EUROPEAN ACADEMIC RESEARCH
Vol. III, Issue 6/ September 2015

Impact Factor: 3.4546 (UIF)
DRJI Value: 5.9 (B+)

ISSN 2286-4822
www.euacademic.org

Some Results on Domination Parameters in Graphs: A Special Reference to 2-Rainbow Edge Domination

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Abstract:

Let G = (V,E) be a graph, and let g be a function that assigns to each edge a set of colors chosen from the power set of {1,2} that is, g:E(G)→𝒫 {1,2}. If for each edge e ∈E(G) such that g(e) = ϕ,we have $\bigcup_{f\in N(e)} g(f)$={1,2}, then g is called 2-Rainbow edge domination function (2REDF) and the weight denoted by w(g) of a function is defined as w(g) = $\sum_{f\in E(G)} |g(f)|$.

The minimum weight of 2REDF is called 2-rainbow edge domination number (2REDN) of G which denoted by γ'_{r2} (G). In this paper we try to determine the value of 2REDN for several classes of graphs.

Key words: Spider graph, 2rainbow edge domination, 2-rainbow edge domination number, bistar, line graph

1. Introduction

The dominating set of a graph G = (V, E) is the subset S⊆ V such that every vertex v ∈V is either an element of S or is adjacent to some element of S.

A dominating set S is a minimal dominating set if no proper subset S′ ⊂ S is a dominating set. The cardinality of minimal dominating set of G is called domination number of G which is denoted by $\gamma(G)$. The open neighborhood N(v) of v ∈V(G) is the set of vertices adjacent to v and the set N[v]=N(v)∪{v} is the closed neighborhood of v. For any number "n" ,⌈n⌉ denotes the smallest integer not less than "n" and ⌊n⌋ denotes the greatest integer not greater than "n". An edge "e" of a graph G is said to be incident with the vertex v if v is an end vertex of e. Two edges e and f which incident with a common vertex v are said to be adjacent. A subset F ⊆ E is an edge dominating set if each edge in E is either in F or is adjacent to an edge in F. An edge dominating set F is called minimal if no proper subset F′ of F is an edge dominating set.

The edge domination number $\gamma'(G)$ is the cardinality of minimal edge dominating set. The open neighborhood of an edge $e \in$ E is denoted as N(e) and it is the set of all edges adjacent to e in G, further N[e]=N(e)∪{e} is the closed neighborhood of "e" in G.

For all terminology and notations related to graph theory not given here we follow [4]. The motivation of domination parameters are obtained from [5] and [6]. This work is mainly based on [1], [2] and [3].

2. 2-Rainbow edge domination function

Let G=(V,E) be a graph and let g be a function that assigns to each edge a set of colors chosen from the power set of {1,2} i.e., g:E(G)→$\mathcal{P}$ {1,2}. If for each edge e ∈E(G) such that g(e) = ϕ,we have $\bigcup_{f\in N(e)} g(f)$={1,2},then g is called 2-Rainbow edge domination function(2REDF) and the weight w(g) of a function is defined as w(g) = $\sum_{f\in E(G)} |g(f)|$.

The minimum weight of 2REDF is called 2-rainbow edge domination number (2REDN) of G which denoted by γ'_{r2} (G).

Theorem 2.1 For any graph G, $\gamma'_{r2}(G) = \gamma_{r2}(L(G))$.

Proof Let G be a graph with edges $e_1, e_2, \ldots, e_q$ and vertices $v_1, v_2, \ldots, v_q$ where e_i corresponding to v_i, i = 1,2,…,q and let H=L(G) and $g: V(H) \to \mathcal{P}(\{1,2\})$ be 2RDF with minimum weight $w(g) = \gamma_{r2}(H) = \gamma_{r2}(L(G)) = t \longrightarrow$ (1)

