



## On Artin cokernel of The Group( $Q_{2m} \times D_3$ ) Where $m = 2p_1 p_2$ and $p_1, p_2$ are prime numbers

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### Abstract:

The main purpose of This paper is to find Artin's character table  $Ar(Q_{2m} \times D_3)$  when  $m$  is even number such that  $m = 2p$ , and  $p$  is a prime number; where  $Q_{2m}$  is denoted to Quaternion group of order  $4m$ , time is said to have only one dimension and space to have three dimension, the mathematical quaternion partakes of both these elements; in technical language it may be said to be "time plus space", or "space plus time" and in this sense it has, or at least involves a reference to four dimensions, and how the one of time of space the three might in the chain of symbols girdled"- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols., 1882, 1885, 1889)), and  $D_3$  is Dihedral group of order 6. In 1962, C.W.Curits & I. Reiner studied Representation Theory of finite groups.

In 1976, I.M.Isaacs studied Characters Theory of finite groups, In 1982 , M.S.Kirdar studied The Factor Group of the Z-Valued class function modulo the group of the Generalized Characters. In 1994 H. H. Abass studies The Factor Group of class function over the group of Generalized Characters of  $D_n$  and found  $\equiv^*(D_n)$ . In 1995, N. R.

Mahmood studies The Cyclic Decomposition of the factor Group  $cf(Q_{2m}, Z)/\bar{R}(Q_{2m})$ . In 2002, K-Sekiguchi studies Extensions and the Irreducibilities of the Induced Characters of Cyclic P-Group. In 2006, A.S.Abid studies Characters Table of Dihedral Group for Odd number.

**Key words:** even number, prime number, Quaternion group, and Dihedral group.

## 1. INTRODUCTION:

Representation Theory is a branch of mathematics that studies abstract algebra structures by Representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication ,In which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication, Moreover, representation and character theory provide applications, no only in other branches of mathematics but also in physics and chemistry.

For a finite group G, the factor group  $\bar{R}(G)/T(G)$  is called the Artin cokernel of G denoted  $AC(G)$ ,  $\bar{R}(G)$ denoted the a belian group generated by Z-valued characters of G under the operation of pointwise addition,  $T(G)$  is a subgroup of  $\bar{R}(G)$  which is generated by Artin's characters.

## 2. PRELIMINARS:[1]: $(3,1)$

### The Generalized Quaternion Group $Q_{2m}$ :

For each positive integer  $m \geq 2$  ,The generalized Quaternion Group  $Q_{2m}$  of order  $4m$  with two generators x and y satisfies  $Q_{2m} = \{x^h y^k, 0 \leq h \leq 2m, 1, k=0, 1\}$

Which has the following properties  $\{x^{2m}=y^4=I, yx^my^{-1}=x^{-m}\}$ .

### Definition :[2](3,2)

Let  $G$  be a finite group, all the characters of group  $G$  induced from a principal character of cyclic subgroup of  $G$  are called Artin characters of  $G$ .

Artin characters of the finite group can be displayed in a table called Artin characters table of  $G$  which is denoted by  $Ar(G)$ ; The first row is  $\Gamma$ -conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized  $|C_G(cl_a)|$  and other rows contains the values of Artin characters .

### Theorem:[5]: (3,3)

The general form of Artin characters table of  $Cp^s$

When  $p$  is a prime number and  $s$  is a positive integer number is given by :-

$$Ar(Cp^s) =$$

$\Gamma$ -classes	[1]	$[x^{ps-1}]$	$[x^{ps-2}]$	...	$[x]$
$ cl_a $	1	1	1	...	1
$ cp^s(cl_a) $	$p^s$	$ps$	$p^s$	...	$p^s$
$\Phi_1$	$p^s$	0	0	...	0
$\Phi_2$	$p^{s-1}$	$p^{s-1}$	0	...	0
:	:	:	:	...	:
$\Phi_k$	$p$	$p$	$p$	...	0
$\Phi_{s+1}$	1	1	1	...	1

Table (1)

### Corollary:[5]: (3,4)

Let  $n = P_1^{a1} \cdot P_2^{a2} \dots P_n^{an}$  where  $g.c.d(P_i, P_j) = 1$  if  $i \neq j$  and  $P_i$  are a prime numbers, and  $a_i$  any positive integer for all  $1 \leq i \leq n$  Then :  
 $Ar(C_m) = Ar(Cp_1^{a1}) \otimes Ar(Cp_2^{a2}) \dots \otimes Ar(Cp_n^{an})$  such that

$$Ar(Cp^1) =$$

$\Gamma$ -class	[1]	$[x]$
$ cl_a $	1	1
$ cp(cl_a) $	$p$	$p$
$\Phi_1$	$p$	0
$\Phi_2$	1	1

Table (2)

Where  $\text{Ar}(\text{C}_{2m})$ ,  $m=2, p$  is  $\text{Ar}(\text{C}_{2.2p}) = \text{Ar}(\text{C}_2^2 p) = \text{Ar}(\text{C}_2^2) \otimes \text{Ar}(p^1)$ .

**Proposition:[4]:**(3,5)

The number of all distinct Artin characters on group  $G$  is equal to the number of  $\Gamma$ -classes on  $G$ . Furthermore, Artin characters are constant on each  $\Gamma$ -classes .

### 3. THE MAIN RESULTS:

**Theorem:(4,1):-** The Artin's character table of the group  $(Q_{4p_1p_2} \times D_3)$  where  $m=2p_1p_2$  such that  $p_1, p_2$  are prime numbers is given as follows:

$$Ar(Q_{4p1p2} \times D_3) =$$

The Artin's character table of matrix from degree  $42 \times 42$  of group  $(Q_{4p1p2} \times D_3)$ :

$F$ -classes of $Q_{(2^m)} \times \{I\}$									
$F$ -classes	$\{[I]\}$	$\{[p^{0^1}I]\}$	$\{[p^{0^2}I]\}$	$\{[p^{0^3}I]\}$	$\{[p^{0^4}I]\}$	$\{[p^{0^5}I]\}$	$\{[p^{0^6}I]\}$	$\{[p^{0^7}I]\}$	$\{[p^{0^8}I]\}$
$ CL_0 $	$\frac{1}{p}$	$\frac{1}{p}$	$\frac{2}{p}$						
$ CL_1 $	$24.2p$	$24.2p$	$24.2p$	$12.2p$	$12.2p$	$12.2p$	$12.2p$	$12.2p$	$12.2p$
$ CL_2 $	$12.2p$	$12.2p$	$12.2p$	$12.2p$	$24$	$24$	$24$	$24$	$24$
$ CL_3 $	$12.2p$	$12.2p$	$12.2p$	$12.2p$	$p$	$p$	$p$	$p$	$p$

$$6Ar(O_{3.2-1.2})$$

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$\Gamma$ -classes of $Q_{2m,2} \times \{1\}$												
$\Gamma$ -classes	[1,1]	[ $x^{2^{2m+2}}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^2, 1$ ]	[ $x^2, 1$ ]	[ $y, 1$ ]
$ Cl_{\alpha} $	3	3	6	6	6	6	6	6	6	6	6	$8p_1p_2$
$ CQ_{2m,2} \times D_3(Cl_{\alpha}) $	$8.2p_1p_2$	$8.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$8$						
$\Phi(1,1)$												
$\Phi(2,1)$												
$\Phi(3,1)$												
$\Phi(4,1)$												
$\Phi(5,1)$												
$\Phi(6,1)$												
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$\Phi(9,1)$												
$\Phi(10,1)$												
$\Phi(11,1)$												
$\Phi(12,1)$												
$\Phi(13,1)$												
$\Phi(14,1)$												

0

$\Gamma$ -classes of $Q_{2m,2} \times \{1\}$												
$\Gamma$ -classes	[1,1]	[ $x^{2^{2m+2}}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^2, 1$ ]	[ $x^2, 1$ ]	[ $y, 1$ ]	[ $y, 1$ ]
$ Cl_{\alpha} $	1	2	2	2	2	2	2	2	2	2	2	$4p_1p_2$
$ CQ_{2m,2} \times D_3(Cl_{\alpha}) $	$24.2p_1p_2$	$24.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$24$
$\Phi(1,1)$												
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$\Phi(10,1)$												
$\Phi(11,1)$												
$\Phi(12,1)$												
$\Phi(13,1)$												
$\Phi(14,1)$												

2Ar( $Q_{2,2p_1p_2}$ )

$\Gamma$ -classes of $Q_{2m,2} \times \{1\}$												
$\Gamma$ -classes	[1,1]	[ $x^{2^{2m+2}}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^2, 1$ ]	[ $x^2, 1$ ]	[ $y, 1$ ]	[ $y, 1$ ]
$ Cl_{\alpha} $	2	2	4	4	4	4	4	4	4	4	4	$8p_1p_2$
$ CQ_{2m,2} \times D_3(Cl_{\alpha}) $	$12.2p_1p_2$	$12.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$6.2p_1p_2$	$24$
$\Phi(1,2)$												
$\Phi(2,2)$												
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$\Phi(5,2)$												
$\Phi(6,2)$												
$\Phi(7,2)$												
$\Phi(8,2)$												
$\Phi(9,2)$												
$\Phi(10,2)$												
$\Phi(11,2)$												
$\Phi(12,2)$												
$\Phi(13,2)$												
$\Phi(14,2)$												

