EUROPEAN ACADEMIC RESEARCH
Vol. III, Issue 7/ October 2015

Impact Factor: 3.4546 (UIF)
DRJI Value: 5.9 (B+)

ISSN 2286-4822
www.euacademic.org

On Artin cokernel of The Group($Q_{2m}\times D_3$) Where m= $2p^h$, *such that* $h \in Z^+$ *and p is prime number*

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Abstract:

The main purpose of This paper is to find Artin's character table $\boldsymbol{Ar(Q_{2m}\times D_3)}$*when m is even number such that* $m=2p^h$*, such that* $h \in Z^+$*, and p is a prime number ;where* Q_{2m} *is denoted to Quaternion group of order 4m, time is said to have only one dimension and space to have three dimension ,the mathematical quaternion partakes of both these elements ; in technical language it may be said to be "time plus space", or "space plus time" and in this sense it has, or at least involves a reference to four dimensions, and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols.,1882,1885,1889)), and* $\boldsymbol{D_3}$ *is Dihedral group of order 6. In 1962, C. W. Curits & I. Reiner studied Representation Theory of finite groups.*

In 1976, I. M. Isaacs studied Characters Theory of finite groups, In 1982, M. S. Kirdar studied The Factor Group of the Z-Valued class function modulo the group of the Generalized Characters, In1994, H. H. Abass studies The Factor Group of class function over the group of Generalized Characters of D_n *and found* $\equiv^*(D_n)$*, In 1995, N. R. Mahmood studies The Cyclic Decomposition of the factor Group* $cf(Q_{2m},Z)/\ \bar{R}(Q_{2m})$*, In 2002, K-Sekiguchi studies Extensions and the*

Irreducibilies of the Induced Characters of Cyclic P-Group. In 2006, A. S. Abid studies Characters Tabl eof Dihedral Group for Odd number.

Key words: even number, prime number, Quaternion group, and Dihedral group.

Introduction:

Representation Theory is a branch of mathematics that studies abstract algebra structures by Representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication ,In which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication, Moreover, representation and character theory provide applications ,no only in other branches of mathematics but also in physics and chemistry.

For a finite group G, the factor group **$\overline{R}(G)/T(G)$** is called the Artin cokernel of **G** denoted **AC(G)**, **$\overline{R}(G)$** denoted the a belian group generated by Z-valued characters of G under the operation of pointwise addition**, T(G)** is a subgroup of **$\overline{R}(G)$** which is generated by Artin's characters

Preliminary: [1]:(3,1)

The Generalized Quaternion Group Q_{2m}:

For each positive integer m≥2, the generalized Quaternion Group **Q_{2m}** of order 4m with two generators x and y satisfies **Q_{2m}** $=\{x^h y^k, 0\le h\le 2m-1, k=0,1\}$

Which has the following properties $\{x^{2m}=y^4=I, yx^my^{-1}=x^{-m}\}$.

Definition :[2](3,2)

Let **G** be a finite group, all the characters of group **G** induced from a principal character of cyclic subgroup of **G** are called Artin characters of **G** .

Artin characters of the finite group can be displayed in a table called Artin characters table of **G** which is denoted by **Ar(G);** The first row is Γ-conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G(cl_a)|$ and other rows contains the values of Artin characters .

The general form of Artin characters table of $\mathbf{C}p^s$

When p is a prime number and s is a positive integer number is given by:-

Ar($\mathbf{C}p^s$)=

Γ-classes	*[1]*	*[x^{ps-1}]*	*[x^{ps-2}]*	…	*[x]*
$\lvert cl_a \rvert$	1	1	1	…	1
$\lvert cp^s(cl_a) \rvert$	p^s	ps	p^s	…	p^s
Φ_1	p^s	0	0	…	0
Φ_2	P^{s-1}	P^{s-1}	0	…	0
⋮	⋮	⋮	⋮	…	⋮
Φ_s	p	p	p	…	0
Φ_{s+1}	1	1	1	…	1

Table (1)

Corollary:[5]:(3,4)

Let $n=P_1^{a1}.P_2^{a2}\ldots P_n^{an}$ where $g.c.d(P_i,P_j)=1$ if $i\neq j$ and P_i are a prime numbers, and a_i any positive integer for all $1\leq i\leq n$ Then :

$Ar(C_m)=Ar(Cp_1^{a1})\otimes Ar(Cp_2^{a2})\ldots\otimes Ar(Cp_n^{an})$ such that

Ar($\mathbf{C}p^1$)=

Γ-class	*[1]*	*[x]*
$\lvert cl_a \rvert$	1	1
$\lvert cp(cl_a) \rvert$	p	p
Φ_1	p	0
Φ_2	1	1

Table(2)

Where **Ar(C_{2m}), m=2.p is Ar($C_{2.2p}$)=Ar($C_2{}^2p$)= Ar($C_2{}^2$)⊗Ar(p^1).**

<u>Proposition:[4]:(3,5)</u>

The number of all distinct Artin characters on group G is equal to the number of Γ-classes on **G** .Furthermore, Artin characters are constant on each Γ-classes .

The main results:

<u>Theorem:(4,1):-</u>

The Artin's character table of the group **($Q_{2.2}p^h \times D_3$)** where m=$2p^h$ such that h $\in Z^+$and p is prime number,it is given as follows:

Ar($Q_{2.2p}{}^h \times D_3$)=

The Artin's character table of group **($Q_{2.2p}{}^h \times D_3$)** :

	Γ-classes of $Q_{2.2p}{}^h \times \{1\}$										
Γ-classes	[I,1]	$[x^{2^h_p},1]$	$[x^{p^h},1]$	$[x^{4p},1]$	$[x^{2p},1]$	$[x^p,1]$	$[x^4,1]$	$[x^2,1]$	[x,1]	[y,1]	[xy,1]
|CL$_a$|	1	1	2	2	2	2	2	2	2	$2(2^h_p)$	$2(2^h_p)$
|C$Q_{2.2p}{}^h \times D_3$(CL$_a$)|	$24(2^h_p)$	$24(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(^h_2p)$	24	24
Φ(I,1) Φ(2,1) Φ(3,1) Φ(4,1) Φ(5,1) Φ(6,1) Φ(7,1) Φ(8,1) Φ(9,1) Φ(10,1) Φ(11,1)	**6Ar($Q_{2.2p}{}^h$)**										

	Γ-classes of $Q_{2.2p^h} \times \{1\}$										
Γ-classes	[I,1]	$[x^{2^h_p},1]$	$[x^{p^h},1]$	$[x^{4p},1]$	$[x^{2p},1]$	$[x^p,1]$	$[x^4,1]$	$[x^2,1]$	[x,1]	[y,1]	[xy,1]
$\|CL_a\|$	1	1	2	2	2	2	2	2	2	$2(2^h_p)$	$2(2^h_p)$
$\|CQ_{2.2p^h}\times D_3(CL_a)\|$	$24(2^h_p)$	$24(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(^h_2p)$	24	24
Φ(I,2) Φ(2,2) Φ(3,2) Φ(4,2) Φ(5,2) Φ(6,2) Φ(7,2) Φ(8,2) Φ(9,2) Φ(10,2) Φ(11,2)	**$2Ar(Q_{2.2p^h})$**										

	Γ-classes of $Q_{2.2p^h} \times \{1\}$										
Γ-classes	[I,1]	$[x^{2^h_p},1]$	$[x^{p^h},1]$	$[x^{4p},1]$	$[x^{2p},1]$	$[x^p,1]$	$[x^4,1]$	$[x^2,1]$	[x,1]	[y,1]	[xy,1]
$\|CL_a\|$	1	1	2	2	2	2	2	2	2	$2(2^h_p)$	$2(2^h_p)$
$\|CQ_{2.2p^h}\times D_3(CL_a)\|$	$24(2^h_p)$	$24(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(^h_2p)$	24	24
Φ(I,3) Φ(2,3) Φ(3,3) Φ(4,3) Φ(5,3) Φ(6,3) Φ(7,3) Φ(8,3) Φ(9,3) Φ(10,3) Φ(11,3)	**$3Ar(Q_{2.2p^h})$**										