Now let $g': E(G) \to \mathcal{P}(\{1, 2\})$. By the definition of line graph, there exist a 1-1 and on-to function between the vertices of line graph of G, i.e., V(H), and edges of G , i.e., E(G) ,such that $g'(e_i) = g(v_i)$, for every $e_i \in E(G)$ and $v_i \in V(G)$. It follows that $g'(e_i) = \emptyset$ implies that $g(v_i) = \emptyset$ for all i=1,2,…,q and since g is a 2RDF; $\bigcup_{ej \in N[ei]} g'(e_j) = \bigcup_{vj \in N[vi]} g(v_i) = \{1,2\}$. Hence $g': E(G) \to \mathcal{P}(\{1, 2\})$ is a 2REDF and $w(g') = \sum_{i=1}^{q} g'(ei) = \sum_{i=1}^{q} g(vi) = t$. But we know that $\gamma'_{r2}(G)$ is the minimum weight of 2REDF g',and hence $\gamma'_{r2}(G)$ should be $\leq t$. It remains to show that $\gamma'_{r2}(G) = t$. Let $h': E(G) \to \mathcal{P}\{1, 2\}$ be a 2REDF in G so that $w(h') = \sum_{i=1}^{q} h'(ei) < t$. It means there exist a 2RDF on H, $h: V(H) \to \mathcal{P}(\{1, 2\})$ defined as $h(v_i) = h'(e_i)$ which implies $w(h) = w(h') < t$,is a contradiction to (1). Hence $\gamma'_{r2}(G) = t$. and $\gamma'_{r2}(G) = \gamma_{r2}(L(G))$. □

Corollary2.2 2- rainbow edge domination function is NP-Complete.

Proposition 2.3 For any graph G = (V, E), $\gamma'_{r2}(G) = \gamma(H \times K_2)$, where H is the line graph of G.

Proof By using previous theorem and the result $\gamma_{rk}(G) = \gamma(G \times K_k)$, if we substitute L(G) instead of G we have $\gamma_{rk}(L(G)) = \gamma(L(G) \times K_k)$ and hence $\gamma'_{r2}(G) = \gamma(L(G) \times K_k)$. If k = 2 and L(G) = H, then $\gamma'_{r2}(G) = \gamma(H \times K_2)$. Hence the proof. □

Proposition 2.4 For any path P_n with n≥3, we have

$$\gamma'_{r2}(P_n) = \begin{cases} \frac{n}{2} & if \quad n \text{ is even} \\ \frac{n+1}{2} & if \quad n \text{ is odd} \end{cases}$$

proof We know that $L(P_n) = P_{n-1}$ and by using the result ($\gamma_{r2}(P_n) = \lfloor \frac{n}{2} \rfloor + 1$) ,we can write , $\gamma'_{r2}(P_n) = \gamma_{r2}(L(P_n)) = \gamma_{r2}(P_{n-1}) = \lfloor \frac{n-1}{2} \rfloor + 1$, and $\lfloor \frac{n-1}{2} \rfloor = \begin{cases} \frac{n-2}{2} & for \quad n \text{ is even} \\ \frac{n-1}{2} & for \quad n \text{ is odd} \end{cases}$

when $n \geq 3$. Hence we get $\gamma'_{r2}(P_n) = \begin{cases} \frac{n-2}{2}+1 & n \text{ is even} \\ \frac{n-1}{2}+1 & n \text{ is odd} \end{cases}$

that is $\gamma'_{r2}(P_n) = \begin{cases} \frac{n}{2} & n \text{ is even} \\ \frac{n+1}{2} & n \text{ is odd} \end{cases}$ □

Proposition 2.5 For any cycle C_n with n vertices

$$\gamma'_{r2}(C_n) = \begin{cases} \frac{n}{2} & if \quad n \equiv 0 \quad (mod\ 4) \\ \frac{n+2}{2} & if \quad n \equiv 2 \quad (mod\ 4) \\ \frac{n+1}{2} & if \quad n \equiv 1 \text{ or } 3 \quad (mod\ 4) \end{cases}$$

Proof We know that $L(C_n) = C_n$. Next, let C_n be a cycle with n vertices, then by using the results : $\gamma_{r2}(C_n) = \lfloor\frac{n}{2}\rfloor + \lceil\frac{n}{4}\rceil - \lfloor\frac{n}{4}\rfloor$ and $\gamma'_{r2}(C_n) = \gamma_{r2}(L(C_n)) = \gamma_{r2}(C_n)$, we have $\gamma'_{r2}(C_n) = \lfloor\frac{n}{2}\rfloor + \lceil\frac{n}{4}\rceil - \lfloor\frac{n}{4}\rfloor$ .This implies that

a) If $n \equiv 0 \pmod 4$ then $\lfloor\frac{n}{2}\rfloor + \lceil\frac{n}{4}\rceil - \lfloor\frac{n}{4}\rfloor = \frac{n}{2}$.