2Ar( $Q_{2,2p_1p_2}$ )

$\Gamma$ -classes of $Q_{2m,2} \times \{1\}$												
$\Gamma$ -classes	[1,1]	[ $x^{2^{2m+2}}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^2, 1$ ]	[ $x^2, 1$ ]	[ $y, 1$ ]	[ $y, 1$ ]
$ Cl_{\alpha} $	3	3	6	6	6	6	6	6	6	6	6	$8p_1p_2$
$ CQ_{2m,2} \times D_3(Cl_{\alpha}) $	$8.2p_1p_2$	$8.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$4.2p_1p_2$	$8$
$\Phi(1,2)$												
$\Phi(2,2)$												
$\Phi(3,2)$												
$\Phi(4,2)$												
$\Phi(5,2)$												
$\Phi(6,2)$												
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$\Phi(9,2)$												
$\Phi(10,2)$												
$\Phi(11,2)$												
$\Phi(12,2)$												
$\Phi(13,2)$												
$\Phi(14,2)$												

0

$\Gamma$ -classes of $Q_{2m,2} \times \{1\}$												
$\Gamma$ -classes	[1,1]	[ $x^{2^{2m+2}}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^2, 1$ ]	[ $x^2, 1$ ]	[ $y, 1$ ]	[ $y, 1$ ]
$ Cl_{\alpha} $	1	1	2	2	2	2	2	2	2	2	2	$4p_1p_2$
$ CQ_{2m,2} \times D_3(Cl_{\alpha}) $	$24.2p_1p_2$	$24.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$12.2p_1p_2$	$24$
$\Phi(1,3)$												
$\Phi(2,3)$												
$\Phi(3,3)$												
$\Phi(4,3)$												
$\Phi(5,3)$												
$\Phi(6,3)$												
$\Phi(7,3)$												
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$\Phi(10,3)$												
$\Phi(11,3)$												
$\Phi(12,3)$												
$\Phi(13,3)$												
$\Phi(14,3)$												

3Ar( $Q_{2,2p_1p_2}$ )

$\Gamma$ -classes of $Q_{2m,2} \times \{1\}$												
$\Gamma$ -classes	[1,1]	[ $x^{2^{2m+2}}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^{2^m}, 1$ ]	[ $y^{2^m}, 1$ ]	[ $x^2, 1$ ]	[ $x^2, 1$ ]	[ $y, 1$ ]	[ $y,$

$\Gamma$ -classes	$\Gamma$ -classes of $Q_{2p_1p_2} \times \{I\}$											
$[I,1]$	$[p^{(2p_1p_2)}, 1]$	$[p^{(2p_1p_2)}, -1]$	$[p^{(2p_1p_2)}, 0]$	$[p^{(2p_1p_2)}, 1]$	$[p^{(2p_1p_2)}, -1]$	$[p^{(2p_1p_2)}, 0]$	$[p^{(2p_1p_2)}, 1]$	$[p^{(2p_1p_2)}, -1]$	$[p^{(2p_1p_2)}, 0]$	$[p^{(2p_1p_2)}, 1]$	$[p^{(2p_1p_2)}, -1]$	$[p^{(2p_1p_2)}, 0]$
$[C_{L_0}]$	2	2	4	4	4	4	4	4	4	4	4	8p_1p_2, 8p_1p_2
$[CQ_{2p_1p_2} \times \{I,1\}]$	12.2p_1p_2, 12.2p_1p_2, 6.2p_1p_2, 24, 24											
$\Phi(1,1)$												
$\Phi(2,1)$												
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$\Phi(10,1)$												
$\Phi(11,1)$												
$\Phi(12,1)$												
$\Phi(13,1)$												
$\Phi(14,1)$												
$\Gamma$ -classes	$\Gamma$ -classes of $Q_{2p_1p_2} \times \{y\}$											
$[I,y]$	$[y^{(2p_1p_2)}, 1]$	$[y^{(2p_1p_2)}, -1]$	$[y^{(2p_1p_2)}, 0]$	$[y^{(2p_1p_2)}, 1]$	$[y^{(2p_1p_2)}, -1]$	$[y^{(2p_1p_2)}, 0]$	$[y^{(2p_1p_2)}, 1]$	$[y^{(2p_1p_2)}, -1]$	$[y^{(2p_1p_2)}, 0]$	$[y^{(2p_1p_2)}, 1]$	$[y^{(2p_1p_2)}, -1]$	$[y^{(2p_1p_2)}, 0]$
$[C_{L_0}]$	3	3	6	6	6	6	6	6	6	6	6	8p_1p_2, 8p_1p_2
$[CQ_{2p_1p_2} \times \{I,y\}]$	8.2p_1, 8.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 4.2p_1, 8, 8											
$\Phi(1,1)$												
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$\Phi(14,1)$												

Table 3

### Proof:

Let  $g_{ij} = (qi, dj) ; q_i \in Q_{4p_1p_2}, d_j \in D_3$

#### Case (I):-

Consider the group  $G = (Q_{4p_1p_2} \times D_3)$  and if  $H$  is a cyclic subgroup of  $(Q_{4p_1p_2} \times \{I\})$  then  $H = \langle (q, 1) \rangle$  and  $\Phi$  the principle character of  $H$  and  $\Phi_j$  Artin's characters of  $Q_{4p_1p_2}, 1 \leq j \leq i+2$ , The cyclic subgroup of  $Q_{4p_1p_2}$  which are  $\langle I \rangle, \langle x^{2p_1p_2} \rangle, \langle x^{p_1p_2} \rangle, \langle x^{4p_2} \rangle, \langle x^{2p_2} \rangle, \langle x^{p_2} \rangle, \langle x^{4p_1} \rangle, \langle x^{2p_1} \rangle, \langle x^{p_1} \rangle, \langle x^4 \rangle, \langle x^2 \rangle, \langle x \rangle, \langle y \rangle, \langle xy \rangle$  and cyclic subgroup of  $D_3$  which are  $\langle I \rangle, \langle r \rangle, \langle s \rangle$ , by using theorem:

$$\Phi_j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} & \text{if } hi \in H \cap cl(g) \\ 0 & \text{if } H \cap cl(g) = \emptyset \end{cases}$$

$H = \langle q, 1 \rangle :-$

$$1:- H_{11} = \langle (I, 1) \rangle \quad \text{if } g = (I, 1) \text{ then } \Phi_{11}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset \quad (g) = \frac{24 \cdot 2p_1p_2}{|CH(g)|}$$

$$.1 = \frac{6|CQ4p_1(1)|}{|C<x>(1)|} .1 = 6.\emptyset_j(I)$$

since  $H \cap cl(I, 1) = (I, 1)$ .

$$2:- H_{21} = \langle (x^{2p_1p_2}, 1) \rangle ; \text{ ( a)if } g = (I, 1) \text{ then } \Phi_{21}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) =$$

$$= \frac{24 \cdot 2p_1p_2}{|CH(g)|} .1 = \frac{6|CQ4p_1p_2(1)|}{|C<x>(x^{2p_1p_2})|} .1$$

$$= 6\emptyset_j(I).$$

$$(b) \text{ if } g = (x^{2p_1p_2}, 1) \text{ then } \emptyset_{2I}(g) = \frac{|CG(g)|}{|CH(g)|} \cdot \emptyset(g) = \frac{24.2p_1p_2}{|CH(g)|}$$

$$\cdot I = \frac{24.2p_1p_2}{|C< x>(x^{2p_1p_2})|} \cdot I = \frac{6|CQ4p(x^{2p_1p_2})|}{|C< x>(x^{2p_1p_2})|} \cdot I =$$

$$= 6\emptyset_j(x^{2p_1p_2}) \quad \text{since} \quad H \cap cl(x^{2p_1p_2}) = (I, 1), (x^{2p_1p_2}, 1)$$

$$\text{otherwise} = 0.$$

3:-  $H_{3I} = < (x^{p_1p_2}, 1) >; (a) \text{ if } g = (I, 1) \text{ then } \emptyset_{3I}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g)$

$$= \frac{24.2p_1p_2}{|CH(g)|} \cdot I =$$

$$= \frac{6|CQ4p_1p_2(g)|}{|C< x>(x^{p_1p_2})|} \cdot I = \frac{6|CQ4p_1p_2|}{|C< x>(x^{p_1p_2})|} \cdot I = 6\emptyset_j(I)$$

(b) if  $g = (x^{2p_1p_2}, 1)$  then  $\emptyset_{3I}(g) = (x^{2p_1p_2}, 1) =$

$$\frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{p_1p_2})|} \cdot I = 6\emptyset_j(x^{2p_1p_2})$$

(c) if  $g = (x^{p_1p_2}, 1)$  then  $\emptyset_{3I}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$

$$\emptyset(g^{-1}) = \frac{12.2p_1p_2}{|C< x>(x^{p_1p_2})|} (1+I) = \frac{3|CQ4p(q)|}{|C< x>(x^{p_1p_2})|} \cdot 2 = 6\emptyset_j(x^{p_1p_2})$$

Since  $H \cap cl(x^{p_1p_2}) = \{(I, 1), (x^{2p_1p_2}, 1), (x^{p_1p_2}, 1)\}$  otherwise = 0.