	Γ-classes of $Q_{2.2p}{}^{h}$ × { r }										
Γ-classes	[I,r]	$[x^{2^h_p}, r]$	$[x^{p^h}$,r]	$[x^{4p}$,r]	$[x^{2p}$,r]	$[x^{p}$,r]	$[x^{4}$,r]	$[x^{2}$,r]	[x,r]	[y,r]	[xy,r]
|CL_a|	2	2	4	4	4	4	4	4	4	$4(2^h_p)$	$4(2^h_p)$
|$CQ_{2.2}{}^{h}{}_{p}$×D_3(CL_a)|	$12\ (2^h_p)$	$12(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(^h_2p)$	24	24
Φ(I,1)											
Φ(2,1)											
Φ(3,1)											
Φ(4,1)											
Φ(5,1)						**0**					
Φ(6,1)											
Φ(7,1)											
Φ(8,1)											
Φ(9,1)											
Φ(10,1)											
Φ(11,1)											

	Γ-classes of $Q_{2.2p}{}^{h}$ × { r }										
Γ-classes	[I,r]	$[x^{2^h_p}, r]$	$[x^{p^h}$,r]	$[x^{4p}$,r]	$[x^{2p}$,r]	$[x^{p}$,r]	$[x^{4}$,r]	$[x^{2}$,r]	[x,r]	[y,r]	[xy,r]
|CL_a|	2	2	4	4	4	4	4	4	4	$4(2^h_p)$	$4(2^h_p)$
|$CQ_{2.2}{}^{h}{}_{p}$×D_3(CL_a)|	$12\ (2^h_p)$	$12(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(^h_2p)$	24	24
Φ(I,2)											
Φ(2,2)											
Φ(3,2)						**2Ar($Q_{2.2p}{}^{h}$)**					
Φ(4,2)											
Φ(5,2)											
Φ(6,2)											
Φ(7,2)											
Φ(8,2)											
Φ(9,2)											
Φ(10,2)											
Φ(11,2)											

	Γ-classes of $Q_{2.2p^h} \times \{ r \}$										
Γ-classes	[I,r]	$[x^{2_p^h}, r]$	$[x^{p^h}$,r]	$[x^{4p}$,r]	$[x^{2p}$,r]	$[x^p$,r]	$[x^4$,r]	$[x^2$,r]	[x,r]	[y,r]	[xy,r]
$\|CL_a\|$	2	2	4	4	4	4	4	4	4	$4(2_p^h)$	$4(2_p^h)$
$\|CQ_{2.2^h p}$×$D_3(CL_a)\|$	$12\ (2_p^h)$	$12(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(^h_2p)$	24	24
Φ(I,3) Φ(2,3) Φ(3,3) Φ(4,3) Φ(5,3) Φ(6,3) Φ(7,3) Φ(8,3) Φ(9,3) Φ(10,3) Φ(11,3)						**0**					

	Γ-classes of $Q_{2.2p^h} \times \{s \}$										
Γ-classes	[I,s]	$[x^{2_p^h}, s]$	$[x^{p^h}$,s]	$[x^{4p}$,s]	$[x^{2p}$,s]	$[x^p$,s]	$[x^4$,s]	$[x^2$,s]	[x,s]	[y,s]	[xy,s]
$\|CL_a\|$	3	3	6	6	6	6	6	6	6	$6\ (2_p^h)$	$6(2_p^h)$
$\|CQ_{2.2^h p}$×$D_3(CL_a)\|$	$8\ (2_p^h)$	$8(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(^h_2p)$	8	8
Φ(I,1) Φ(2,1) Φ(3,1) Φ(4,1) Φ(5,1) Φ(6,1) Φ(7,1) Φ(8,1) Φ(9,1) Φ(10,1) Φ(11,1)						**0**					

	Γ-classes of $Q_{2.2p}{}^{h}$ × {s}										
Γ-classes	[I,s]	$[x^{2_p^h}, s]$	$[x^{p^h}$,s]	$[x^{4p}$,s]	$[x^{2p}$,s]	$[x^{p}$,s]	$[x^{4}$,s]	$[x^{2}$,s]	[x,s]	[y,s]	[xy,s]
|CL$_a$|	3	3	6	6	6	6	6	6	6	6 (2_p^h)	6(2_p^h)
|CQ$_{2.2}{}^{h}{}_{p}$×D$_3$(CL$_a$)|	8 (2_p^h)	8(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4$(\frac{h}{2}p)$	8	8
Φ(I,2) Φ(2,2) Φ(3,2) Φ(4,2) Φ(5,2) Φ(6,2) Φ(7,2) Φ(8,2) Φ(9,2) Φ(10,2) Φ(11,2)	**0**										

	Γ-classes of $Q_{2.2p}{}^{h}$ × {s}										
Γ-classes	[I,s]	$[x^{2_p^h}, s]$	$[x^{p^h}$,s]	$[x^{4p}$,s]	$[x^{2p}$,s]	$[x^{p}$,s]	$[x^{4}$,s]	$[x^{2}$,s]	[x,s]	[y,s]	[xy,s]
|CL$_a$|	3	3	6	6	6	6	6	6	6	6 (2_p^h)	6(2_p^h)
|CQ$_{2.2}{}^{h}{}_{p}$×D$_3$(CL$_a$)|	8 (2_p^h)	8(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4(2_p^h)	4$(\frac{h}{2}p)$	8	8
Φ(I,3) Φ(2,3) Φ(3,3) Φ(4,3) Φ(5,3) Φ(6,3) Φ(7,3) Φ(8,3) Φ(9,3) Φ(10,3) Φ(11,3)	**Ar($Q_{2.2p}{}^{h}$)**										

Table(3)

Proof:-

Let g_{ij}=(qi,dj) ; qi $\in Q_{2.2p^h}$,dj $\in D_3$

Case (I):-

Consider the group G=($Q_{2.2p^h}\times D_3$) and if H is a cyclic subgroup of ($Q_{2.2p^h}\times\{I\}$) then H=<(q,1)> and Φ the principle character of H and Φ_j Artin's characters of $Q_{2.2p^h}$,1≤j≤i+2 ,The cyclic subgroup of $Q_{2.2^h p}$ which are {<I>},{< $x^{2^h_p}$ >},{< x^{p^h} >},
{< x^{4p} >} ,{< x^{2p} >},{< x^p >},
{< x^4 >},{< x^2 >} ,{x} ,{<y>},{<xy>}

and cyclic subgroup of D_3 which are {<I>},{<r>},{<s>}, by using theorem:

$$\Phi j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} \sum_{i=1}^{n} \emptyset(hi) & if\ hi \in H \cap cl(g) \\ 0 & if\ H \cap cl(g) = \emptyset \end{cases}$$

H=<q,1>:-

1:-H_{11} =<(I,1)> if g=(I,1) then $\emptyset_{11}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset$ (g)=$\frac{24.2p^h}{|CH(g)|}$ .1=$\frac{6|CQ2m(1)|}{|C<x>(1)|}$ .1=6.$\emptyset_j$(1)
since H∩cl(I,1)=(I,1)

2:-H_{21} =<($x^{2^h_p}$,1)> ; (a)if g=(I,1)then$\emptyset_{21}$ (g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$ =$\frac{24.2p^h}{|CH(g)|}$ .1=$\frac{6|CQ2m(1)|}{|C<x>\left(x^{2^h_p}\right)|}$ .1= 6$\emptyset_j(I)$

(b)if g=($x^{2^h_p}$,1) then $\emptyset_{21}$(g)=$\frac{|CG(g)|}{|CH(g)|}.\emptyset(g)$ = $\frac{24.2p^h}{|CH(g)|}$ .1=$\frac{24.2p^h}{|C<x>\left(x^{2^h_p}\right)|}$ .1=$\frac{6|CQ2m\left(x^{2^h_p}\right)|}{|C<x>\left(x^{2^h_p}\right)|}$ .1=6.$\emptyset_j(x^{2^h_p})$

since H∩ $cl(x^{2^h_p})$ =(I,1),($x^{2^h_p}$, 1) otherwise=0.

3:-H_{31} =< (x^{p^h}, 1) >;(a) if g=(I,1) then $\emptyset_{31}$(g)=$\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$ =$\frac{24.2p^h}{|CH(g)|}.1$ =
=$\frac{6|CQ2m(g)|}{|C<x>(x^{p^h})}$.1=$\frac{6|CQ2m|}{|C<x>\left(x^{p^h}\right)|}$.1=6.$\emptyset_j$ (I)

(b) if g=($x^{2^h_p}$,1) then $\emptyset_{31}$(g)=$\left(x^{2^h_p},1\right)$ = $\frac{|CG(g)|}{|CH(g)|}\emptyset(g)$=$\frac{24.2p^h}{|C<x>\left(x^{p^h}\right)|}$.1=6.$\emptyset_j(x^{2^h_p})$

(c)if $g=(x^{p^h},1)$ *then* $\emptyset_{31}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2p^h}{|C<x>(x^{p^h})|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^{p^h})|}.2=6.\emptyset_j(x^{p^h})$

Since $H\cap cl\left(x^{p^h}\right)=\left\{(I,1),\left(x^{\frac{2^h}{p}},1\right),(x^{p^h},1)\right\}$ *otherwise=0.*

4:-$H_{41}=<(x^{4p},1)>$; (a) if $g=(I,1)$ then $\emptyset_{41}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{4p})|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^{4p})|}.1=6.\emptyset_j(I)$.