b) If $n \equiv 2 \pmod 4$ then $\lfloor\frac{n}{2}\rfloor + \lceil\frac{n}{4}\rceil - \lfloor\frac{n}{4}\rfloor = \frac{n}{2} + \frac{n+2}{4} - \frac{n-2}{4} = \frac{2n+n+2-n+2}{4} = \frac{2n+4}{4} = \frac{n+2}{2}$

c) If $n \equiv 1 \pmod 4$ then $\lfloor\frac{n}{2}\rfloor + \lceil\frac{n}{4}\rceil - \lfloor\frac{n}{4}\rfloor = \frac{n-1}{2} + \frac{n+3}{4} - \frac{n-1}{4} = \frac{2n-2+n+3-n+1}{4} = \frac{2n+2}{4} = \frac{n+1}{2}$ **d)** If $n \equiv 3 \pmod 4$ then $\lfloor\frac{n}{2}\rfloor + \lceil\frac{n}{4}\rceil - \lfloor\frac{n}{4}\rfloor = \frac{n-1}{2} + \frac{n+1}{4} - \frac{n-3}{4} = \frac{2n-2+n+1-n+3}{4} = \frac{2n+2}{4} = \frac{n+1}{2}$.□

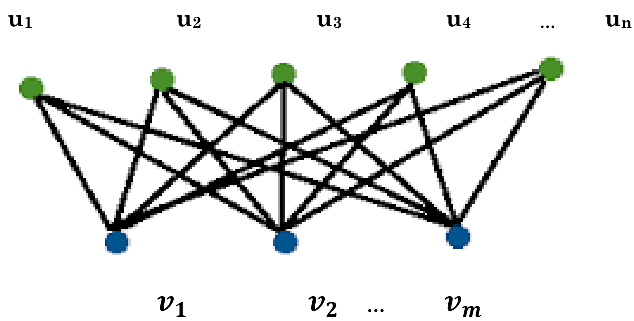
Proposition 2.6 For any star $K_{1,n}$, $n \geq 2$, $\gamma'_{r2}(K_{1,n}) = 2$

proof It is known that the line graph of $K_{1,n}$ is the complete graph K_{n+1} and hence $\gamma'_{r2}(K_{1,n}) = \gamma_{r2}(L(K_{1,n})) = \gamma_{r2}(K_{n+1}) = 2$. □

Theorem 2.7 If $G \cong K_{m,n}$ for $2 \leq m \leq n$, then

$$\gamma_{r2}(G) = \begin{cases} 2(m-1) & \text{if} \quad m = n \\ 2m & \text{otherwise} \end{cases}$$

Proof Let the bipartite sets of G are A=$\{v_1, v_2, \ldots, v_m\}$ and B=$\{u_1, u_2, \ldots, u_n\}$ and the edges $e_{11} = v_1u_1$, $e_{22} = v_2u_2$, ..., $e_{ij} = v_iu_j$; i= 1,...,m and j=1,2,...,n as shown in the following figure



Now, let g: E(G) → $\mathcal{P}(\{1,2\})$ defined as;

$$g(e_{ij}) = \begin{cases} \{1\} & \text{for} \quad i = j = 1 \\ \{2\} & \text{for} \quad i = j = 2 \\ \{1,2\} & \text{for} \quad e_{ij} \text{ where } i = j \text{ and } i, j \neq 1,2 \\ \emptyset & \text{otherwise} \end{cases}$$

Clearly $e_{12} = \emptyset$ and its neighbors are e_{11} and e_{22} .Hence $\bigcup_{j=1}^{m} g(e_{1j}) = \{1,2\}$and e_{11} and e_{22} belongs to $N[e_{1j}]$.

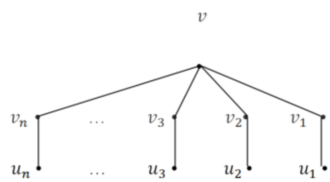
Since e_{21}=∅ and its neighbors are e_{11} and e_{22} hence $\bigcup_{j=1}^{n} g(e_{2j})$ =$\{1,2\}$ *and* e_{11} *and* e_{22} belongs to $N[e_{2j}]$.

Moreover for every e_{ij}, i=3,4,…,m and j =1,2,…,n . It is clear that $\bigcup_{e_{ii} \in N[e_{ij}]} g(e_{ij})$=$\{1,2\}$; for $i = 3,4, \ldots, m$ *and* $j = 1,2, \ldots, n$. It follows that g:E(G) → $\mathcal{P}(\{1,2\})$ is 2REDF.