4:-  $H_{4I} = < (x^{4p_2}, 1) >; (a) \text{ if } g = (I, 1) \text{ then } \emptyset_{4I}(g) =$

$$\emptyset_{4I}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{4p_2})|} \cdot I = \frac{6|CQ4p(q)|}{|C< x>(x^{4p_2})|} \cdot I = 6\emptyset_j(I).$$

(b) if  $g = (x^{4p_2}, 1)$  then  $\emptyset_{4I}(g) =$

$$\frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{|C< x>(x^{4p_2})|} (1+I) = \frac{3|CQ4p(q)|}{|C< x>(x^{4p_2})|} \cdot 2 =$$

$$6\emptyset_j(x^{4p_2})$$

Since  $H \cap cl(x^{4p_2}) = \{(I, 1), (x^{4p_2}, 1)\}$ . Otherwise = 0 and  $\emptyset(g) = \emptyset(g^{-1}) = 1$ .

5:-  $H_{5I} = < (x^{2p_2}, 1) >; (a) \text{ if } g = (I, 1) \text{ then } \emptyset_{5I}(g) =$

$$\emptyset_{5I}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{2p_2})|} \cdot I = \frac{6|CQ4p(q)|}{|Cx>(x^{2p_2})|} \cdot I = 6\emptyset_j(I)$$

(b) if  $g = (x^{2p_1p_2}, 1)$  then  $\emptyset_{5I}(g) =$

$$\emptyset_{5I}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{2p_2})|} \cdot I = \frac{6|CQ4p(q)|}{|C< x>(x^{2p_2})|} \cdot I = 6\emptyset_j(x^{2p_1p_2}).$$

(c) if  $g = (x^{4p_2}, 1)$  then  $\emptyset_{5I}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$

$$\emptyset(g^{-1})) = \frac{12.2p_1p_2}{|C< x>(x^{2p_2})|} (1+I) = \frac{3|CQ4p(q)|}{|C< x>(x^{2p_2})|} \cdot 2 = 6\emptyset_j(x^{4p_2}).$$

$$(d) \quad \text{if } g=(x^{2p^2}, 1) \quad \text{then} \quad \emptyset_{51}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \\ (g^{-1})) = \frac{12.p_1p_2}{|C< x>(x^{2p^2})|}(1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^{2p^2})|}.2 = 6 \emptyset_j(x^{2p^2}).$$

Since  $H \cap cl(x^{2p^2}) = \{(I, 1), (x^{2p^1p^2}, 1), (x^{4p^2}, 1), (x^{2p^2}, 1)\}$ .  
Otherwise = 0. and  $\emptyset(g) = \emptyset(g^{-1})$ .

$$6: H_{61} = <(x^{p^2}, 1>; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \\ \emptyset_{61}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{p^2})|}.1 = \frac{6|CQ4p(q)|}{|C< x>(x^{p^2})|}.1 = 6 \emptyset_j(I)$$

$$(b) \quad \text{if } g=(x^{2p^1p^2}, 1) \quad \text{then} \\ \emptyset_{61}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{p^2})|}.1 = \frac{6|CQ4p(q)|}{|C< x>(x^{p^2})|}.1 = 6 \emptyset_j(x^{2p^1p^2}).$$

$$(c) \quad \text{if } g=(x^{p^1p^2}, 1) \quad \text{then} \quad \emptyset_{61}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \\ \frac{12.2p_1p_2}{|C< x>(x^{p^2})|}(1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^{p^2})|}.2 = 6 \emptyset_j(x^{p^1p^2})$$

$$(d) \quad \text{if } g=(x^{4p^2}, 1) \quad \text{then} \quad \emptyset_{61}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \\ \emptyset(g^{-1})) = \frac{12.2p_1p_2}{|C< x>(x^{p^2})|}(1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^{p^2})|}.2 = 6 \emptyset_j(x^{4p^2}).$$

$$(e) \quad \text{if } g=(x^{2p^2}, 1) \quad \text{then} \quad \emptyset_{61}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \\ \emptyset(g^{-1})) = \frac{12.2p_1p_2}{|C< x>(x^{p^2})|}(1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^{p^2})|}.2 = 6 \emptyset_j(x^{2p^2}).$$

$$(F) \quad \text{If } g=(x^{p^2}, 1) \quad \text{then} \quad \emptyset_{61}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \\ \emptyset(g^{-1})) = \frac{12.2p_1p_2}{|C< x>(x^{p^2})|}(1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^{p^2})|}.2 = 6 \emptyset_j(x^{p^2}).$$

Since

$$H \cap cl(x^{p^2}) = \{(I, 1), (x^{2p^1p^2}, 1), (x^{p^1p^2}, 1), (x^{4p^2}, 1), (x^{2p^2}, 1), (x^{p^2}, 1)\} \quad \emptyset(g) = \\ \emptyset(g^{-1}) = 1 \text{ otherwise } = 0.$$

$$7: H_{71} = <(x^{4p^1}, 1>; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \\ \emptyset_{71}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C< x>(x^{4p^1})|}.1 = \frac{6|CQ4p(q)|}{|C< x>(x^{4p^1})|}.1 = 6 \emptyset_j(I)$$

$$(b) \quad \text{if } g=(x^{4p^1}, 1) \quad \text{then} \quad \emptyset_{71}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \\ \emptyset(g^{-1})) = \frac{12.2p_1p_2}{|C< x>(x^{4p^1})|}(1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^{4p^1})|}.2 = 6 \emptyset_j(x^{4p^1}).$$

Since  $H \cap cl(x^{4p^1}) = \{(I, 1), (x^{4p^1}, 1)\}$ . Otherwise = 0 and  $\emptyset(g) = \emptyset(g^{-1}) = 1$ .

$$8:-H_{8I}=\langle(x^{2p_1}, 1)\rangle; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then } \emptyset_{8I}(g)=\frac{|CG(g)|}{|CH(g)|} \emptyset(g)=\frac{24.2p_1p_2}{|C<x>(x^{2p_1})|}, I=\frac{6|CQ4p(q)|}{|C<x>(x^{2p_1})|}, I=6 \emptyset_j(I)$$

b) if  $g=(x^{2p_1p_2}, 1)$  then  $\emptyset_{8I}(g)=\frac{|CG(g)|}{|CH(g)|} \emptyset(g)=\frac{24.2p_1p_2}{|C<x>(x^{2p_1})|}, I=\frac{6|CQ4p(q)|}{|C<x>(x^{2p_1})|}, I=6\emptyset_j(x^{2p_1p_2}).$

(c) if  $g=(x^{4p_1}, 1)$  then  $\emptyset_{8I}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{|C<x>(x^{2p_1})|}(1+1)=\frac{3|CQ4p(q)|}{|C<x>(x^{2p_1})|}, 2=6\emptyset_j(x^{4p_1}).$

(d) If  $g=(x^{2p_1}, 1)$  then  $\emptyset_{8I}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{|C<x>(x^{2p_1})|}(1+1)=\frac{3|CQ4p(q)|}{|C<x>(x^{2p_1})|}, 2=6\emptyset_j(x^{2p_1}).$

Since

$$H \cap cl(x^{2p_1})=\{(I, 1), (x^{2p_1p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1)\}.$$

Otherwise=0 and  $\emptyset(g) = \emptyset(g^{-1})$

$$9:-H_{9I}=\langle(x^{p_1}, 1)\rangle; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then } \emptyset_{9I}(g)=\frac{|CG(g)|}{|CH(g)|} \emptyset(g)=\frac{24.2p_1p_2}{|C<x>(x^{p_1})|}, I=\frac{6|CQ4p(q)|}{|C<x>(x^{p_1})|}, I=6 \emptyset_j(I)$$

b) if  $g=(x^{2p_1p_2}, 1)$  then  $\emptyset_{9I}(g)=\frac{|CG(g)|}{|CH(g)|} \emptyset(g)=\frac{24.2p_1p_2}{|C<x>(x^{p_2})|}, I=\frac{6|CQ4p(q)|}{|C<x>(x^{p_2})|}, I=6\emptyset_j(x^{2p_1p_2}).$

(c) if  $g=(x^{p_1p_2}, 1)$  then  $\emptyset_{9I}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{|C<x>(x^{p_1})|}(1+1)=\frac{3|CQ4p(q)|}{|C<x>(x^{p_1})|}, 2=6\emptyset_j(x^{p_1p_2})$