.(b) if $g=(x^{4p},1)$ then $\emptyset_{41}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x4p)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^{4p})|}.2=6.\emptyset_j(x^{4p})$

Since $H\cap cl(x^{4p})=\{(I,1),(x^{4p},1)\}$. *Otherwise=0 and* $\emptyset(g)=\emptyset(g^{-1})=1$.

5:-$H_{51}=<(x^{2p},1)>$;(a) if $g=(I,1)$ then $\emptyset_{51}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{2p})|}.1=\frac{6|CQ2m(q)|}{|Cx>(x^{2p})|}.1=6\emptyset_j(I)$

(b) if $g=(x^{\frac{2^h}{p}},1)$ then $\emptyset_{51}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{2p})|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^{2p})|}.1=6.\emptyset_j(x^{\frac{2^h}{p}})$.

(c) if $g=(x^{4p},1)$then $\emptyset_{51}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^{2p})|}.2=6\,\emptyset_j(x^{4p})$

(d) if $g=(x^{2p},1)$ then $\emptyset_{51}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+(g^{-1}))=\frac{12.2p^h}{|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^{2p})|}.2=6\,\emptyset_j(x^{2p})$.

Since $H\cap cl(x^{2p})=\left\{(I,1),\left(x^{\frac{2^h}{p}},1\right),(x^{4p},1),,(x^{2p},1)\right\}$. *Otherwise=0.and* $\emptyset(g)=\emptyset(g^{-1})$.

6:-$H_{61}=<(x^{p},1)>$; (a)if $g=(I,1)$ then $\emptyset_{61}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{p})|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^{p})|}.1=6\,\emptyset_j(I)$

(b) if $g=(x^{\frac{2^h}{p}},1)$ then $\emptyset_{61}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{p})|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^{p})|}.1=6.\emptyset_j(x^{\frac{2^h}{p}})$.

(c) if $g=(x^{p^h},1)$ then $\emptyset_{61}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2p^h}{|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^p)|}.2=6.\emptyset_j(x^{p^h})$

(d) if $g=(x^{4p},1)$ then $\emptyset_{61}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^p)|}.2=6.\emptyset_j(x^{4p})$.

(e) if $g=(x^{2p},1)$ then $\emptyset_{61}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{|c<X>(x^p)|}.2=6$ $\emptyset_j(x^{2p})$.

(F) If $g=(x^p,1)$ then $\emptyset_{61}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^p)|}.2=6\emptyset_j(x^p)$.

Since

$H\cap cl(x^p)=\left\{(I.1),\left(x^{2\frac{h}{p}},1\right),\left(x^{p^h},1\right),(x^{4p},1),(x^{2p},1),(x^p,1)\right\}$

$\emptyset(g)=\emptyset(g^{-1})=1$ *otherwise =0.*

7:-H_{71}=<(x^4,1)>; (a)if g=(I,1) then $\emptyset_{71}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{4p})|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^{4p})|}.1=6\ \emptyset_j(I)$

(b) if $g=(x^{4p},1)$ then $\emptyset_{71}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^{4p})|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^{4p})|}.2=6.\emptyset_j(x^{4p})$.

Since $H\cap cl(x^{4p})=\{(I,1),(x^{4p},1)\}$. Otherwise=0 and $\emptyset(g)=\emptyset(g^{-1})=1$.

8:-H_{81}=<(x^2,1)>; (a)if g=(I,1) then $\emptyset_{81}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^2)|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^2)|}.1=6\ \emptyset_j(I)$

(b) if $g=(x^{2\frac{h}{p}},1)$ then $\emptyset_{81}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^2)|}.1=\frac{6|CQ2m(q)|}{|C<x>(x^2)|}.1=6.\emptyset_j(x^{2\frac{h}{p}})$.

(c)) if $g=(x^{4p},1)$ then $\emptyset_{81}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^2)|}.2=6.\emptyset_j(x^{4p})$.

(d) If $g=(x^{2p},1)$ then $\emptyset_{81}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^2)|}.2=6$ $\emptyset_j(x^{2p})$

(e) if $g=(x^4,1)$ then $\emptyset_{81}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+$
$\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^2)|}.2=6.\emptyset_j(x^4)$.

(f) if $g=(x^2,1)$ then $\emptyset_{81}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+$
$\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x^2)|}.2=6.\emptyset_j(x^2)$.

Since $H\cap cl(x^2)=\left\{(I,1),\left(x^{2\frac{h}{p}},1\right),(x^{4p},1),,(x^{2p},1),(x^4,1),(x^2,1)\right\}$.

Otherwise=0.

$and\emptyset(g)=\emptyset(g^{-1})$

9:-H_{121}=<(x,1)>; (a)if $g=(I,1)$ then $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{|C<x>(x)|}.1=6.\ \emptyset_j(I)$.

(b) if $g=(x^{2\frac{h}{p}},1)$ then $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{|C<x>(x)|}.1=6.\emptyset_j(x^{2\frac{h}{p}})$.

(c) if $g=(x^{p^h},1)$ then $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=$
$\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x)|}.2=6.\emptyset_j(x^{p^h})$

(d) if $g=(x^{4p},1)$ then $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+$
$\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x)|}.2=6.\emptyset_j(x^{4p})$.

(e) if $g=(x^{2p},1)$ then $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|c<X>(x)|}.2=6$ $\emptyset_j(x^{2p})$.

(F) If $g=(x^p,1)$ then $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x)|}.2=6\ \emptyset_j(x^p)$.

(g) *if* $g=(x^4,1)$ *then* $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x)|}.2=6.\emptyset_j(x^4)$.

(h) *If* $g=(x^2,1)$ *then* $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x)|}.2=6\ \emptyset_j(x^2)$

(i) *If* $g=(x,1)$ *then* $\emptyset_{91}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{|C<x>(x)|}.2=6\ \emptyset_j(x)$

Since

$H\cap cl(x)=$

$\left\{(I,1),\left(x^{2\frac{h}{p}},1\right),\left(x^{p^h},1\right),(x^{4p},1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1),(x,1)\right\}$

Otherwise=0.and $\emptyset(g)=\emptyset(g^{-1})$

10:- $H_{101}=<(y,1)>$ *;(a) if* $g=(I,1)$ *then* $\emptyset_{101}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{4}.1=6\ \emptyset_{i+1}(I)$.

(b) if $g=(y^2,1)=\left(x^{2\frac{h}{p}},1\right)$ *then* $\emptyset_{101}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{242p^h}{4}.1=6\ \emptyset_{i+1}(x^{2\frac{h}{p}})$.

(c) if $g=(y,1)$ *or* $(y^3,1)$ *then* $\emptyset_{101}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{24}{4}(1+1)=6.2=12$.

Since $H\cap cl(g)=\{(I,1),(y^2,1),(y,1)\}$ *and* $\emptyset(g)=\emptyset(g^{-1})$*otherwise =0.*

11:- $H_{111}=<(xy,1)>$ *; (a) if* $g=(I,1)$ *then* $\emptyset_{111}(g)=\frac{|CG(G)|}{|cH(g)|}\emptyset(g)=\frac{242p^h}{4}.1=6\ \emptyset_{i+2}(I)$.

(b) if $g=((xy)^2,1)=(x^{2\frac{h}{p}},1)$ *then* $\emptyset_{111(g)}=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{242p^h}{4}.1=6\ \emptyset_{i+2}(x^{2\frac{h}{p}})$.

(c) if $g=(xy,1)$ *or* $((xy)^3,1)$ *then* $\emptyset_{111}(g)=|\ \frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right)=\frac{24}{4}(1+1)=6.2=12$.