Since (m-2) edges have assigned to {1, 2} and two edges have assigned to {1} or {2} w(g) = 2m-2 =2(m-1). Hence γ'_{r2} (G) ≤ 2(m-1) for m= n and m,n≥ 2. If m≠n; m edges have the weight {1,2} and hence γ'_{r2} (G) =2m.

Definition Let G be the complete bipartite graph $K_{1,n}$. The graph which obtained by subdivision of every edge once is called spider graph of 2n edges and 2n+1 vertices.

Theorem 2.8 For any spider graph G, γ'_{r2}(G) = n+1.
Proof Consider the spider graph G given in the following figure



Let $vv_1 = e_{11}, vv_2 = e_{12}, \ldots, vv_n = e_{1n}$ and $v_1u_1 = e_{21}$, $v_2u_2 = e_{22}, \ldots, v_nu_n = e_{2n}$. Let g: E (G) → $\mathcal{P}(\{1, 2\})$ defined as follows

$$g(e_{ij}) = \begin{cases} \{1,2\} & \text{for } i = j = 1 \\ \{1\} \text{ or } \{2\} & \text{for } e_{2j}, j = 2,3,\ldots,n \\ \emptyset & otherwise \end{cases}$$

for any $e_{1j} = \emptyset$; $j =$ 2,3,…,n; $\bigcup_{j=1}^{n} g(e_{1j}) = \{1,2\}$ and e_{1j}, $e_{2j} \in N[e_{2j}]$. Also $e_{21} = \emptyset$ and $\bigcup_{i=1,2} g(e_{i1}) = \{1,2\}$ .Hence g is a 2REDF and w(g)=(n-1)+2=n+1 and γ'_{r2} (G)≤n+1 →(1) .

Now, we show that γ'_{r2} (G)=n+1. Suppose not. That is γ'_{r2} (G) < n+1. That is, min(w(g)) < n+1.It means that the edge e_{1j}, e_{2j} j=1,2,..,n can be assign in many cases;

Case 1 If $e_{2j} = \emptyset$ for j=1,2,..,n , then e_{1j}'s, j=1,2,..,n, should assign to {1,2} and hence w(g) = 2n , is a contradiction to (1).

Case 2 If $e_{2j} = \{1,2\}$ for j=1,2,..,n , then e_{1j}'s , j=1,2,..,n, should assign to$\emptyset$ and hence w(g) = 2n , is a contradiction to (1).

Case 3 If $e_{21} = \emptyset$, $e_{11} = \{1,2\}$ and $e_{2j} = \{1\}$ or $\{2\}$ for j= 2,3,…,n, then w(g) = (n-1) +2= n+1, is a contradiction to (1).

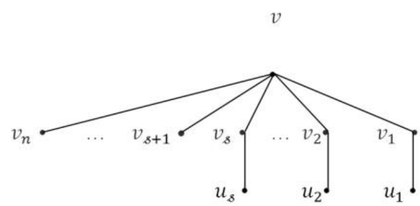
Case 4 If we map e_{2t}, for $1 < t < n$ to $\emptyset$ then the edges e_{1t}, $1 < t < n$ must be map to {1,2}and hence w(g) = 2t+(n-t) = n+t,is a contradiction to (1).

Case 5 If e_{1j}'s and e_{2j}'s ={1} or {2} for j=1,2,…,n , that is all edges assign to {1} or {2} then w(g) = 2n , is a contradiction to (1).

In all cases we get contradiction and hence γ'_{r2} (G)=n+1 for any spider graph G.

Definition If we subdivide s edges $1\leq s \leq n-1$ of $K_{1,n}$ then the resultant graph is called wounded spider.

Theorem 2.9 For any wounded spider graph G with $n + s$ edges , $\gamma'_{r2}(G)= s+1$.