(d) if  $g=(x^{4p_1}, 1)$  then  $\emptyset_{9I}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{|C<x>(x^{p_1})|}(1+1)=\frac{3|CQ4p(q)|}{|C<x>(x^{p_1})|}, 2=6\emptyset_j(x^{4p_1})$

(e) If  $g=(x^{2p_1}, 1)$  then  $\emptyset_{9I}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{|C<x>(x^{p_1})|}(1+1)=\frac{3|CQ4p(q)|}{|C<x>(x^{p_1})|}, 2=6\emptyset_j(x^{2p_1})$

(f) If  $g=(x^{p_1}, 1)$  then  $\emptyset_{9I}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{|C<x>(x^{p_1})|}(1+1)=\frac{3|CQ4p(q)|}{|C<x>(x^{p_1})|}, 2=6\emptyset_j(x^{p_1}).$

*Since*

$$H \cap cl(x^{p^1}) = \{(I, 1), (x^{2p^1p^2}, 1), (x^{p^1p^2}, 1), (x^{4p^1}, 1), (x^{2p^1}, 1), (x^{p^1}, 1)\} \quad \emptyset(g) = \emptyset(g^{-1}) = 1 \text{ otherwise } = 0.$$

$$10:-H_{101} = \langle x^4, 1 \rangle \quad ; \quad (a \quad \quad ) \text{if} \quad g = (I, 1) \quad \text{then} \\ \emptyset_{101}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p1p2}{|C < x > (x^4)|} \cdot 1 = \frac{6|CQ4p(q)|}{|C < x > (x^4)|} \cdot 1 = 6 \emptyset_j(I).$$

$$(b) \quad \text{if } g = (x^{4p^2}, 1) \quad \text{then} \quad \emptyset_{101}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p1p2}{|C< x>(x^4)|} (1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^4)|} \cdot 2 = 6 \cdot \emptyset_j(x^{4p^2}).$$

$$(c) \quad if \quad g = (x^{4p_1}, 1) \quad then \quad \emptyset_{101}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \\ \frac{12 \cdot 2p_1 p_2}{|C < x > (x^4)|} (1+1) = \frac{3|CQ4p(q)|}{|C < x > (x^4)|}, 2 = 6. \emptyset_j(x^{4p_1}).$$

$$(d) \quad \text{if} \quad g = (x^4, 1) \quad ) \quad \text{then} \quad \emptyset_{101}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1}) = \\ \frac{12.2p_1p_2}{|C<_x>(x^4)|} (1+1) = \frac{3|CQ4p(q)|}{|C<_x>(x^4)|}, 2 = 6. \emptyset_j(x^4) .$$

Since  $H \cap cl(x^4) = \{(I, 1), (x^{4p_2}, 1), (x^{4p_1}, 1), (x^4, 1)\}$ .

Otherwise = 0. and  $\phi(g) = \phi(g^{-1})$

$$11:-H_{111}=\langle x^2, 1 \rangle \quad ; \quad (a \quad \quad ) if \quad \quad g=(I, 1) \quad \quad \text{then} \\ \emptyset_{111}(g)=\frac{|CG(g)|}{|CH(g)|} \emptyset(g)=\frac{24.2p1p2}{|C< x>(x^2)|}.1=\frac{6|CQ4p(q)|}{|C< x>(x^2)|}.1=6 \emptyset_j(I).$$

$$(b) \quad \text{if } g = (x^{2p_1 p_2}, 1) \quad \text{then} \\ \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{|C < x > (x^2)|} \cdot 1 = \frac{6|CQ4p(q)|}{|C < x > (x^2)|} \cdot 1 = 6 \cdot \emptyset_j(x^{2p_1 p_2}).$$

$$(c) \quad ) \quad if \quad g = (x^{4p^2}, 1) \quad then \quad \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p1p2}{|C_{<x>}(x^2)|} (1+1) = \frac{3|CQ4p(q)|}{|C_{<x>}(x^2)|} \cdot 2 = 6 \cdot \emptyset_j(x^{4p^2}).$$

$$(d) \quad \text{If } g = (x^{2p^2}, 1) \quad \text{then} \quad \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p^1p^2}{|C<x>(x^2)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x^2)|} \cdot 2 = 6 \emptyset_j(x^{2p^2}).$$

$$(e) \quad \text{if} \quad g = (x^{4p_1}, 1) \quad \text{then} \quad \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1}) = \\ \frac{12.2p_1p_2}{|C< x>(x^2)|} (1+1) = \frac{3|CQ4p(q)|}{|C< x>(x^2)|}. 2 = 6. \emptyset_j(x^{4p_1}).$$

$$(f) \quad \text{If } g = (x^{2p_1}, 1) \quad \text{then} \quad \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x^2)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x^2)|} \cdot 2 = 6 \emptyset_j(x^{2p_1})$$

$$(g) \quad \text{if } g = (x^4, 1) \quad ) \quad \text{then} \quad \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x^2)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x^2)|} \cdot 2 = 6 \emptyset_j(x^4)$$

$$(h) \quad \text{if } g = (x^2, 1) \quad \text{then} \quad \emptyset_{111}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x^2)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x^2)|} \cdot 2 = 6 \emptyset_j(x^2)$$

Since

$$H \cap cl(x^2) =$$

$$\{(I, 1), (x^{2p_1 p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (x^4, 1), (x^2, 1)\}$$

Otherwise = 0. and  $\emptyset(g) = \emptyset(g^{-1})$ .

$$12:- H_{121} = \langle (x, 1) \rangle ; \quad (a) \quad \text{if } g = (I, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{|C<x>(x)|} \cdot 1 = \frac{6|CQ4p(q)|}{|C<x>(x)|} \cdot 1 = 6 \emptyset_j(I).$$

$$b) \quad \text{if } g = (x^{2p_1 p_2}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{|C<x>(x)|} \cdot 1 = \frac{6|CQ4p(q)|}{|C<x>(x)|} \cdot 1 = 6 \emptyset_j(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (x^{p_1 p_2}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^{p_1 p_2})$$

$$(d) \quad \text{if } g = (x^{4p_2}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^{4p_2}).$$

$$(e) \quad \text{if } g = (x^{2p_2}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^{2p_2}).$$

$$(F) \quad \text{If } g = (x^{p_2}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^{p_2}).$$

$$(g) \quad \text{if } g = (x^{4p_1}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^{4p_1}).$$

$$(h) \quad \text{If } g = (x^{2p_1}, 1) \quad \text{then} \quad \emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12.2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 =$$

$$6 \emptyset_j(x^{2p_1})$$

$$(i) \quad \text{If } g = (x^{p_1}, 1) \quad \text{then}$$

$$\emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x^{p_1})|} \cdot 2 =$$

$$6 \emptyset_j(x^{p_1}).$$

$$(j) \quad \text{if } g = (x^4, 1) \quad \text{then}$$

$$\emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^4)$$

$$(k) \quad \text{if } g = (x^2, 1) \quad \text{then}$$

$$\emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x^2)$$

$$(l) \quad \text{if } g = (x, 1) \quad \text{then}$$

$$\emptyset_{121}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{|C<x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{|C<x>(x)|} \cdot 2 = 6 \emptyset_j(x).$$

Since

$$H \cap cl(x) =$$

$$\{(I, 1), (x^{2p_1 p_2}, 1), (x^{p_1 p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (x^{p_1}, 1), (x^4, 1), (x^2, 1), (x, 1)\}$$

Otherwise = 0. and  $\emptyset(g) = \emptyset(g^{-1})$ .

$$13:- H_{131} = \langle(y, 1) \rangle ; (a) \quad \text{if } g = (I, 1)$$

$$\text{then } \emptyset_{131}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{4} \cdot 1 = 6 \emptyset_{i+1}(I).$$

$$(b) \quad \text{if } g = (y^2, 1) = (x^{2p}, 1) \text{ then } \emptyset_{131}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{4} \cdot 1 = 6$$

$$\emptyset_{i+1}(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (y, 1) \quad \text{or} \quad (y^3, 1) \quad \text{then} \quad \emptyset_{131}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{24}{4} (1+1) = 6 \cdot 2 = 12.$$

$$\text{Since } H \cap cl(g) = \{(I, 1), (y^2, 1), (y, 1)\} \quad \text{and} \quad \emptyset(g) = \emptyset(g^{-1}) \text{ otherwise} = 0.$$

$$14:- H_{141} = \langle(xy, 1) \rangle ; (a) \quad \text{if } g = (I, 1) \quad \text{then}$$

$$\emptyset_{141}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{4} \cdot 1 =$$

$$6 \emptyset_{i+2}(I).$$

(b) if  $g=((xy)^2, 1)=(x^{2p}, 1)$  then  $\emptyset_{141(g)} = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{4} \cdot 1 = 6 \emptyset_{i+2}(x^{2p_1 p_2})$ .