Since $H\cap cl(xy)=\{(I,1),(xy^2,1),(xy,1)\}$ *and* $\emptyset(g)=\emptyset(g^{-1})=1$

Case (II):-

Consider the group G=($Q_{2.2p^h}$ ×D_3) and if H is a cyclic subgroup of ($Q_{2.2p^h}$×{r}) then H=<(q,r)> and ∅ the principle character of H and ∅j Artin's character of $Q_{2.2p^h}$, 1≤j≤i+2 ,by using theorem:-

$$\Phi j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} \sum_{i=1}^{n} \emptyset(hi) & if\ hi \in H \cap cl(g) \\ 0 & if\ H \cap cl(g) = \emptyset \end{cases}$$

H=<(q,r)

1:H_{12}=<(I,r)> (a) if g=(I,1) then

$\emptyset_{12}=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(1)|}.1=\frac{6|CQ2m(1)|}{3|C<x>(1)|}.1=2.\emptyset_j(1)$

(b):-If g=(I,r)or=(I,r^2) then

$\emptyset_{12}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(1)|}(1+1)=\frac{3|CQ2m(1)|}{3|C<x>(1)|}.2=2.\emptyset_j(q)$

Since H⌒cl(g)=$\{(I,1),(I,r),(I,r^2)\}$ *and* $\emptyset(g) = \emptyset(g^{-1}) = 1$ *othrewise=0.*

2:-H_{22}=<($x^{2^h_p}$,r)> (a) if g=(I,1) then $\emptyset_{22}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{242p^h.}{|C<x>\left(x^{2^h_p}\right)|}(1)=\frac{6|CG2m(q)|}{3|C<x>\left(x^{2^h_p}\right)|}(1)=2.\emptyset_j(I)$

(b) if g=($x^{2^h_p}$,1) then $\emptyset_{22}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{2^h_p})|}(1)=\frac{6|CQ2m\left(x^{2^h_p}\right)|}{3|c<x>\left(x^{2^h_p}\right)|}(1)=2.\emptyset_j(x^{2^h_p})$

(c) if g=(I,r) then$\emptyset_{22}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(q)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(q)|}.2=2.\emptyset_j(q)$

(d) if g=($x^{2^h_p}$,r) then$\emptyset_{22}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(q)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(q)|}.2=2.\emptyset_j(q).$

Since H⌒cl(g)=$\left\{(I,1),\left(x^{2^h_p},1\right),(I,r),(x^{2^h_p},r)\right\}$*.and* $\emptyset(g) = \emptyset(g^{-1}) = 1$*.othrewise=0*

3:-H_{32} =<(x^{p^h},r)> (a) if g=(I,1) then

$\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>\left(x^{p^h}\right)|}.1=\frac{6|CQ2m(q)|}{3|C<x>\left(x^{p^h}\right)|}.1=2.\emptyset_j(I).$

(b) if g=($x^{2^h_p}$,1) then$\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{|C<x>\left(x^{p^h}\right)|}.1=\frac{6|CQ2m(q)|}{3|C<x>\left(x^{p^h}\right)|}.1=2\emptyset j(x^{2^h_p})$

(c) if $g=(x^{p^h},1)$ then $\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^{p^h})|}(1+1)=\frac{3|CQ4p|}{3|C<x>(x^{p^h})|}.2=2.\emptyset_j(x^{p^h})$.

(d) if $g=(I,r)$ then $\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^{p^h})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{p^h})|}.2=2.\emptyset_j(q)$.

(e) if $g=(x^{2\frac{h}{p}},r)$ then $\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{|C<x>(x^{p^h})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{p^h})|}.2=2.\emptyset j(q)$.

(f) if $g=x^{p^h}$ $(,r)$ then $\emptyset_{32}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^{p^h})|}(1+1+1+1)=\frac{6.4(2p^h)}{3|C<x>(x^{p^h})|}=\frac{6|CQ2m(q)|}{3|C<x>(x^{p^h})|}=2.\emptyset j(q)$.

since $H\cap cl(g)=\{(I,1),(x^{2\frac{h}{p}},1),(x^{p^h},1),(I,r),(x^{2\frac{h}{p}},r),(x^{2\frac{h}{p}},r)\}$.and $\emptyset(g)=\emptyset((g^{-1})=1$.othrewise=0.

4:-$H_{42}=<(x^{4p},r)>$ (a) if $g=(I,1)$ then $\emptyset_{42(g)}=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{242p^h.}{|C<x>(x^{4p})|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^{4p})|}.1=2.\emptyset_j(I)$.

(b) if $g=(x^{4p},1)$ then $\emptyset_{42(g)}=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^{4p})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{4p})|}.2=2.\emptyset_j(x^{4p})$.

(c) if $g=(I,r)$ then $\emptyset_{42(g)}=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^{4p})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{4p})|}.2=2.\emptyset_j(q)$.

(d) if $g=(x^{4p},r)$ then

$\emptyset_{42(g)}$

$=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=$

$\frac{6.2p^h}{3|C<x>(x^{4p})|}(1+1+1+1)=\frac{6.4(2p)}{3|C<x>(x^{4p})|}=\frac{6|CQ2m(q)|}{3|C<x>(x^{4p})|}=2.\emptyset_j(q)$

since $H\cap cl(g)=\{(I,1),(x^{4p},1),(I,r),(x^{4p},r)\}$and $\emptyset(g)=\emptyset((g^{-1})=1$.otherwise=0.

5:-H_{52}=<x^{2p}, (,r)> (a) if g=(I,1) then $\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^{2p})|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^{2p})|}.1=2.\emptyset_j(I)$.

(b) if g=($x^{2\frac{h}{p}}$, 1) then $\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p1p2}{|C<x>(x^{2p2})|}.1=\frac{6|CQ4p|}{3|C<x>(x^{2p2})|}.1=2.\emptyset_j\left(x^{2\frac{h}{p}}\right)$.

(c) if g=(x^{4p},1) then $\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1})=\frac{12.2p^h}{3|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{2p})|}.2=2..\emptyset_j(x^{4p})$.

(d) if g=(x^{2p},1) then $\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{122p^h.}{3|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{2p})|}.2=2.\emptyset_j(x^{2p})$.

(e) if g=(I,r) then $\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{2p})|}.2=2..\emptyset_j(q)$.

(f) if g=($x^{2\frac{h}{p}}$,r) the$\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^{2p})|}.2=2..\emptyset_j(q)$.

(g) if g=(x^{4p},r) then

$\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{62p^h.}{3|C<x>(x^{2p})|}(1+1+1+1)=\frac{6.4(2p^h.)}{3|C<x>(x^{2p})|}=\frac{6|CQ2m(q)|}{3|C<x>(x^{2p})|}.2=2..\emptyset_j(q)$

(h) if g=(x^{2p}, r) then

$\emptyset_{52}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^{2p})|}(1+1+1+1)=\frac{6.4(2p1p2)}{3|C<x>(x^{2p})|}=\frac{6|CQ2m(q)|}{3|C<x>(x^{2p})|}2=2.\emptyset_j(q)$.

since

H∩

cl(g)=

$\left\{(I,1),\left(x^{2\frac{h}{p}},1\right),(x^{4p},1),(x^{2p},1),(I,r),\left(x^{2\frac{h}{p}},r\right),(x^{4p},r),(x^{2p},r)\right\}$

and Ø(g)=Ø((g^{-1}) = 1.othrewise=0.

6:-H_{62}=<(x^p,r)> (a) if g=(I,1) then $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^p)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^p)|}=2.\emptyset_j(I)$

(b) *if* $g=(x^{2_p^h},1)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^p)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^p)|}=2$ $.\emptyset_j(x^{2_p^h})$.

(c) *if* $g=(x^{p^h},1)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^p)|}.2=2.\emptyset_j(x^{p^h})$

(d) *if* $g=(x^{4p},1)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^p)|}.2=2.\emptyset_j(x^{4p})$.

(e) *if* $g=(x^{2p},1)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h.}{3|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^p)|}.2=2.\emptyset_j(x^{2p})$.

(f) *if* $g=(x^p,1)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^p)|}.2=2.\emptyset_j(x^p)$.

(g) *if* $g=(I,r)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^p)|}.2=2$ $.\emptyset_j(q)$.

(h) *if* $g=(x^{2_p^h},r)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^p)|}.2=2.\emptyset_j(q)$.

(i) *if* $g=(x^{p^h},r)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^p)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^p)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^p)|}=2.\emptyset_j(q)$.

(j) *if* $g=(x^{4p},r)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^p)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^p)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^p)|}=2.\emptyset_j(q)$.

(k) *if* $g=(x^{2p},r)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^p)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^p)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^p)|}=2.\emptyset_j(q)$.

(l) *if* $g=(x^p,r)$ *then* $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^p)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^p)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^p)|}=2.\emptyset_j(q)$.

