based on Branson [1968-1977] formulae of the effective moment of inertia for Steel-RC:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \,. \qquad \textbf{(1)}$$

where, I_e is the effective moment of inertia, M_{cr} is the cracked section moment, I_g is the total moment of inertia, M_a is the maximum moment in member at the deflection stage, I_{cr} is the cracked section moment of inertia.

ACI 440R-96 proposed new formulae of the effective moment of inertia for FRP-RC:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \,. \qquad \textbf{(2)}$$

where, β_d is a reduction coefficient related to the reduced tension-stiffening exhibited by FRP-RC members. Based on evaluation of experimental results

$$\beta_d = \frac{1}{5}\left(\frac{\rho_f}{\rho_{fb}}\right) \leq 1 \,. \qquad \textbf{(3)}$$

This equation is valid only if $M_a \geq M_{cr}$. If $M_a \leq M_{cr}$, than $I_e = I_g$ and if $M_a \approx M_{cr}$, or slightly less, than $I_e = I_{cr}$, because shrinkage and temperature may cause section cracking.

Some other models based on ACI give this formula:

$$I_e = \frac{I_g}{\beta} \cdot \left(\frac{M_{cr}}{M_{max}}\right)^3 + \alpha \cdot I_{cr} \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3 \right] \leq I_g . \quad \textbf{(4)}$$

where, α and β are coefficients of bond properties of FRP and

$$\frac{1}{\beta} = \alpha^* \left(\frac{E_{frp}}{E_s} + 1\right) \leq 1 . \quad \textbf{(5)}$$

where, α^* is a coefficient that increases with the bond quality. When no experimental results are given than α^*= 0, 5 and α = 1.

EuroCode 2 propose a simplified model for Steel-RC

$$v = v_1 \, \gamma + v_2 \cdot (1 - \gamma) \text{ where } \gamma = \beta \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m . \quad \textbf{(6)}$$

Here, v_1 and v_2 are calculated taking into account that the moment of inertia of the section is constant and respectively I_1 and I_2 (or I_g and I_{cr}). For steel bars the coefficients β and m, are to be taken as given in the table.

Table 1. Coefficient β and m for steel bars

Model	β	m
EC2	1,0	2,0
CEB	0,8	1,0

For FRP bars, these coefficients must be reconsidered through experimental tests.

EC2 hasn't proposed yet a model for FRP-RC deflection design, but the Italian Code CNR-DT 203/2006, based on experimental tests, shows that the model for EC2 can be suitable for FRP-RC too. Therefore the EC2 equations to compute deflection "f" must be reconsidered:

$$f = f_1 \cdot \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m + f_2 \cdot \left[1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m \right] . \quad \textbf{(7)}$$

f_1 = deflection of uncracked section

f_2 = deflection of cracked section

β_1 = 0,5 coefficient of bond properties of FRP bars

β_2=coefficient of the duration of loading: β_2=1 for short-time loads and β_2=0,5 for long-time cycling loads.

M_{max} = the maximum moment acting on the examined element

M_{cr} = the cracking moment calculated at the same cross section of M_{max}

m = 2

In the last two decades a number of researchers tried to adjust the Branson's equation, comparing to experimental results of FRP-RC member tests. The experimental results show that Branson's equation overestimated the moment of inertia I_e and underestimates the deflection, because the Branson's equation was calibrated for RC beams where $I_g/I_{cr} \leq 3$ [Bischoff 2005], but not for most members that have $5 \leq \frac{I_g}{I_{cr}} \leq 25$ [Bischoff 2009].

Also the bond behavior of FRP bars and concrete differs from the bond behavior of steel and concrete.