(c) if  $g=(xy, 1)$  or  $((xy)^3, 1)$  then  $\emptyset_{141}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{24}{4} (1+1) = 6 \cdot 2 = 12$ .

Since  $H \cap cl(xy) = \{(I, 1), (xy^2, 1), (xy, 1)\}$  and  $\emptyset(g) = \emptyset(g^{-1}) = 1$

### Case (II):-

Consider the group  $G = (\mathbf{Q}_{4p_1 p_2} \times \mathbf{D}_3)$  and if  $H$  is a cyclic subgroup of  $(\mathbf{Q}_{4p_1 p_2} \times \{r\})$  then  $H = \langle(q, r) \rangle$  and  $\emptyset$  the principle character of  $H$  and  $\emptyset_j$  Artin's character of  $\mathbf{Q}_{4p_1 p_2}$ ,  $1 \leq j \leq i+2$ , by using theorem:-

$$\emptyset_j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} \sum_{i=1}^n \emptyset(hi) & \text{if } hi \in H \cap cl(g) \\ 0 & \text{if } H \cap cl(g) = \emptyset \end{cases}$$

$H = \langle(q, r) \rangle$

$$1: - H_{12} = \langle(I, r) \rangle \quad (a) \quad \text{if } g = (I, 1) \quad \text{then} \quad \emptyset_{12} = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{|C \times \langle(1) \rangle|} \cdot 1 = \frac{6|CQ4p(1)|}{3|C \times \langle(1) \rangle|} \cdot 1 = 2 \cdot \emptyset_j(1)$$

$$(b) \quad \text{If } g = (I, r) \text{ or } (I, r^2) \quad \text{then}$$

$$\emptyset_{12}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{|C \times \langle(1) \rangle|} (1+1) = \frac{3|CQ4p(1)|}{3|C \times \langle(1) \rangle|} \cdot 2 = 2\emptyset_j(q)$$

Since  $H \cap cl(g) = \{(I, 1), (I, r), (I, r^2)\}$  and  $\emptyset(g) = \emptyset(g^{-1}) = 1$  otherwise = 0.

$$2: - H_{22} = \langle(x^{2p_1 p_2}, r) \rangle \quad (a) \quad \text{if } g = (I, 1) \quad \text{then}$$

$$\emptyset_{22}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{|C \times \langle(x^{2p_1 p_2}) \rangle|} (1) = \frac{6|CQ4p(q)|}{3|C \times \langle(x^{2p_1 p_2}) \rangle|} (1) = 2 \cdot \emptyset_j(I)$$

$$(b) \quad \text{if } g = (x^{2p_1 p_2}, 1) \quad \text{then} \quad \emptyset_{22}(g)$$

$$= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{|C \times \langle(x^{2p_1 p_2}) \rangle|} (1) = \frac{6|CQ4p(x^{2p_1 p_2})|}{3|C \times \langle(x^{2p_1 p_2}) \rangle|} (1) = 2 \cdot \emptyset_j(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (I, r) \quad \text{then} \quad \emptyset_{22}(g)$$

$$= \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{3|C \times \langle(q) \rangle|} (1+1) = \frac{3|CQ4p(q)|}{3|C \times \langle(q) \rangle|} \cdot 2 = 2 \cdot \emptyset_j(q).$$

$$(d) \quad \text{if } g = (x^{2p_1 p_2}, r) \quad \text{then} \quad \emptyset_{22}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{3|C \times \langle(q) \rangle|} (1+1) = \frac{3|CQ4p(q)|}{3|C \times \langle(q) \rangle|} \cdot 2 = 2 \cdot \emptyset_j(q).$$

Since  $H \cap cl(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (I, r), (x^{2p_1 p_2}, r)\}$ . and  $\emptyset(g) = \emptyset(g^{-1}) = 1$ . otherwise = 0.

3:-  $H_{32} = \langle (x^{p_1 p_2}, r) \rangle$  (a) if  $g = (I, 1)$  then

$$\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{|C<\alpha>(x^{p_1 p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{3|C<\alpha>(x^{p_1 p_2})|} \cdot 1 = 2 \cdot \emptyset_j(I).$$

(b) if  $g = (x^{2p_1 p_2}, 1)$  then  $\emptyset_{32}(g)$

$$= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{|C<\alpha>(x^{p_1 p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{3|C<\alpha>(x^{p_1 p_2})|} \cdot 1 = 2 \cdot \emptyset_j(x^{2p_1 p_2}).$$

(c) if  $g = (x^{p_1 p_2}, 1)$  then  $\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) =$

$$\frac{12.2p_1 p_2}{3|C<\alpha>(x^{p_1 p_2})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x^{p_1 p_2})|} \cdot 2 = 2 \cdot \emptyset_j(x^{p_1 p_2}).$$

(d) if  $g = (I, r)$  then  $\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) +$

$$\emptyset(g^{-1}) = \frac{12.2p_1 p_2}{3|C<\alpha>(x^{p_1 p_2})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x^{p_1 p_2})|} \cdot 2 = 2 \cdot \emptyset_j(q).$$

(e) if  $g = (x^{2p_1 p_2}, r)$  then  $\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) =$

$$\frac{12.2p_1 p_2}{3|C<\alpha>(x^{p_1 p_2})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x^{p_1 p_2})|} \cdot 2 = 2 \cdot \emptyset_j(q).$$

(f) if  $g = (x^{p_1 p_2}, r)$  then  $\emptyset_{32}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) =$

$$\frac{6.2p_1 p_2}{3|C<\alpha>(x^{p_1 p_2})|} (1+1+1+1) = \frac{6.4(2p_1 p_2)}{3|C<\alpha>(x^{p_1 p_2})|} = \frac{6|CQ4p(q)|}{3|C<\alpha>(x^{p_1 p_2})|} =$$

$2 \cdot \emptyset_j(q)$ . since  $H \cap$

$cl(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (x^{p_1 p_2}, 1), (I, r), (x^{2p_1 p_2}, r), (x^{p_1 p_2}, r)\}$ . and

$\emptyset(g) = \emptyset(g^{-1}) = 1$ . otherwise = 0.

4:-  $H_{42} = \langle (x^{4p_2}, r) \rangle$  (a) if  $g = (I, 1)$  then  $\emptyset_{42}(g) =$

$$\frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{|C<\alpha>(x^{4p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{3|C<\alpha>(x^{4p_2})|} \cdot 1 = 2 \cdot \emptyset_j(I).$$

(b) if  $g = (x^{4p_2}, 1)$  then  $\emptyset_{42}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) +$

$$\emptyset(g^{-1}) = \frac{12.2p_1 p_2}{3|C<\alpha>(x^{4p_2})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x^{4p_2})|} \cdot 2 = 2 \cdot \emptyset_j(x^{4p_2}).$$

(c) if  $g = (I, r)$  then  $\emptyset_{42}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) =$

$$\frac{12.2p_1 p_2}{3|C<\alpha>(x^{4p_2})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x^{4p_2})|} \cdot 2 = 2 \cdot \emptyset_j(q).$$

(d) if  $g = (x^{4p_2}, r)$  then  $\emptyset_{42}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) =$

$$\frac{6.2p_1 p_2}{3|C<\alpha>(x^{4p_2})|} (1+1+1+1) = \frac{6.4(2p)}{3|C<\alpha>(x^{4p_2})|} = \frac{6|CQ4p(q)|}{3|C<\alpha>(x^{4p_2})|} = 2 \cdot \emptyset_j(q).$$

since  $H \cap cl(g) = \{(I, 1), (x^{4p_2}, 1), (I, r), (x^{4p_2}, r)\}$  and  
 $\emptyset(g) = \emptyset((g^{-1})) = 1$ . otherwise = 0.

5:-  $H_{52} = \langle(x^{2p_2}, r) \rangle$  (a) if  $g = (I, 1)$  then

$$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<\mathbf{x}>(x^{2p_2})|} \cdot I = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} \cdot I = 2 \cdot \emptyset_j(I).$$

(b) if  $g = (x^{2p_1p_2}, 1)$  then

$$\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<\mathbf{x}>(x^{2p_2})|} \cdot I = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} \cdot I = 2 \cdot \emptyset_j(x^{2p_1p_2}).$$

(c) if  $g = (x^{4p_2}, 1)$  then  $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\mathbf{x}>(x^{2p_2})|} (I+I) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} \cdot 2 = 2 \cdot \emptyset_j(x^{4p_2}).$

(d) if  $g = (x^{2p_2}, 1)$  then  $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\mathbf{x}>(x^{2p_2})|} (I+I) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} \cdot 2 = 2 \cdot \emptyset_j(x^{2p_2}).$

(e) if  $g = (I, r)$  then  $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\mathbf{x}>(x^{2p_2})|} (I+I) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} \cdot 2 = 2 \cdot \emptyset_j(q).$

(f) if  $g = (x^{2p_1p_2}, r)$  the  $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\mathbf{x}>(x^{2p_2})|} (I+I) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} \cdot 2 = 2 \cdot \emptyset_j(q).$

(g) if  $g = (x^{4p_2}, r)$  then  $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<\mathbf{x}>(x^{2p_2})|} (I+I+I+I) = \frac{6.4(2p_1p_2)}{3|C<\mathbf{x}>(x^{2p_2})|} = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} = 2 \cdot \emptyset_j(q).$

(h) if  $g = (x^{2p_2}, r)$  then  $\emptyset_{52}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<\mathbf{x}>(x^{2p_2})|} (I+I+I+I) = \frac{6.4(2p_1p_2)}{3|C<\mathbf{x}>(x^{2p_2})|} = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_2})|} = 2 \cdot \emptyset_j(q).$  since  $H \cap$

$cl(g) =$

$$\{(I, 1), (x^{2p_1p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (I, r), (x^{2p_1p_2}, r), (x^{4p_2}, r), (x^{2p_2}, r)\}$$

and  $\emptyset(g) = \emptyset((g^{-1})) = 1$ . otherwise = 0.