Since

$H\cap cl(g) =$
$\left\{(I,1),\left(x^{2\frac{h}{p}},1\right),\left(x^{p^h},1\right),(x^{4p},1),(x^{2p},1),,(I,r),\left(x^{2\frac{h}{p}},r\right),\left(x^{p^h},r\right),(x^{4p},r),(x^{2p},r),\right\}$

And $\emptyset(g)=\emptyset(g^{-1}) = 1$.*othrewise=0.*

7:-H_{72}=<(x^4,r)>, *(a) if g=(I,1) then* $\emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^4)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^4)|}=2\,.\emptyset_j(I)$

(b) if g=(x^4*,1) then* $\emptyset_{72}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^4)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^4)|}.2=\ 2\,.\emptyset_j(x^4)$

(c) if g=(I,r) then $\emptyset_{72}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^4)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^4)|}.2=\ 2\,.\emptyset_j(q)$.

(d) if g=(x^{4p}*,r) then*

$\emptyset_{72}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x^4)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^4)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^4)|}2=\ 2\,.\emptyset_j(q)$.

since $H\cap cl(g)=\{(I,1),(x^{4p},1),(I,r),(x^{4p},r)\}$*and* $\emptyset(g)=\emptyset((g^{-1})=1$.*othrewise=0.*

8:-H_{82}=<(x^2,r)>; *(a) if g=(I,1) then* $\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^2)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^2)|}=2\,.\emptyset_j(I)$

(b) if g=($x^{2\frac{h}{p}},1$*)then* $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x^2)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x^2)|}=2\,.\emptyset_j(x^{2\frac{h}{p}})$.

(c) if g=(x^{4p}*,1) then* $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^2)|}.2=2\,.\emptyset_j(x^{4p})$

(d) if g=($x^{2p},1$*) then* $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^2)|}.2=2\,.\emptyset_j(x^{2p})$.

(e) if $g=(x^4,1)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{12.2p^h}{3|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^2)|}.2= 2.\emptyset_j\ (x^4)$.

(f) if $g=(x^2,1)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{12.2p^h}{3|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^2)|}.2= 2.\emptyset_j\ (x^2)$.

(g) if $g=(I,r)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{12.2p^h}{3|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^2)|}.2= 2\emptyset_j\ (q)$.

(h) if $g=(x^{2^h_p},r)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{12.2p^h}{3|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x^2)|}.2= 2\ \emptyset_j(q)$.

(i) if $g=(x^{4p},r)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{6.2p^h}{3|C<x>(x^2)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^2)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^2)|}.2= 2.\ \emptyset_j(q)$.

(j) if $g=(x^{2p},r)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{6.2p^h}{3|C<x>(x^2)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^2)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^2)|}2= 2.\ \emptyset_j\ (q)$.

(k) if $g=(x^4,r)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{6.2p^h}{3|C<x>(x^2)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^2)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^2)|}2= 2\ .\emptyset_j(q)$.

(l) if $g=(x^2,r)$ then $\emptyset_{82}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))= \frac{6.2p^h}{3|C<x>(x^2)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x^2)|}=\frac{6|CQ2m(q)|}{3|C<x>(x^2)|}2= 2\ .\emptyset_j(q)$.

Since

$H\cap cl(g)=$

$\left\{(I,1),\left(x^{2^h_p},1\right),(x^{4p},1),(x^{2p},1),(x^4,1),(x^2,1),(I,r),\left(x^{2^h_p},r\right),(x^{4p},r),(x^{2p},r),(x^4,r),(x^2,r)\right\}$

and $\emptyset(g)=\emptyset((g^{-1})=1$.othrewise=0.

9:-H_{92}=<(x,r)>; (a) if $g=(I,1)$ then $\emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x)|}=2\ .\emptyset_j(I)$

(b) if $g=(x^{2^h_p},1)$then $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{3|C<x>(x)|}=2\ .\emptyset_j(x^{2^h_p})$.

(c) *if* $g=(x^{p^h},1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x^{p^h})$

(d) *if* $g=(x^{4p},1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x^{4p})$

(e) *if* $g=(x^{p2},1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x^{p2})$.

(f) *if* $g=(x^p,1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x^p)$

(g) *if* $g=(x^4,1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x^4)$

(h) *if* $g=(x^2,1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x^2)$

(i) *if* $g=(x,1)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(x)$

(j) *if* $g=(I,r)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$.

(k) *if* $g=(x^{2_p^h},r)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{3|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(l) *if* $g=(x^{p^h},r)$ *then*

$\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(m) *if* $g=(x^{4p},r)$ *then*

$\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(n) if $g=(x^{2p},r)$ *then*

$\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(o)if $g=(x^{p},r)$ *then*

$\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(p) if $g=(x^4,r)$ *then*

$\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(q) if $g=(x^2,r)$ *then*

$\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)|}{3|C<x>(x)|}.2=2.\emptyset_j(q)$

(r) if $g=(x,r)$ *then* $\emptyset_{92}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{6.2p^h}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p^h)}{3|C<x>(x)|}=\frac{6|CQ2m(q)|}{3|C<x>(x)|}2=2\,\emptyset_j(q)$

Since

$H\cap cl(g)=\{(I,1),(x^{2p^h},1),(x^{p^h},1),(x^{4p},1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1),$
$,(x,1),(I,r),(x^{2p^h},r),$
$(x^{p^h},r),(x^{4p},r),(x^{2p},r),(x^p,r),(x^4,r),(x^2,r),(x,r)\}$ *And* $\emptyset(g)=\emptyset((g^{-1})=1$*.othrewise=0.*

10:- $H_{102}=<(y,r)>$ *(a) if* $g=(I,1)$ *then* $\emptyset_{102}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h.}{12}.1=2.(2p)=2.\emptyset_{i+1}(I)$.

(b) if g=(y^2,1) or ($x^{2^h_p}$,1) then $\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h.}{12}.1 = 2.(2p) = 2.\emptyset_{i+1}(x^{2^h_p})$.

(c) if g=(y,1) or (y^3,1) then $\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{24}{12}(1+1) = 4$.

(d) if g=(I,r) then $\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p^h.}{12}.(1+1) = 2.(2p) = 2.\ \emptyset_{i+1}(q)$.

(e) if g=(y^2,r) or ($x^{2^h_p}$,r) then $\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p^h}{12}(1+1) = 2.(2p) = 2.\ \emptyset_{i+1}(q)$.

(f) if g=((y,r) or (y^3,r) then $\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12}{12}(1+1+1+1) = 4$.

Since $H \cap cl(g) = \{(I,1),(y^2,1),(y,1),(I,r),(y^2,r),(y,r)\}$ *And* $\emptyset(g) = \emptyset((g^{-1}) = 1.othrewise = 0$.

11:-H_{112}=<(xy,r)> (a) if g=(I,1) then $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h.}{12}.1 = 2.(2p) = 2.\emptyset_{i+2}(I)$.

(b) if g=(($xy)^2$,1)=($x^{2^h_p}$,1) then $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h.}{12}.1 = 2.(2p) = 2.\emptyset_{i+2}(x^{2^h_p})$.

(c) if g=(xy,1) or ($(xy)^3$,1) then $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{24}{12}(1+1) = 4$.

(d) if g=(I,r) then $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p^h.}{12}(1+1) = 2.(2p) = 2.\ \emptyset_{i+2}(q)$.

(e) if g=(($xy)^2$,r)= ($x^{2^h_p}$,r) then $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p^h.}{12}(1+1) = 2.(2p) = 2.\ \emptyset_{i+2}(q)$.

(f) if g=(xy,r) or(($xy)^3$,r) then $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12}{12}(1+1+1+1) = 4$.