Proof Let $vv_1 = e_{11}, vv_2 = e_{12}, \dots, vv_n = e_{1n}$ and $v_1u_1 = e_{21}$, $v_2u_2 = e_{22}, \dots, v_su_s = e_{2s}$, Let g:E(G) $\rightarrow$ $\mathcal{P}(\{1,2\})$ defined as follows

$$g(e_{ij}) = \begin{cases} \{1,2\} & \text{for } i = j = 1 \\ \{1\} \text{ or } \{2\} & \text{for } e_{2j}, j = 2,3,\dots,n \\ \emptyset & otherwise \end{cases}$$

We have $e_{1j} = \emptyset \; for \; j = 2, 3, \dots, n$ and $\bigcup_{j=1}^{n} g(e_{1j}) = \{1,2\}$. Also $e_{21} = \emptyset$ and $\bigcup_{i=1,2} g(e_{i1}) = \{1, 2\}$. Hence g is a 2REDF and w (g) = (s -1) +2 = s+1 implies $\gamma'_{r2}(G)\leq s + 1$. If we us the technique used as in the previous theorem, we can prove $\gamma'_{r2}(G)= s + 1$.

Definition: Let G=(V,E) be a graph and g:E(G) $\rightarrow \mathcal{P}(\{1,2\})$ be a 2REDF with minimum weight then g can partition the edges of G into the following sets:

$E_0 = \{ e_i \in E(G); g(e_i) = \emptyset \;, i = 1,2,\dots,n\}$

${}^1E_1 = \{ e_i \in E(G); g(e_i) = \{1\} \;, i = 1,2,\dots,n\}$

${}^2E_1 = \{ e_i \in E(G); g(e_i) = \{2\} \;, i = 1,2,\dots,n\}$

$E_2 = \{ e_i \in E(G); g(e_i) = \{1,2\} \;, i = 1,2,\dots,n\}$

Theorem 2.10 Let G be a graph and g:E(G) $\rightarrow \mathcal{P}(\{1,2\})$ be 2REDF with w(g) = $\gamma'_{r2}(G)$ and g partitioned the edges in to E_0 ,

${}^1E_1$, ${}^2E_1$, E_2, then no edge in E_2 is adjacent to any edge in ${}^1E_1$, ${}^2E_1$.

Proof we define $g:E(G) \rightarrow \mathcal{P}(\{1,2\})$ with smallest weight w(g) = γ'_{r2} (G). Suppose e, $e' \in E(G)$,where $e \in E_2$ and $e' \in {}^1E_1$, ${}^2E_1$ .On the contrary suppose e be adjacent to e' and g(e) = {1,2} , and g(e') = {1} or {2} , we can construct $g':E(G) \rightarrow \mathcal{P}(\{1,2\})$ by defining $g'(e') = \emptyset$ which implies g′ is a 2REDF in G and w(g′) < w (g) which contradiction. □

Definition A bistar is a tree obtained from the graph K_2 with two vertices u *and* v by attaching m pendent edges in u and n pendent edges in v and denoted by $B(m,n)$.

Theorem 2.11 For any connected graph G, γ'_{r2} (G) =2 if and only if $G \cong B(m,n)$ where m and n not both zero.

Proof If $G \cong B(m,n)$ and ($m,n \neq 0$), then by theorem 2.1 and fact that line graph of B (m,n) is two complete graphs say K_m and K_n with a common vertex we have γ'_{r2} (G) =2 (Note that if m,n=0 then G is complete bipartite graph and γ'_{r2} (G) =2). Conversely if γ'_{r2} (G)=2, then there exist a function g: $E(G) \rightarrow \mathcal{P}(\{1,2\})$ such that w(g) = 2 , that is $|E_0| + |{}^1E_1| + |{}^2E_1| + |E_2| = 2$ then

If $|E_2| \neq 0$ then $|E_2| = 2$ and $|E_0| + |{}^1E_1| + |{}^2E_1| = 0$ hence $G \cong B(m,n)$.

If $|E_2| = 0$ then $|E_0| + |{}^1E_1| + |{}^2E_1| = 2$ either $|E_0| = 0$ and $|{}^1E_1| = 0$ and $|{}^2E_1| = 2$ or $|E_0| = 0$ and $|{}^1E_1| = 2$ and $|{}^2E_1| = 0$ or $|E_0| = 0$ and $|{}^1E_1| = |{}^2E_1| = 1$ and all cases $G \cong P_3 \cong B(0,1)$. Hence $G \cong B(m,n)$. □

Observation 2.12 For any graph G, $1 \leq \gamma'_{r2}$ (G) $\leq q$ **,** where q is the number of edges in G.