Benmokrane [1996], added two reduction factors and adjusted this equation:

$$I_e = \alpha \cdot I_{cr} + \left(\frac{I_g}{\beta} - \alpha \cdot I_{cr}\right) \cdot \left(\frac{M_{cr}}{M_{max}}\right)^3 \leq I_g \,. \qquad \textbf{(8)}$$

From experimental data α=0.84 and β=7, because of the nature of FRP reinforcement, with larger deflection and greater reduction of compressed concrete section when applied M_{cr}.

Faza and Gangarao [1992], proposed a model for two concentrated point loads based on the assumption that a concrete section between the point loads is fully cracked, while the end sections are partially cracked.

$$I_m = \frac{23 \cdot I_{cr} \cdot I_e}{8 \cdot I_{cr} + 15 \cdot I_e} \quad \text{where} \quad I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \,. \quad \textbf{(9)}$$

The maximum deflection is calculated as follows:

$$\Delta_{max} = \frac{23 \cdot P \cdot L^3}{648 \cdot E_c \cdot I_e} \,. \qquad \textbf{(10)}$$

Toutanji and Saafi [2000] adjusted the ratio M_{cr}/M_a to take into account the modulus of elasticity of FRP bars *(E_{frp})* and the reinforcement ratio *(ρ_f)*. They took a set of 13 GFRP-RC beams with a ratio $13 \leq \frac{I_g}{I_{cr}} \leq 25$. The model proposed was:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^m \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] \cdot I_{cr} \leq I_g \,. \qquad \textbf{(11)}$$

where: $m = 6 - \frac{10 \cdot E_f}{E_s} \cdot \rho_f,$ if $\frac{E_f}{E_s} \cdot \rho_f < 0.3$ and $m = 3,$ if $\frac{E_f}{E_s} \cdot \rho_f \geq 0.3$

Brown and Bartholomew [1996], used the same model based on tests of two GFRP-RC beams with the ratio $\frac{I_g}{I_{cr}} \approx 11$ and $m = 5$, while *Al-Sayed [2000]* proposed $m = 5{,}5$.

ACI 440.1 R-01 adopted the modification proposed by *GAO [1998]:*

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \;. \qquad \textbf{(12)}$$

where, β_d = 0,6 based on Masmoudi (1998) and Theriault & Benmokrane [1998] studies. They recommended,

$$\beta_d = \alpha_b \left(\frac{E_{frp}}{E_s} + 1\right) \leq I_g \,. \qquad \textbf{(13)}$$

α_b is a bond dependent coefficient: α_d =0.5 for GFRP [GAO 1998] and later based on experimental tests of 48 GFRP-RC and the amount of the longitudinal reinforcement:

$$\alpha_b = 0{,}064 \cdot \left(\frac{\rho_f}{\rho_{fb}}\right) + 0{,}13 \,. \qquad \textbf{(14)}$$

Recently, *Rafi and Nadjai [2009],* introduced γ factor, that reduces the portion of cracked moment of inertia:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot \frac{I_{cr}}{\gamma_{\;cr}} \leq I_g \;. \qquad \textbf{(15)}$$

$$\gamma = 0{,}86 \cdot \left(1 + \frac{E_f}{400}\right) . \qquad \textbf{(16)}$$

$$\beta_d = \frac{1}{5}\left(\frac{\rho_f}{\rho_{fb}}\right) . \qquad \textbf{(17)}$$

ISIS Canada [2001], based on *Ghali and Azarnejad [1999],* when service load level is less that cracked moment, M_{cr}, the immediate deflection can be evaluated using the transformed moment of inertia, I_t, instead of effective moment of inertia, I_e , used when service moments exceed the cracked moment.