Since $H \cap cl(g) = \{(I,1),((xy)^2,1)(xy,1),(I,r),((xy)^2,r),(xy,r)\} and$
$\emptyset(g) = \emptyset((g^{-1}) = 1.othrewise = 0$

Case (III):-

Consider the group G=($Q_{2.2p^h} \times D_3$) and if H is a cyclic subgroup of ($Q_{2.2p^h}\times\{s\}$) then H=<(q,s)> and ∅ the principle character of H and ∅j Artin's character of $Q_{2.2p^h}$, 1≤j≤i+2 ,by using theorem:-

$$\Phi j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} \sum_{i=1}^{n} \emptyset(hi) & if\ hi \in H \cap cl(g) \\ 0 & if\ H \cap cl(g) = \emptyset \end{cases}$$

H=<(q,s)>

1:-H_{13}=<(I,s)> (a) if g=(I,1) then $\emptyset_{13}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(I)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(I)|}.1=3.\emptyset_j(I).$

(b) if g=(I,s) then $\emptyset_{13}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(I)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(I)|}.1=\emptyset_j(q).$

Since H∩cl(g)=$\{(I,1),(I,s)\}$ *othrewise* $= 0$

2:-H_{23}=<($x^{2^h_p}$,s)> (a) if g=(I,1) then $\emptyset_{23}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{2|C<x>\left(x^{2^h_p}\right)|}.1=\frac{6|CQ2m(q)|}{2|C<x>\left(x^{2^h_p}\right)|}.1=3.\emptyset_j(I).$

(b) if g=($x^{2^h_p}$,1) then $\emptyset_{23}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{2|C<x>\left(x^{2^h_p}\right)|}.1=\frac{6|CQ2m(q)|}{2|C<x>\left(x^{2^h_p}\right)|}.1 =3.\emptyset_j(x^{2^h_p}).$

(c) if g=(I,s) then $\emptyset_{23}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>\left(x^{2^h_p}\right)|}.1=\frac{2|CQ2m(q)|}{2|C<x>\left(x^{2^h_p}\right)|}.1=\emptyset_j(q).$

(d) if g=($x^{2^h_p}$,s) then $\emptyset_{23}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>\left(x^{2^h_p}\right)|}.1=\frac{2|CQ2m(q)|}{2|C<x>\left(x^{2^h_p}\right)|}.1=\emptyset_j(q).$

Since H∩cl(g)=$\left\{(I,1),\left(x^{2^h_p},1\right),(I,s),(x^{2^h_p},s)\right\}$ *And* $\emptyset(g) = \emptyset((g^{-1}) =1$ *othrewise* $= 0.$

3:-H_{33}=<x^{p^h} (,s)> (a) if g=(I,1) then $\emptyset_{33}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>\left(x^{p^h}\right)|}.1=\frac{6|CQ2m(q)|}{2|C<x>\left(x^{p^h}\right)|}.1=3.\emptyset_j(I).$

(b) if $g=(x^{2^h_p},1)$ then $\emptyset_{33}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x^{p^h})|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^{p^h})|}.1=3.\emptyset_j(x^{p^h})$.

(c) if $g=(x^{p^h},1)$ then $\emptyset_{33}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^{p^h})|}(1+1)=\frac{3|CQ4p|}{2|C<x>(x^{p^h})|}.2=3.\emptyset_j(x^{p^h})$.

(d) if $g=(I,s)$ then $\emptyset_{33}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^{p^h})|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^{p^h})|}.1=\emptyset_j(q)$.

(e) if $g=(x^{2^h_p},s)$ then $\emptyset_{33}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^{p^h})|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^{p^h})|}.1=\emptyset_j(q)$.

(f) if $g=(x^{p^h},s)$ then $\emptyset_{33}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^{p^h})|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^{p^h})|}.2=\emptyset_j(q)$.

Since $H\cap cl(g)=$
$\{(I,1),(x^{2^h_p},1),(x^{p^h},1),(I,s),(x^{2^h_p},s),(x^{p^h},s)\}$ And $\emptyset(g)=\emptyset((g^{-1})=1$ othrewise $=0$.

4:-H_{43}=<(x^{4p},s)> (a) if $g=(I,1)$ then $\emptyset_{43}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x^{4p})|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^{4p})|}.1=3.\emptyset_j(I)$.

(b)if $g=(x^{4p},1)$ then $\emptyset_{43}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^{4p})|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^{4p})|}.2=3.\emptyset_j(x^{4p})$.

(c) if $g=(I,s)$ then $\emptyset_{43}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^{4p})|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^{4p})|}.1=\emptyset_j(q)$.

(d) if $g=(x^{4p},s)$ then $\emptyset_{43}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^{4p})|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^{4p})|}.2=\emptyset_j(q)$.

Since $H\cap cl(g)=\{(I,1),(x^{4p},1),(I,s),(x^{4p},s)\}$ And $\emptyset(g)=\emptyset((g^{-1})=1$ othrewise $=0$.

5:- H_{53}=<(x^{2p},s)> (a) if g=(I,1) then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x^{2p})|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^{2p})|}.1=3.\emptyset_j(I)$.

(b) if g=$(x^{2^h_p}, 1)$ then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x^{2p})|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^{2p})|}.1=3.\emptyset_j(x^{2^h_p})$.

(c) if g=$(x^{4p},1)$ then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^{4p})|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^{4p})|}.2=3.\emptyset_j(x^{4p})$.

(d) if g=$(x^{2p},1)$ then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^{2p})|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^{2p})|}.2=3.\emptyset_j(x^{2p})$.

(e) if g=(I,s) then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^{2p})|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^{2p})|}.1=\emptyset_j(q)$.

(f) if g=$(x^{2^h_p},s)$ then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^{2p})|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^{2p})|}.1=\emptyset_j(q)$.

(g) if g=(x^{4p},s) then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^{2p})|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^{2p})|}.2=\emptyset_j(q)$.

(h) if g=(x^{2p},s) then $\emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^{2p})|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^{2p})|}.2=\emptyset_j(q)$.

Since

$H \cap cl(g)=$

$\left\{(I,1),\left(x^{2^h_p},1\right),(x^{4p},1),(x^{2p},1),(I,s),\left(x^{2^h_p},s\right),(x^{4p},s),(x^{2p},s)\right\}$

And $\emptyset(g) = \emptyset((g^{-1}) = 1$ othrewise $= 0$.

6:- H_{63}=<(x^{p},s)> (a) if g=(I,1) then $\emptyset_{63}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x^{p})|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^{p})|}.1=3.\emptyset_j(I)$..

(b) if g=$(x^{2^h_p}, 1)$ then $\emptyset_{63}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x^{p})|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^{p})|}.1=3.\emptyset_j(x^{2^h_p})$

(c) if $g=(x^{p^h},1)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12..2p^h}{2|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^p)|}.2=3.\emptyset_j(x^{p^h})$.

(d) if $g=(x^{4p},1)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^p)|}.2=3.\emptyset_j(x^{4p})$.

(e) if $g=(x^{2p},1)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^p)|}.2=3.\emptyset_j(x^{2p})$.

(f) if $g=(x^p,1)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^p)|}.2=3.\emptyset_j(x^p)$.

(g) if $g=(I,s)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8..2p^h}{2|C<x>(x^p)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^p)|}.1=\emptyset_j(q)$.

(h) if $g=(x^{2^h_p},s)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^p)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^p)|}.1=\emptyset_j(q)$.

(i) if $g=(x^{p^h},s)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^p)|}.2=\emptyset_j(q)$.

(j) if $g=(x^{4p},s)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^p)|}.2=\emptyset_j(q)$.

(k) if $g=(x^{2p},s)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^p)|}.2=\emptyset_j(q)$.

(l) if $g=(x^p,s)$ then $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^p)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^p)|}.2=\emptyset_j(q)$.

Since $H\cap cl(g)=$
$\{(I,1),(x^{2^h_p},1),(x^{p^h},1),(x^{4p},1),(x^{2p},1),(x^p,1),(I,s),(x^{2^h_p},s),(x^{p^h},s),(x^{4p},s),(x^{2p},s),(x^p,s)\}$

And $\emptyset(g)=\emptyset((g^{-1})=1$ *othrewise* $=0$.

7:-H_{73}=<(x⁴,s)>; (a) if g=(I,1) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{2|C<x>(x^4)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^4)|}.1=3.\emptyset_j(I)$.

(b) if g=(x^{4p},1) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^4)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^4)|}.2=3.\emptyset_j(x^{4p})$

(c) if g=(x^4,1) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^4)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^4)|}.2=3.\emptyset_j(x^4)$

(d) if g=(I,s) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^4)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^4)|}.1=\emptyset_j(q)$.

(e) if g=(x^{4p1},s) then $\emptyset_{73}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^4)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^4)|}.2=\emptyset j(q)$.

Since H∩cl(g)={(I,1),(x^{4p},1),(x^4,1),(I,s),(x^{4p},s),(x^4,s)} And$\emptyset(g)=\emptyset((g^{-1})=1$ *othrewise* $=0$

8:-H_{83}=<(x²,s)>; (a) if g=(I,1) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{2|C<x>(x^2)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^2)|}.1=3.\emptyset_j(I)$.

(b) if g=($x^{2^h_p}$,1) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h.}{2|C<x>(x^2)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x^2)|}.1=3.\emptyset_j(x^{2^h_p})$.

(c) if g=(x^{4p},1) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^{4p})$

(d) if g=(x^{2p},1) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^{2p})$

(e) if g=(x^4,1) then $\emptyset_{83}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^4)$

(f) if g=(x^2,1) then $\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^2)$

(g) if g=(I,s) then $\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^2)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^2)|}.1=\emptyset_j(q)$.