6:-  $H_{62} = \langle(x^{p_2}, r) \rangle$  (a) if  $g = (I, 1)$  then  $\emptyset_{62}(g) =$

$$\frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<\mathbf{x}>(x^{p_2})|} \cdot I = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{p_2})|} = 2 \cdot \emptyset_j(I)$$

(b) if  $g = (x^{2p_1p_2}, 1)$  then

$$\emptyset_{62}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<\mathbf{x}>(x^{p_2})|} \cdot I = \frac{6|CQ4p|}{3|C<\mathbf{x}>(x^{p_2})|} = 2 \cdot \emptyset_j(x^{2p_1p_2}).$$

- (c) if  $g=(x^{p_1p_2}, 1)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{3|C<x>(x^{p_2})|}.2=2.\emptyset_j(x^{p_1p_2})$
- (d) if  $g=(x^{4p_2}, 1)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{3|C<x>(x^{p_2})|}.2=2.\emptyset_j(x^{4p_2}).$
- (e) if  $g=(x^{2p_2}, 1)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{3|C<x>(x^{p_2})|}.2=2\emptyset_j(x^{2p_2}).$
- (f) if  $g=(x^{p_2}, 1)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{3|C<x>(x^{p_2})|}.2=2.\emptyset_j(x^{p_2}).$
- (g) if  $g=(I, r)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{3|C<x>(x^{p_2})|}.2=2\emptyset_j(q).$
- (h) if  $g=(x^{2p_1p_2}, r)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{12.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{3|C<x>(x^{p_2})|}.2=2.\emptyset_j(q).$
- (i) if  $g=(x^{p_1p_2}, r)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x^{p_2})|}=\frac{6|CQ4p(q)|}{3|C<x>(x^{p_2})|}=2.\emptyset_j(q).$
- (j) if  $g=(x^{4p_2}, r)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x^{p_2})|}=\frac{6|CQ4p(q)|}{3|C<x>(x^{p_2})|}=2.\emptyset_j(q).$
- (k) if  $g=(x^{2p_2}, r)$  then  $\emptyset_{62}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x^{p_2})|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x^{p_2})|}=\frac{6|CQ4p(q)|}{3|C<x>(x^{p_2})|}=2.\emptyset_j(q).$

$$(l) \quad \text{if } g=(x^{p^2}, r) \quad \text{then} \quad \emptyset_{62}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^{p^2})|}(1+1+1+1) \frac{6.4(2p_1p_2)}{3|C<x>(x^{p^2})|} = \frac{6|CQ4p(q)|}{3|C<x>(x^{p^2})|} = 2. \emptyset_j(q).$$

Since

$$H \cap cl(g) = \{(I, 1), (x^{2p_1p_2}, 1), (x^{p_1p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (I, r), (x^{2p_1p_2}, r), (x^{p_1p_2}, r), (x^{4p_2}, r), (x^{2p_2}, r), \}$$

And  $\emptyset(g) = \emptyset(g^{-1}) = 1$ . otherwise = 0.

$$7:- H_{72} = \langle(x^{4p_1}, r) \rangle, \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \\ \emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<x>(x^{4p_1})|} \cdot 1 = \frac{6|CQ4p(q)|}{3|C<x>(x^{4p_1})|} = 2 \cdot \emptyset_j(I) \\ (b) \quad \text{if } g=(x^{4p_1}, 1) \quad \text{then} \\ \emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^{4p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{4p_1})|} \cdot 2 = 2 \cdot \emptyset_j(x^{4p_1}).$$

$$(c) \quad \text{if } g=(I, r) \quad \text{then} \\ \emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^{4p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{4p_1})|} \cdot 2 = 2 \cdot \emptyset_j(q).$$

$$(d) \quad \text{if } g=(x^{4p_1}, r) \quad \text{then} \quad \emptyset_{72}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^{4p_1})|} (1+1+1+1) \frac{6.4(2p_1p_2)}{3|C<x>(x^{4p_1})|} = \frac{6|CQ4p(q)|}{3|C<x>(x^{4p_1})|} \\ \text{since} \quad H \cap cl(g) = \{(I, 1), (x^{4p_1}, 1), (I, r), (x^{4p_1}, r)\} \text{and} \\ \emptyset(g) = \emptyset(g^{-1}) = 1 \text{. otherwise = 0.}$$

$$8:- H_{82} = \langle(x^{2p_1}, r) \rangle; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \quad \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<x>(x^{2p_1})|} \cdot 1 = \frac{6|CQ4p(q)|}{3|C<x>(x^{2p_1})|} = 2 \cdot \emptyset_j(I) \\ (b) \quad \text{if } g=(x^{2p_1p_2}, 1) \quad \text{then} \\ \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<x>(x^{2p_1})|} \cdot 1 = \frac{6|CQ4p(q)|}{3|C<x>(x^{2p_1})|} = 2 \cdot \emptyset_j(x^{2p_1p_2}). \\ (c) \quad \text{if } g=(x^{4p_1}, 1) \quad \text{then} \\ \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^{2p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{2p_1})|} \cdot 2 = 2 \cdot \emptyset_j(x^{4p_1})$$

$$(d) \quad \text{if } g=(x^{2p_1}, 1) \quad \text{then} \quad \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^{2p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{2p_1})|} \cdot 2 = 2 \cdot \emptyset_j(x^{2p_1}).$$

$$(e) \quad \text{if } g = (I, r) \quad \text{then} \quad \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<\mathbf{x}>(x^{2p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_1})|}, 2 =$$

$$2 \cdot \emptyset_j(q).$$

$$(f) \quad \text{if } g = (x^{2p_1 p_2}, r) \quad \text{then}$$

$$\emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<\mathbf{x}>(x^{2p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_1})|}, 2 =$$

$$2 \cdot \emptyset_j(q).$$

$$(g) \quad \text{if } g = (x^{4p_1}, r) \quad \text{then} \quad \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6.2p_1 p_2}{3|C<\mathbf{x}>(x^{2p_1})|} (1+1+1+1) = \frac{6.4(2p_1 p_2)}{3|C<\mathbf{x}>(x^{2p_1})|} = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_1})|}$$

$$.2 = 2 \cdot \emptyset_j(q)$$

$$(h) \quad \text{if } g = (x^{2p_1}, r) \quad \text{then} \quad \emptyset_{82}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6.2p_1 p_2}{3|C<\mathbf{x}>(x^{2p_1})|} (1+1+1+1) = \frac{6.4(2p_1 p_2)}{3|C<\mathbf{x}>(x^{2p_1})|} = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{2p_1})|}$$

$$.2 = 2 \cdot \emptyset_j(q)$$

Since

$$H \cap cl(g) =$$

$$\{(I, 1), (x^{2p_1 p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (I, r), (x^{2p_1 p_2}, r), (x^{4p_1}, r), (x^{2p_1}, r)\}$$

and

$$\emptyset(g) = \emptyset(g^{-1}) = 1. otherwise = 0.$$

$$9:- H_{92} = <(x^{p_1}, r)> \quad ; \quad (a) \quad \text{if } g = (I, 1) \quad \text{then}$$

$$\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{|C<\mathbf{x}>(x^{p_1})|}, 1 = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{p_1})|} = 2 \cdot \emptyset_j(I)$$

$$(b) \quad \text{if } g = (x^{2p_1 p_2}, 1) \quad \text{then}$$

$$\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{|C<\mathbf{x}>(x^{p_1})|}, 1 = \frac{6|CQ4p(q)|}{3|C<\mathbf{x}>(x^{p_1})|} = 2 \cdot \emptyset_j(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (x^{p_1 p_2}, 1) \quad \text{then} \quad \emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<\mathbf{x}>(x^{p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{p_1})|}, 2 = 2 \cdot \emptyset_j(x^{p_1 p_2})$$

$$2 \cdot \emptyset_j(x^{4p_1}) \quad (d) \quad \text{if } g = (x^{4p_1}, 1) \quad \text{then} \quad \emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<\mathbf{x}>(x^{p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{p_1})|}, 2 =$$

$$(e) \quad \text{if } g = (x^{2p_1}, 1) \quad \text{then} \quad \emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<\mathbf{x}>(x^{p_1})|} (1+1) = \frac{3|CQ4p(q)|}{3|C<\mathbf{x}>(x^{p_1})|}, 2 = 2 \cdot \emptyset_j(x^{2p_1}).$$