Mota [2006] examined a number of the suggested formulations for I_e and found an equation that provided the most conservative results over the entire range of experimental results of test specimens.

$$I_e = \frac{I_t \cdot I_{cr}}{I_{cr} + \left[1 - \frac{1}{2}\left(\frac{M_{cr}}{M_a}\right)^2\right] \cdot (I_t - I_{cr})} \;. \qquad \textbf{(18)}$$

Where, I_t is the moment of inertia of a non-cracked concrete section, and

$$I_{cr} = \frac{b\cdot(k\cdot d)^3}{3} + n_{frp}\cdot A_{frp}\cdot(d-k\cdot d)^2 \; . \qquad \textbf{(19)}$$

$$k = \sqrt{\left(\rho\cdot n_{frp}\right)^2 + 2\cdot\rho\cdot n_{frp}} - \rho\cdot n_{frp} \; . \qquad \textbf{(20)}$$

$$n_{frp} = \frac{E_{frp}}{E_c} \; . \qquad \textbf{(21)}$$

$$\rho = \frac{A_{frp}}{b\cdot d} \; . \qquad \textbf{(22)}$$

"b" represents the width of cross-section (mm) and "d" the depth to FRP layer (mm).

CAN/CSA-S806 [2002] used *Razqapur* methodology which assumes that tension stiffening is insignificant in cracked regions on FRP-RC beams, using $E_c \cdot I_g$ when $M_a < M_{cr}$, and $E_c \cdot I_{cr}$ when $M_a > M_{cr}$, to integrate the curvature *M/EI* along the beam span. This leads to a simple expression for beam deflection δ_{max}, for a four-point bending configuration with two point loads at a distance a from the supports, assuming L_g, the distance that the beam is uncracked:

$$\delta_{max} = \frac{PL^3}{24E_cI_{cr}}\left[3\left(\frac{a}{L}\right) - 4\left(\frac{a}{L}\right)^3 - 8\left(1-\frac{I_{cr}}{I_g}\right)\left(\frac{L_g}{L}\right)^3\right]. \qquad \textbf{(23)}$$

Bischoff [2005], Bischoff [2007a], Bischoff and Scanlon [2007], proposed an equation derived from integration of curvatures along the beam taking into account the tension-stiffening effect:

$$I_e = \frac{I_{cr}}{1-\left(1-\frac{I_{cr}}{I_g}\right)\cdot\left(\frac{M_{cr}}{M_a}\right)^2} \; . \qquad \textbf{(24)}$$

Abdalla, *El-Badry* and *Rizkalla* introduced a model similar to EC2-CEB, suggesting α=0.85 and β=0.5

$$v = \left(\frac{M_{cr}}{M_a}\right)\cdot\beta\cdot v_1 + \left[1-\beta\left(\frac{M_{cr}}{M_a}\right)\right]\cdot\alpha\cdot v_2 \; . \qquad \textbf{(25)}$$

But *Abdalla [2002]* gave also a model based on ACI:

$$I_e = \frac{I_g\cdot I_{cr}}{I_{cr}\cdot\xi + 1{,}15\cdot I_g(1-\xi)} \; . \qquad \textbf{(26)}$$

where $\xi = \frac{0{,}5M_{cr}}{M_a}$. The coefficient of 1.15 (or better 1/0.85), takes into account the reduction of tension-stiffening effect in the fully cracked FRP concrete section.