(h) if g=(x^{2p^h},s) then$\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x^2)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x^2)|}.1=\emptyset_j(q)$.

(i) if g=(x^{4p},s) then $\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^2)|}.2=\emptyset_j(q)$.

(j) if g=(x^{2p},s) then $\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^2)|}.2=\emptyset_j(q)$.

(k) if g=(x^4,s) then $\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^2)|}.2=\emptyset_j(q)$.

(l) if g=(x^2,s) then $\emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x^2)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x^2)|}.2=\emptyset_j(q)$.

Since

$H\cap cl(g)=\{(I,1),(x^{2p^h},1),(x^{4p},1),(x^{2p},1),(x^4,1),(x^2,1),(I,s),(x^{2p^h},s),(x^{4p},s),(x^{2p},s),(x^4,s),(x^2,s)\}$

And$\emptyset(g) = \emptyset((g^{-1}) = 1$ othrewise = 0

9:-H_{93}=<(x,s)>; (a) if g=(I,1) then $\emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{242p^h.}{2|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x)|}.1=3.\emptyset_j(I)$.

(b) if g=(x^{2p^h},1) then $\emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x)|}.1=3.\emptyset_j(x^{2p^h})$.

(c) if g=(x^{p^h},1) then $\emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{2|C<x>(x)|}.1=\frac{6|CQ2m(q)|}{2|C<x>(x)|}.1=3.\emptyset_j(x^{p^h})$.

(d) if $g=(x^{4p},1)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x)|}.2=3.\emptyset_j(x^{4p})$.

(e) if $g=(x^{2p},1)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x)|}.2=3.\emptyset_j(x^{2p})$.

(f) if $g=(x^{p},1)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x)|}.2=3.\emptyset_j(x^{p})$.

(g) if $g=(x^{4},1)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x)|}.2=3.\emptyset_j(x^{4})$

(h) if $g=(x^{2},1)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x)|}.2=3.\emptyset_j(x^{2})$

(i) if $g=(x,1)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p^h}{2|C<x>(x)|}(1+1)=\frac{3|CQ2m(q)|}{2|C<x>(x)|}.2=3.\emptyset_j(x)$

(j) if $g=(I,s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x)|}.1=\emptyset_j(q)$.

(k) if $g=(x^{2p^h},s)$ then$\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{2|C<x>(x)|}.1=\frac{2|CQ2m(q)|}{2|C<x>(x)|}.1=\emptyset_j(q)$.

(l) if $g=(x^{p^h},s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

(m) if $g=(x^{4p},s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

(n) if $g=(x^{2p},s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

(o) if $g=(x^p,s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

(p) if $g=(x^4,s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

(q) if $g=(x^2,s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

(r) if $g=(x,s)$ then $\emptyset_{93}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p^h}{2|C<x>(x)|}(1+1)=\frac{|CQ2m(q)|}{2|C<x>(x)|}.2=\emptyset_j(q)$.

Since $H\cap cl(g)=\{(I,1),(x^{2p^h},1),(x^{p^h},1),(x^{4p},1),(x^{2p},1),(x^p,1),(x^4,1),(x^2,1),(x,1),(I,s),(x^{2p^h},s),(x^{p^h},s),(x^{4p},s),(x^{2p},s),(x^p,s),(x^4,s),(x^2,s),(x,s)\}$ and $\emptyset(g)=\emptyset((g^{-1})=1$ othrewise $=0$

10:- $H_{103}=<(y,s)>$; (a) if $g=(I,1)$ then $\emptyset_{103}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{8}.1=3.\emptyset_{i+1}(I)$.

(b) if $g=(y^2,1)=(x^{2p^h},1)$ then $\emptyset_{103}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p^h}{8}.1=3.\emptyset_{i+1}(x^{2p^h})$.

(c) if $g=(y,1)$ or $(y^3,1)$ then $\emptyset_{103}(g)=\frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right)=\frac{24}{8}(1+1)=6$.

(d) if $g=(I,s)$ then $\emptyset_{103}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{8}.1=\emptyset_{i+1}(q)$.

(e) if $g=(y^2,s)=(x^{2p^h},s)$ then $\emptyset_{103}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{8}.1=\emptyset_{i+1}(q)$.

(f) if $g=(y,s)$ or (y^3,s) then $\emptyset_{103}(g)=\frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right)=\frac{8}{8}(1+1)=2$.

Since $H\cap cl(g)=\{(I,1),(y^2,1),(y,1),(I,s),(y^2,s),(y,s)\}$ And $\emptyset(g)=\emptyset((g^{-1})=1$ othrewise $=0$.

11:-H_{113}=<(xy,s)>; a) if g=(I,1) then $\emptyset_{113}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{8}.1=3.\emptyset_{i+2}(I)$.

(b) if g=((xy)²,1) =$(x^{2\frac{h}{p}},1)$ then $\emptyset_{113}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p^h}{8}.1=3.\emptyset_{i+2}(x^{2\frac{h}{p}})$.

(c) if g=(xy,1) or ((xy)³,1) then $\emptyset_{113}(g)= \frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right) = \frac{24}{8}(1+1)=6$.

(d) if g=(I,s) then $\emptyset_{113}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{8}.1=\emptyset_{i+2}(q)$.

(e) if g=((xy)²,s) =$(x^{2\frac{h}{p}},s)$ then $\emptyset_{113}(g)= \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p^h}{8}.1=\emptyset_{i+2}(q)$.

(f) if g=(xy,s) or ((xy)³,s) then $\emptyset_{113}(g)= \frac{|CG(g)|}{|CH(g)|}\left(\emptyset(g)+\emptyset(g^{-1})\right) =\frac{8}{8}(1+1)=2$. Since H∩cl(g) $=\{(I,1),((xy)^2,1),(xy,1),(I,s),((xy)^2,s),(xy,s)\}$ And $\emptyset(g) = \emptyset((g^{-1})$ =1 othrewise = 0.

Note: $(xy)^2=y^2$ since $(xy)^2=xyxy= xyxy.y^2y^2=xyxy^3y^2=x(yxy^3)y^2= xx^{-1}y^2=y^2$.

Example:-(4,2)

*Let p =5 , h = 2 ; m = $2.p^h$ =2.5^2 =50 , such that h is any positive integer and p is prime number, Q_{2m} =Q_{100} To find Artin's character of the group **(Q_{100} × D_3)** the cyclic subgroup of **Q_{100}** which are*

{<I>},{<x^{50}>},{<x^{25} >},{<x^{20} >} ,{<x^{10} >},{<x^5 >}, {<x^4 >},{<x^2 >} ,{<x>} ,{<y>},{<xy>} and cyclic subgroup of D_3 which are {<I>},{<r>},{<s>} ,

∴ The cyclic subgroup of **(Q_{100} × D_3)** which are

{<I,1>},{<x^{50},1>},{<x^{25},1 >},{<x^{20},1 >} ,{<x^{10} ,1>},{<x^5,1 >}, {<x^4 ,1 >},{<x^2,1 >} ,{x,1} ,{<y,1>},{<xy,1>} ,

{<I,r >},{<x^{50},r>},{<x^{25},r >},{<x^{20},r >} ,{<x^{10} ,r >},{<x^5,r >} , {<x^4 ,r >},{<x^2,r >} ,{x,r } ,{<y,r >},{<xy,r >} ,

{<I,s >},{< x^{50}, s>},{< x^{25}, s >},{< x^{20}, s >} ,{< x^{10} ,s >},{< x^5, s >},{< x^4 ,s >},{< x^2, s >} ,{<x,s> } ,{<y,s >},{<xy,s >} ,

by using theorem:-

$$\Phi j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} \sum_{i=1}^{n} \emptyset(hi) & if\ hi \in H \cap cl(g) \\ 0 & if\ H \cap cl(g) = \emptyset \end{cases}$$

Then $\boldsymbol{Ar(Q_{100} \times D_3) = Ar(Q_{2^2.5^2} \times D_3) = Ar(Q_{2^2.5^2}) \otimes Ar(D_3) =}$