- (f) if  $g = (x^{p^1}, 1)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{p^1})|} \cdot 2 = 2 \cdot \emptyset_j(x^{p^1}).$
- (g) if  $g = (I, r)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{p^1})|} \cdot 2 = 2 \cdot \emptyset_j(q).$
- (h) if  $g = (x^{2p^1p^2}, r)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^{p^1})|} \cdot 2 = 2 \cdot \emptyset_j(q)$
- (i) if  $g = (x^{p^1p^2}, r)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1+1+1) \frac{6.4(2p^1p^2)}{3|C<x>(x^{p^1})|} = \frac{6|CQ4p(q)|}{3|C<x>(x^{p^1})|} = 2 \cdot \emptyset_j(q).$
- (j) if  $g = (x^{4p^1}, r)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1+1+1) \frac{6.4(2p^1p^2)}{3|C<x>(x^{p^1})|} = \frac{6|CQ4p(q)|}{3|C<x>(x^{p^1})|} \cdot 2 = 2 \cdot \emptyset_j(q)$
- (k) if  $g = (x^{2p^1}, r)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1+1+1) \frac{6.4(2p^1p^2)}{3|C<x>(x^{p^1})|} = \frac{6|CQ4p(q)|}{3|C<x>(x^{p^1})|} \cdot 2 = 2 \cdot \emptyset_j(q)$
- (l) if  $g = (x^{p^1}, r)$  then  $\emptyset_{92}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1 p_2}{3|C<x>(x^{p^1})|}(1+1+1+1) \frac{6.4(2p^1p^2)}{3|C<x>(x^{p^1})|} = \frac{6|CQ4p(q)|}{3|C<x>(x^{p^1})|} \cdot 2 = 2 \cdot \emptyset_j(q)$

Since  $H \cap cl(g) =$

$$\{(I, 1), (x^{2p^1p^2}, 1), (x^{p^1p^2}, 1), (x^{4p^1}, 1), (x^{2p^1}, 1), (x^{p^1}, 1), (I, r), (x^{2p^1p^2}, r), (x^{p^1p^2}, r), (x^{4p^1}, r), (x^{2p^1}, r), (x^{p^1}, r)\}$$

And  $\emptyset(g) = \emptyset(g^{-1}) = 1.0$  otherwise = 0.

10:-  $H_{101} = \langle (x^4, r) \rangle$ ; (a) if  $g = (I, 1)$  then  $\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{3|C<x>(x^4)|} \cdot I = \frac{6|CQ4p(q)|}{3|C<x>(x^4)|} = 2 \cdot \emptyset_j(I)$

$$(b) \quad \text{if } g=(x^{4p^2}, 1) \quad \text{then}$$

$$\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^4)|}(1+1) =$$

$$\frac{3|CQ4p(q)|}{3|C<x>(x^4)|}.2 = 2. \emptyset_j(x^{4p^2}) \quad (c) \quad \text{if } g=(x^{4p^1}, 1) \quad \text{then}$$

$$\emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^4)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^4)|}.2 =$$

$$2. \emptyset_j(x^{4p^1})$$

$$(d) \text{ if } g=(x^4, 1) \text{ then } \emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^4)|}(1+1) =$$

$$\frac{3|CQ4p(q)|}{3|C<x>(x^4)|}.2 = 2. \emptyset_j(x^4)$$

$$(e) \quad \text{if } g=(I, r) \quad \text{then} \quad \emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^4)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^4)|}.2 =$$

$$2 \emptyset_j(q).$$

$$(f) \quad \text{if } g=(x^{4p^2}, r) \quad \text{then} \quad \emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^4)|}(1+1+1+1) \frac{6.4(2p_1p_2)}{3|C<x>(x^4)|} = \frac{6|CQ4p(q)|}{3|C<x>(x^4)|}$$

$$.2 = 2 \emptyset_j(q).$$

$$(g) \quad \text{if } g=(x^{4p^1}, r) \quad \text{then} \quad \emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^4)|}(1+1+1+1) \frac{6.4(2p_1p_2)}{3|C<x>(x^4)|} = \frac{6|CQ4p(q)|}{3|C<x>(x^4)|}$$

$$.2 = 2 \emptyset_j(q).$$

$$(h) \quad \text{if } g=(x^4, r) \quad \text{then} \quad \emptyset_{102}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^4)|}(1+1+1+1) \frac{6.4(2p_1p_2)}{3|C<x>(x^4)|} = \frac{6|CQ4p(q)|}{3|C<x>(x^4)|}$$

$$.2 = 2 \emptyset_j(q).$$

Since

$$H \cap cl(g) =$$

$$\{(I, 1), (x^{4p^2}, 1), (x^{4p^1}, 1), (x^4, 1), (I, r), (x^{4p^2}, r), (x^{4p^1}, r), (x^4, r)\}$$

And  $\emptyset(g) = \emptyset(g^{-1}) = 1.0 \text{ otherwise} = 0.$

$$11:- H_{112} = \langle (x^2, r) \rangle; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then}$$

$$\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<x>(x^2)|}.1 = \frac{6|CQ4p(q)|}{3|C<x>(x^2)|} = 2 \emptyset_j(I)$$

$$(b) \quad \text{if } g=(x^{2p_1p_2}, 1) \text{ then}$$

$$\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{|C<x>(x^2)|}.1 = \frac{6|CQ4p(q)|}{3|C<x>(x^2)|} = 2 \emptyset_j(x^{2p_1p_2}).$$

- (c) if  $g=(x^{4p^2}, 1)$  then  
 $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) =$   
 $\frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(x^{4p^2})$
- (d) if  $g=(x^{2p^2}, 1)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$   
 $\emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(x^{2p^2})$
- (e) if  $g=(x^{4p^1}, 1)$  then  
 $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 =$   
 $2. \emptyset_j(x^{4p^1})$
- (f) if  $g=(x^{2p^1}, 1)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$   
 $\emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(x^{2p^1})$
- (g) if  $g=(x^4, 1)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(x^4)$
- (h) if  $g=(x^2, 1)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(x^2)$
- (i) if  $g=(I, r)$  then  
 $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(q).$
- (k) if  $g=(x^{2p_1p_2}, r)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$   
 $\emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<x>(x^2)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<x>(x^2)|}.2 = 2. \emptyset_j(q)$
- (l) if  $g=(x^{4p^2}, r)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$   
 $\emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^4)|}(1+1+1+1) = \frac{6.4(2p_1p_2)}{3|C<x>(x^2)|} = \frac{6|CQ4p(q)|}{3|C<x>(x^2)|}$   
 $.2 = 2. \emptyset_j(q).$
- (m) if  $g=(x^{2p^2}, r)$  then  $\emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$   
 $\emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<x>(x^2)|}(1+1+1+1) = \frac{6.4(2p_1p_2)}{3|C<x>(x^2)|} = \frac{6|CQ4p(q)|}{3|C<x>(x^2)|}$   
 $.2 = 2. \emptyset_j(q).$

$$(n) \quad \text{if } g=(x^{4p_1}, r) \quad \text{then } \emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6 \cdot 2p_1p_2}{3|C< x>(x^2)|} (1+1+1+1) \frac{6 \cdot 4(2p_1p_2)}{3|C< x>(x^2)|} = \frac{6|CQ4p(q)|}{3|C< x>(x^2)|}$$

$$.2 = 2 \cdot \emptyset_j(q).$$

$$(o) \quad \text{if } g=(x^{2p_1}, r) \quad \text{then } \emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6 \cdot 2p_1p_2}{3|C< x>(x^2)|} (1+1+1+1) \frac{6 \cdot 4(2p_1p_2)}{3|C< x>(x^2)|} = \frac{6|CQ4p(q)|}{3|C< x>(x^2)|}$$

$$.2 = 2 \cdot \emptyset_j(q).$$

$$(p) \quad \text{if } g=(x^4, r) \quad \text{then } \emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6 \cdot 2p_1p_2}{3|C< x>(x^2)|} (1+1+1+1) \frac{6 \cdot 4(2p_1p_2)}{3|C< x>(x^2)|} = \frac{6|CQ4p(q)|}{3|C< x>(x^2)|}$$

$$.2 = 2 \cdot \emptyset_j(q).$$

$$(q) \quad \text{if } g=(x^2, r) \quad \text{then } \emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{6 \cdot 2p_1p_2}{3|C< x>(x^2)|} (1+1+1+1) \frac{6 \cdot 4(2p_1p_2)}{3|C< x>(x^2)|} = \frac{6|CQ4p(q)|}{3|C< x>(x^2)|}$$