Table 2. Different design models

Reference	**Models for I_e**
ACI 318 R -95 (1995) Branson formulae	$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g$
Benmokrane (1996)	$I_e = \frac{I_g}{7} \cdot \left(\frac{M_{cr}}{M_{max}}\right)^3 + 0{,}84 \cdot I_{cr} \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] \leq I_g$
ACI 440.1R -03 (2003)	$I_e = \frac{I_g}{\beta} \cdot \left(\frac{M_{cr}}{M_{max}}\right)^3 + \alpha \cdot I_{cr} \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] \leq I_g \; \frac{1}{\beta} = \alpha^* \left(\frac{E_{frp}}{E_s} + 1\right) \leq 1$, α^*= 0,5 and α = 1
Yost (2003) based on ACI	$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \beta_d = a_b \left(\frac{E_{frp}}{E_s} + 1\right) \; a_b = 0{,}064 \left(\frac{\rho_{frp}}{\rho_{fb}}\right) + 0{,}13$
ACI 440.1R -06 (2006)	$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr} \leq I_g \qquad \beta_d = \frac{1}{5}\left(\frac{\rho_{frp}}{\rho_{fb}}\right) \leq 1$
Rafi & Nadjai (2009)	$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot \beta_d \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot \frac{I_{cr}}{\gamma} \leq I_g \, \gamma = \left(0{,}0017 \frac{\rho_{frp}}{\rho_{fb}} + 0{,}8541\right) \cdot \left(1 + \frac{E_{frp}}{2 \cdot E_s}\right) \leq 1 \quad \gamma = 0{,}86 \cdot \left(1 + \frac{E_f}{400}\right)$ (GPa)
EC2-CEB, Italian Code CNR-DT 203/2006	$f = f_1 \cdot \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m + f_2 \cdot \left[1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m\right] \qquad m = 2$
Toutanji & Saafi (2000)	$I_e = \left(\frac{M_{cr}}{M_a}\right)^m \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] \cdot I_{cr} \leq I_g$ $m = 6 - \frac{10 \cdot E_f}{E_s} \cdot \rho_f$ if $\frac{E_f}{E_s} \cdot \rho_f < 0.3$ so m = 5.98 and m = 3 if $\frac{E_f}{E_s} \cdot \rho_f \geq 0.3$
Alsayed Model A (2000)	$I_e = \left(\frac{M_{cr}}{M_a}\right)^m \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] \cdot I_{cr} \leq I_g \qquad m = 5{,}5$
Alsayed Model B (2000)	$I_e = I_{cr}$ for $\frac{M_a}{M_{cr}} \geq 3$ $I_e = \left[1.4 - \frac{2}{15}\left(\frac{M_a}{M_{cr}}\right)\right] \cdot I_{cr}$ for $1 < \frac{M_a}{M_{cr}} < 3$
Bischoff (2005,2007) & Scanlon (2007)	$I_e = \frac{I_{cr}}{1 - \left(1 - \frac{I_{cr}}{I_g}\right) \cdot \left(\frac{M_{cr}}{M_a}\right)^2}$
Abdalla based on ACI (2002)	$I_e = \frac{I_g \cdot I_{cr}}{I_{cr} \cdot \xi + 1{,}15 \cdot I_g (1 - \xi)} \qquad \xi = \frac{0{,}5 M_{cr}}{M_a}$
Abdalla, Rizkalla & El Badry (EC2)	$v = \left(\frac{M_{cr}}{M_a}\right) \cdot \beta \cdot v_1 + \left[1 - \beta\left(\frac{M_{cr}}{M_a}\right)\right] \cdot \alpha \cdot v_2$ α = 0,85 dhe ß = 0,5 $v_1 = f_1, v_2 = f_2, v = f$
ISIS Canada (2001) & Mota (2006)	$I_e = \frac{I_t \cdot I_{cr}}{I_{cr} + \left[1 - \frac{1}{2}\left(\frac{M_{cr}}{M_a}\right)^2\right] \cdot (I_t - I_{cr})}$
Hall & Ghali (2000)	$I_m = \frac{I_t \cdot I_{cr}}{\left[I_t + \beta_1 \cdot \beta_2 \left(\frac{M_{cr}}{M_a}\right)^2 \cdot (I_{cr} - I_t)\right]} \beta_1 = 1{,}0 \; for\ ribbed\ bars \beta_2 = 0{,}5 \; for\ sustained\ loading$

3 The Numerical Example

We have made calculations for a simply supported, normal weight interior beam with a span length $l = 4\,m$ and $f_c' = 30\,MPa$. It is designed to carry a service live load of $w_{LL} = 6\,kN/m$ and a superimposed service dead load of $w_{SDL} = 3\,kN/m$. The cross section of the beam is to be taken as 250 mm x 400 mm. 4 Ø 16 GFRP bars are selected as main beam reinforcement and Ø 9.5 GFRP bars are selected as shear beam reinforcement. Material properties of GFRP bars are: tensile strength f_{fu}^*= 320 MPa, rupture strain ε_{fu}^*=0.014 and Modulus of elasticity E_f = 44 800 MPa.