	Γ-classes of $Q_{2.2p^h} \times \{1\}$										
Γ-classes	[I,1]	$[x^{2\frac{h}{p}},1]$	$[x^{p^h},1]$	$[x^{4p},1]$	$[x^{2p},1]$	$[x^p,1]$	$[x^4,1]$	$[x^2,1]$	[x,1]	[y,1]	[xy,1]
$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	2	2	$2(2\frac{h}{p})$	$2(2\frac{h}{p})$
$\lvert CQ_{2.2p^h} \times D_3(CL_\alpha) \rvert$	$24(2\frac{h}{p})$	$24(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(\frac{h}{2}p)$	24	24
Φ(I,1)											
Φ(2,1)											
Φ(3,1)											
Φ(4,1)											
Φ(5,1)											
Φ(6,1)	$6Ar(Q_{2.2p^h})$										
Φ(7,1)											
Φ(8,1)											
Φ(9,1)											
Φ(10,1)											
Φ(11,1)											

	Γ-classes of $Q_{2.2p^h} \times \{1\}$										
Γ-classes	[I,1]	$[x^{2\frac{h}{p}},1]$	$[x^{p^h},1]$	$[x^{4p},1]$	$[x^{2p},1]$	$[x^p,1]$	$[x^4,1]$	$[x^2,1]$	[x,1]	[y,1]	[xy,1]
$\lvert CL_\alpha \rvert$	1	1	2	2	2	2	2	2	2	$2(2\frac{h}{p})$	$2(2\frac{h}{p})$
$\lvert CQ_{2.2p^h} \times D_3(CL_\alpha) \rvert$	$24(2\frac{h}{p})$	$24(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(2\frac{h}{p})$	$12(\frac{h}{2}p)$	24	24
Φ(I,2)											
Φ(2,2)											
Φ(3,2)											
Φ(4,2)											
Φ(5,2)											
Φ(6,2)											
Φ(7,2)	$2Ar(Q_{2.2p^h})$										
Φ(8,2)											
Φ(9,2)											
Φ(10,2)											
Φ(11,2)											

	Γ-classes of $Q_{2.2p^h}$ × { 1 }										
Γ-classes	[I,1]	$[x^{2^h_p},1]$	$[x^{p^h},1]$	$[x^{4p},1]$	$[x^{2p},1]$	$[x^{p},1]$	$[x^{4},1]$	$[x^{2},1]$	[x,1]	[y,1]	[xy,1]
|CL_α|	1	1	2	2	2	2	2	2	2	$2(2^h_p)$	$2(2^h_p)$
|$CQ_{2.2p^h}$×$D_3(CL_\alpha)$|	$24(2^h_p)$	$24(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12(2^h_p)$	$12({}^h_2p)$	24	24
Φ(I,3) Φ(2,3) Φ(3,3) Φ(4,3) Φ(5,3) Φ(6,3) Φ(7,3) Φ(8,3) Φ(9,3) Φ(10,3) Φ(11,3)	**3Ar($Q_{2.2p^h}$)**										

	Γ-classes of $Q_{2.2p^h}$ × { r }										
Γ-classes	[I,r]	$[x^{2^h_p},r]$	$[x^{p^h},r]$	$[x^{4p},r]$	$[x^{2p},r]$	$[x^{p},r]$	$[x^{4},r]$	$[x^{2},r]$	[x,r]	[y,r]	[xy,r]
|CL_α|	2	2	4	4	4	4	4	4	4	$4(2^h_p)$	$4(2^h_p)$
|$CQ_{2.2^h{}_p}$×$D_3(CL_\alpha)$|	$12\ (2^h_p)$	$12(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6({}^h_2p)$	24	24
Φ(I,1) Φ(2,1) Φ(3,1) Φ(4,1) Φ(5,1) Φ(6,1) Φ(7,1) Φ(8,1) Φ(9,1) Φ(10,1) Φ(11,1)	**0**										

	Γ-classes of $Q_{2.2p^h}$ × { r }										
Γ-classes	[I,r]	$[x^{2^h_p},r]$	$[x^{p^h},r]$	$[x^{4p},r]$	$[x^{2p},r]$	$[x^{p},r]$	$[x^{4},r]$	$[x^{2},r]$	[x,r]	[y,r]	[xy,r]
|CL_α|	2	2	4	4	4	4	4	4	4	$4(2^h_p)$	$4(2^h_p)$
|$CQ_{2.2^h{}_p}$×$D_3(CL_\alpha)$|	$12\ (2^h_p)$	$12(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6(2^h_p)$	$6({}^h_2p)$	24	24
Φ(I,2) Φ(2,2) Φ(3,2) Φ(4,2) Φ(5,2) Φ(6,2) Φ(7,2) Φ(8,2) Φ(9,2) Φ(10,2) Φ(11,2)	**2Ar($Q_{2.2p^h}$)**										

	Γ-classes of $Q_{2.2p^h}$ × { r }										
Γ-classes	[I,r]	$[x^{2_p^h}, r]$	$[x^{p^h},r]$	$[x^{4p},r]$	$[x^{2p},r]$	$[x^{p},r]$	$[x^{4},r]$	$[x^{2},r]$	[x,r]	[y,r]	[xy,r]
$\|CL_\alpha\|$	2	2	4	4	4	4	4	4	4	$4(2_p^h)$	$4(2_p^h)$
$\|CQ_{2.2^h p}\times D_3(CL_\alpha)\|$	$12(2_p^h)$	$12(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(2_p^h)$	$6(_2^h p)$	24	24
Φ(I,3) Φ(2,3) Φ(3,3) Φ(4,3) Φ(5,3) Φ(6,3) Φ(7,3) Φ(8,3) Φ(9,3) Φ(10,3) Φ(11,3)	0										

	Γ-classes of $Q_{2.2p^h}$ × {s }										
Γ-classes	[I,s]	$[x^{2_p^h}, s]$	$[x^{p^h},s]$	$[x^{4p},s]$	$[x^{2p},s]$	$[x^{p},s]$	$[x^{4},s]$	$[x^{2},s]$	[x,s]	[y,s]	[xy,s]
$\|CL_\alpha\|$	3	3	6	6	6	6	6	6	6	$6(2_p^h)$	$6(2_p^h)$
$\|CQ_{2.2^h p}\times D_3(CL_\alpha)\|$	$8(2_p^h)$	$8(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(_2^h p)$	8	8
Φ(I,1) Φ(2,1) Φ(3,1) Φ(4,1) Φ(5,1) Φ(6,1) Φ(7,1) Φ(8,1) Φ(9,1) Φ(10,1) Φ(11,1)	0										

	Γ-classes of $Q_{2.2p^h}$ × {s}										
Γ-classes	[I,s]	$[x^{2_p^h}, s]$	$[x^{p^h},s]$	$[x^{4p},s]$	$[x^{2p},s]$	$[x^{p},s]$	$[x^{4},s]$	$[x^{2},s]$	[x,s]	[y,s]	[xy,s]
$\|CL_\alpha\|$	3	3	6	6	6	6	6	6	6	$6(2_p^h)$	$6(2_p^h)$
$\|CQ_{2.2^h p}\times D_3(CL_\alpha)\|$	$8(2_p^h)$	$8(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(2_p^h)$	$4(_2^h p)$	8	8
Φ(I,2) Φ(2,2) Φ(3,2) Φ(4,2) Φ(5,2) Φ(6,2) Φ(7,2) Φ(8,2) Φ(9,2) Φ(10,2) Φ(11,2)	0										

	Γ-classes of $Q_{2.2p^h} \times \{s\}$										
Γ-classes	$[I,s]$	$[x^{2^{\frac{h}{p}}},s]$	$[x^{p^h},s]$	$[x^{4p},s]$	$[x^{2p},s]$	$[x^p,s]$	$[x^4,s]$	$[x^2,s]$	$[x,s]$	$[y,s]$	$[xy,s]$
$\lvert CL_\alpha \rvert$	3	3	6	6	6	6	6	6	6	$6(2^{\frac{h}{p}})$	$6(2^{\frac{h}{p}})$
$\lvert CQ_{2.2^h p}\times D_3(CL_\alpha) \rvert$	$8(2^{\frac{h}{p}})$	$8(2^{\frac{h}{p}})$	$4(2^{\frac{h}{p}})$	$4(2^{\frac{h}{p}})$	$4(2^{\frac{h}{p}})$	$4(2^{\frac{h}{p}})$	$4(2^{\frac{h}{p}})$	$4(2^{\frac{h}{p}})$	$4(^{h}_{2}p)$	8	8
$\Phi(1,3)$	$Ar(Q_{2.2p^h})$										
$\Phi(2,3)$											
$\Phi(3,3)$											
$\Phi(4,3)$											
$\Phi(5,3)$											
$\Phi(6,3)$											
$\Phi(7,3)$											
$\Phi(8,3)$											
$\Phi(9,3)$											
$\Phi(10,3)$											
$\Phi(11,3)$											

Table 4

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