Proof For any non trivial graph G, if the function ; $g:E(G) \rightarrow \mathcal{P}(\{1,2\})$ defined as g(e_i **) =** {1} or {2} then w(g) = q. Also if G = K_2 then γ'_{r2} (K_2)=1. Hence $1 \leq \gamma'_{r2}$ (G) $\leq q$. □

Proposition 2.13 For any graph G, $\gamma'(G) \leq \gamma'_{r2}(G) \leq 2\gamma'(G)$.
Proof Since $\gamma'_{r2}(G) = \gamma(L(G)\times K_2)$ and by Vizing's conjecture ; $\gamma(G\times H) \geq \gamma(G).\gamma(H)$. But, we have $\gamma'_{r2}(G) = \gamma_{r2}(L(G)) = \gamma(L(G)\times K_2) \geq \gamma(L(G)).\gamma(K_2) = \gamma'(G)$ and hence $\gamma'_{r2}(G) \geq \gamma'(G)$.
For the upper bound,
Let S be the minimum edge dominating set in G and let g:E(G) $\rightarrow \mathcal{P}(\{1,2\})$ defined as $g(e_i) = \begin{cases} \{1,2\} \text{ for } e_i \in S & i = 1,2,\dots,n \\ \emptyset & \text{otherwise} \end{cases}$
It is clear that g is 2REDF and w (g) = $2\gamma'(G)$. Thus $\gamma'_{r2}(G) \leq 2\gamma'(G)$ and hence $\gamma'(G) \leq \gamma'_{r2}(G) \leq 2\gamma'(G)$. □
Definition A graph G is called 2-rainbow edge graph if $\gamma'_{r2}(G) = 2\gamma'(G)$.
Example P_3 is a 2-rainbow edge graph.
Notation The definition can be generalize to k-rainbow edge graph.

Theorem 2.14 For any graph G, $\gamma'_{r2}(G) \leq q - \Delta'(G) + 1$ where $\Delta'(G)$ is the maximum edge degree in G.
Proof Let G= (V, E) be a graph with n vertices with maximum edge degree $\Delta'(G)$. Let $f \in E(G)$. Without lose of generality we let $\deg(f) = \Delta'(G)$, and g: E (G) $\rightarrow \mathcal{P}(\{1, 2\})$ be defined as;

$$g(e_i) = \begin{cases} \{1,2\} & \text{for } e_i = f, i = 1,2,\dots,n \\ \{1\} \text{ or } \{2\} & \text{for } e_i \notin N(f) \\ \emptyset & \text{for } e_i \in N(f) \end{cases}$$

Since for $e_i \in N(f)$, $g(e_i) = \emptyset$, $\cup_{e_i\in N[f]} g(e_i) = \{1,2\}$.Then it is clear that g is 2REDF and w(g) = $(q - 1 - \Delta'(G)) + 2 = q - \Delta'(G) + 1$ hence $\gamma'_{r2}(G) \leq q - \Delta'(G) + 1$.□
Remark The bound is sharp for $G\cong P_3$.□
Corollary 2.15 For any graph G, $\gamma'_{r2}(G) \leq q - \delta'(G) + 1$.□

Theorem 2.16 Let G(V,E) be a graph without isolated vertices $\gamma(G) = \gamma'_{r2}(G)=1$ if and only if $G=K_2$.
Proof: If $G=K_2$ then it is clear that $\gamma(G) = \gamma'_{r2}(G)=1$. Conversely, if $\gamma(G) = \gamma'_{r2}(G)$, then the 2REDF with minimum

weight is g:E(G) $\rightarrow \mathcal{P}(\{1,2\})$ defined as ; g(e) ={1} or {2} , that is there is only one edge in G. Since G has no isolated vertices, G=K_2. □

Theorem 2.17: γ'_{r2} (G) = q if and only if G≅ mP_2 or mP_3 where m≥ 1.

Proof: If G= mP_2**,** then γ'(G) = m, similarly if G= mP_3 then γ'(G) =2m= q.

Conversely, if γ'_{r2} (G) = q and g:E(G) $\rightarrow \mathcal{P}(\{1,2\})$, be a 2-REDF with w(g) = q**,** we have two cases;

case 1 If g(e) ≠∅ for every $e \in E(G)$ then G=K_2 or mK_2 **.**

case 2 If there exist at least one edge e_i ∈ E(G) such that g(e_i) **=** {1,2} then G=K_3 or mK_3 **.** □

Acknowledgement

The second author wishes to thank university of Kerala for providing all facilities to do this research work.

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