4 The Result Comparison

All the analysis results taken from different methods are put in **Table 3.**

Reference	I_e	Δ_{LT}
ACI 318R-95 (1995), Branson	$13{,}08 \cdot 10^8\,mm^4$	$1{,}36mm$
Benmokrane (1996)	$1{,}887 \cdot 10^8\,mm^4$	$9{,}45mm$
ACI 440.1R-03 (2003)	$8{,}011 \cdot 10^8\,mm^4$	$2{,}22mm$
Yost (2003) based on ACI	$3{,}128 \cdot 10^8\,mm^4$	$5{,}70mm$
ACI 440.1R -06 (2006)	$2{,}641 \cdot 10^8\,mm^4$	$6{,}75mm$
Rafi & Nadjai (2009)	$2{,}642 \cdot 10^8\,mm^4$	$6{,}75mm$
EC2-CEB, Italian Code CNR-DT 203/2006		$8{,}72mm$
Toutanji & Saafi (2000)	$12{,}89 \cdot 10^8\,mm^4$	$1{,}39mm$
Alsayed Model A (2000)	$12{,}83 \cdot 10^8\,mm^4$	$1{,}38mm$
Alsayed Model B (2000)	$1{,}649 \cdot 10^8\,mm^4$	$10{,}82mm$
Bischoff (2005,2007) & Scanlon (2007)	$11{,}81 \cdot 10^8\,mm^4$	$1{,}51mm$
Abdalla based on ACI (2002)	$2{,}075 \cdot 10^8\,mm^4$	$8{,}60mm$
Abdalla, Rizkalla & El Badry (EC2)		$5{,}34mm$
ISIS Canada (2001) & Mota (2006)	$2{,}514 \cdot 10^8\,mm^4$	$7{,}48mm$
Hall & Ghali (2000)	$2{,}514 \cdot 10^8\,mm^4$	$7{,}48mm$

It is calculated the mid span displacements of a GFRP reinforced concrete beam, based on different equations for effective moments of inertia I_e, using different levels of loading (only different service dead loads, while the service live load is maintained constant), in normal reinforced ratio ($\frac{\rho_{frp}}{\rho_{fb}} \approx 1$) and in high reinforced ratio ($\frac{\rho_{frp}}{\rho_{fb}} \geq 2.5$). The results of the most used methods are included in comparative charts in order to find out some theoretical conclusions.

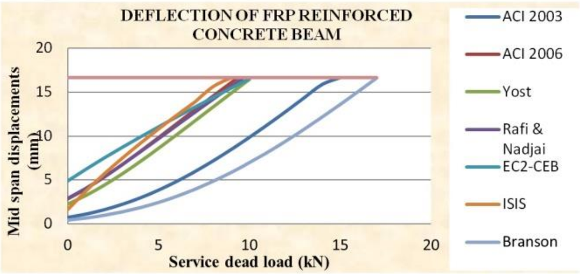

Fig. 1. Relation between service dead load and mid span displacement for $\frac{\rho_{frp}}{\rho_{fb}} \approx 1$

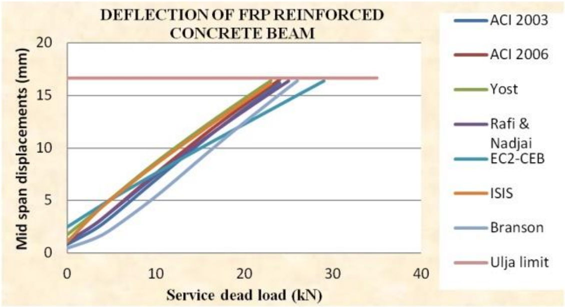

Fig. 2. Relation between service dead load and mid span displacement for $\frac{\rho_{frp}}{\rho_{fb}} \approx 2.53$

5 Summary and conclusions

Based on the numerical examples we used and the charts above, we can see that the most reliable methods, those with the most consistent theoretical results, are: ACI 440.1R-06 (2006), Yost (2003), Rafi & Nadjai (2009) based on ACI and ISIS Canada (2001& Canadian Code (CAN/CSA-S806 2002) a similar method, but at the same time some different to ACI Code. We can see also that the original Branson's equation is just an envelope for the other methods, for short beams we get smaller deflections than other methods and for longer ones we get bigger displacement than other methods. This because Branson's equation doesn't takes into account the type of FRP used as reinforcement, so it hasn't used any reduction factor based on the $\frac{E_{frp}}{E_s}$ ratios, or even on the quantity of the reinforcement used so the $\frac{\rho_{frp}}{\rho_{fb}}$ ratios, as other methods do. So this equation is more generalized and conservative. Some other methods use only one of these coefficients so they are conservative too.

Based on a lot of experimental results, the reinforcement ratio and the elastic modulus of FRP bars are the most significant variables to calculate deflections, especially the $\frac{\rho_{frp}}{\rho_{fb}}$ ratio, because the $\frac{E_{frp}}{E_s}$ ratio doesn't have a significant effect. Yost [2003] based on ACI, Rafi & Nadjai [2009] and also ACI 440.1R -06 [2006] use both reduction factors taking into account these variables and give good results, so this three methods, based on ACI code, gives more accurate results for beams with low and high reinforcement ratios. This is also shown in the charts where ACI 440.1R -06 [2006] preserve the same load ratios for the same deflection for $\frac{\rho_{frp}}{\rho_{fb}} = 1$ and $\frac{\rho_{frp}}{\rho_{fb}} = 2.5$, (see the load ratio for the same deflection $\frac{23\ kN}{9\ kN} = 2{,}55 \approx \frac{\rho_{frp}}{\rho_{fb}} = 2.5$). At Yost and Rafi & Nadjai, this ratio is 2-2.5.

Also, Hall& Gali, ISIS Canada [2001] and the Canadian Code [CAN/CSA-S806 2002], using different equation instead of Branson's one, where is introduced I_t, the moment of inertia of a non-cracked section transformed to concrete taking into account not only the reinforcement and modulus of elasticity ratios (but not in $\frac{\rho_{frp}}{\rho_{fb}}$ form), but also a coefficient characterizing the bond properties of reinforcement bars, gives accurate and similar results as the others above.

EC2-CEB, Italian Code CNR-DT 203/2006 gives good results that don't depend on the effective moment of inertia, but only on displacements of non-cracked cross concrete section and the cracked section. It takes into account the bond properties of reinforcement and the type of load: short time loads or long-time cycling loads, but doesn't depend on reinforcement and modulus of elasticity ratios.

As shown in the chart 2, if reinforcement ratios increases, especially for $\frac{\rho_{frp}}{\rho_{fb}} > 2.5$, than deflections decreases. This happens because the cracking moment of the beam I_{cr} increases and few cracks appear at the same level of loading, in fact for high reinforcement ratios $I_{cr} > I_e$, so is used I_{cr} instead of I_e. The compressive strength of concrete increases too, but this effect on beam's deflection is not considerable in high reinforcement ratios.

In this study is used only one type of GFRP bar but exist different types of FRP reinforcements with very large properties, so different results are taken based on different methods. For this reason a lot of researchers, based on a large numbers of experimental tests are modifying and optimizing Branson's equation, so that the predicted values of deflection approach the experimental values. New models are going to be developed based on experimental results and elaborated genetic algorithm used to evaluate the effects of several parameters and reevaluate the power "m" in Branson's equation.

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