$$.2 = 2 \cdot \emptyset_j(q).$$

Since

$$H \cap cl(g) = \{(I, 1), (x^{2p_1p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{4p_2}, 1), (x^{2p_1}, 1), (x^4, 1), (x^2, 1), (I, r), (x^{2p_1p_2}, r),$$

$$(x^{4p_2}, r), (x^{2p_2}, r), (x^{4p_1}, r), (x^{2p_1}, r), (x^4, r), (x^2, r)\} \quad \text{and } \emptyset(g) = \emptyset(g^{-1}) = 1. \text{ otherwise} = 0.$$

$$12:- H_{122} = <(x, r)>; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \quad \emptyset_{122}(g) =$$

$$\frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1p_2}{|C< x>(x)|} \cdot 1 \frac{6|CQ4p(q)|}{3|C< x>(x)|} = 2 \cdot \emptyset_j(I)$$

$$\text{if } g=(x^{2p_1p_2}, 1) \text{ then } \emptyset_{112}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1p_2}{|C< x>(x)|} \cdot 1 \frac{6|CQ4p(q)|}{3|C< x>(x)|} = 2 \emptyset_j(x^{2p_1p_2}).$$

$$c) \quad \text{if } g=(x^{p_1p_2}, 1) \quad \text{then} \quad \emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1})) = \frac{12 \cdot 2p_1p_2}{3|C< x>(x)|} (1+1) = \frac{3|CQ4p(q)|}{3|C< x>(x)|}. 2 = 2 \cdot \emptyset_j(x^{p_1p_2})$$

$$(d) \quad \text{if } g=(x^{4p_2}, 1) \quad \text{then} \quad \emptyset_{122}(g) =$$

$$\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1p_2}{3|C< x>(x)|} (1+1) =$$

$$\frac{3|CQ4p(q)|}{3|C< x>(x)|}. 2 = 2 \cdot \emptyset_j(x^{4p_2})$$

if  $g=(x^{p^2}, 1)$  then

$$\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x^{p^2}). \quad (e)$$

(f) if  $g=(x^{4p_1}, 1)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x^{4p_1})$

(g) if  $g=(x^{2p_1}, 1)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x^{2p_1})$

(i) if  $g=(x^{p_1}, 1)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x^{p_1})$

(j) if  $g=(x^4, 1)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x^4)$

(k) if  $g=(x^2, 1)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x^2)$

(l) if  $g=(x, 1)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(x)$

(m) if  $g=(I, r)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(q)$ .

(n) if  $g=(x^{2p_1p_2}, r)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{3|C<\alpha>(x)|}(1+1) = \frac{3|CQ4p(q)|}{3|C<\alpha>(x)|}.2 = 2 \cdot \emptyset_j(q)$

(o) if  $g=(x^{p_1p_2}, r)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<\alpha>(x)|}(1+1+1+1) = \frac{6.4(2p_1p_2)}{3|C<\alpha>(x)|} = \frac{6|CQ4p(q)|}{3|C<\alpha>(x)|}$   
 $.2 = 2 \cdot \emptyset_j(q)$

(p) if  $g=(x^{4p_2}, r)$  then  $\emptyset_{122}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{6.2p_1p_2}{3|C<\alpha>(x)|}(1+1+1+1) = \frac{6.4(2p_1p_2)}{3|C<\alpha>(x)|} = \frac{6|CQ4p(q)|}{3|C<\alpha>(x)|}$

.2= 2 . $\emptyset_j(q)$

$$(q) \quad \text{if } g=(x^{2p^2}, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|}$$

.2= 2 . $\emptyset_j(q)$

$$(r) \text{if } g=(x^{p^2}, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|}$$

.2= 2 . $\emptyset_j(q)$

$$(s) \quad \text{if } g=(x^{4p^1}, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|} 2= 2 .\emptyset_j(q)$$

$$(t) \quad \text{if } g=(x^{2p^1}, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|}$$

.2= 2 . $\emptyset_j(q)$

$$(u) \quad \text{if } g=(x^{p^1}, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|}$$

.2= 2 . $\emptyset_j(q)$

$$(v) \quad \text{if } g=(x^4, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|}$$

.2= 2 . $\emptyset_j(q)$

$$(w) \quad \text{if } g=(x^2, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|}$$

.2= 2 . $\emptyset_j(q)$

$$(x) \quad \text{if } g=(x, r) \quad \text{then } \emptyset_{122}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) +$$

$$\emptyset(g^{-1}))=\frac{6.2p_1p_2}{3|C<x>(x)|}(1+1+1+1)\frac{6.4(2p_1p_2)}{3|C<x>(x)|}\frac{-6|CQ4p(q)|}{3|C<x>(x)|} 2= 2 .\emptyset_j(q)$$

Since

$$H \cap cl(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (x^{p_1 p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (x^{p_1}, 1), (x^4, 1), (x^2, 1), (x, 1), (I, r), (x^{2p_1 p_2}, r),$$

$$(x^{p_1 p_2}, r), (x^{4p_2}, r), (x^{2p_2}, r), (x^{p_2}, r), (x^{4p_1}, r), (x^{2p_1}, r), (x^{p_1}, r), (x^4, r), (x^2, r), (x, r)\}$$

And  $\emptyset(g) = \emptyset(g^{-1}) = 1$ . otherwise = 0.

$$13:- H_{132} = \langle(y, r) \rangle \quad (a) \quad \text{if } g = (I, 1) \quad \text{then } \emptyset_{132}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{12} \cdot 1 = 2 \cdot (2p) = 2 \cdot \emptyset_{i+1}(I).$$

$$(b) \quad \text{if } g = (y^2, 1) \quad \text{or } (x^{2p_1 p_2}, 1) \quad \text{then } \emptyset_{132}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{12} \cdot 1 = 2 \cdot (2p) = 2 \cdot \emptyset_{i+1}(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (y, 1) \quad \text{or } (y^3, 1) \quad \text{then } \emptyset_{132}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{24}{12} (1+1) = 4.$$

$$(d) \quad \text{if } g = (I, r) \quad \text{then } \emptyset_{132}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{12} (1+1) = 2 \cdot (2p) = 2 \cdot \emptyset_{i+1}(q).$$

$$(e) \quad \text{if } g = (y^2, r) \quad \text{or } (x^{2p_1 p_2}, r) \quad \text{then } \emptyset_{132}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{12} (1+1) = 2 \cdot (2p) = 2 \cdot \emptyset_{i+1}(q).$$

$$(f) \quad \text{if } g = ((y, r) \quad \text{or } (y^3, r)) \quad \text{then } \emptyset_{132}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12}{12} (1+1+1+1) = 4.$$

Since  $H \cap cl(g) = \{(I, 1), (y^2, 1), (y, 1), (I, r), (y^2, r), (y, r)\}$  And  $\emptyset(g) = \emptyset(g^{-1}) = 1$ . otherwise = 0.

$$14:- H_{142} = \langle(xy, r) \rangle \quad (a) \quad \text{if } g = (I, 1) \quad \text{then } \emptyset_{142}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{12} \cdot 1 = 2 \cdot (2p) = 2 \cdot \emptyset_{i+2}(I).$$

$$(b) \quad \text{if } g = ((xy)^2, 1) = (x^{2p_1 p_2}, 1) \quad \text{then } \emptyset_{142}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{12} \cdot 1 = 2 \cdot (2p) = 2 \cdot \emptyset_{i+2}(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (xy, 1) \quad \text{or } ((xy)^3, 1) \quad \text{then } \emptyset_{142}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{24}{12} (1+1) = 4.$$

$$(d) \quad \text{if } g = (I, r) \quad \text{then } \emptyset_{142}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{12} (1+1) = 2 \cdot (2p) = 2 \cdot \emptyset_{i+2}(q).$$

$$(e) \text{ if } g=((xy)^2, r) = (x^{2p_1 p_2}, r) \text{ then } \emptyset_{142}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) \\ = \frac{12 \cdot 2p_1 p_2}{12} (1 + 1) = 2 \cdot (2p) = 2 \cdot \emptyset_{i+2}(q).$$

$$(f) \text{ if } g=(xy, r) \text{ or } ((xy)^3, r) \text{ then } \emptyset_{142}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) \\ = \frac{12}{12} (1+1+1+1) = 4.$$

Since

$$H \cap cl(g) = \{(I, 1), ((xy)^2, 1)(xy, 1), (I, r), ((xy)^2, r), (xy, r)\} \text{ And } \emptyset(g) = \emptyset((g^{-1})) = 1. \text{ otherwise } = 0$$

### Case (III):-

Consider the group  $G = (\mathbf{Q}_{4p_1 p_2} \times \mathbf{D}_3)$  and if  $H$  is a cyclic subgroup of  $(\mathbf{Q}_{4p_1 p_2} \times \{s\})$  then  $H = \langle(q, s) \rangle$  and  $\emptyset$  the principle character of  $H$  and  $\emptyset_j$  Artin's character of  $\mathbf{Q}_{4p_1 p_2}$ ,  $1 \leq j \leq i+2$ , by using theorem:-

$$\emptyset_j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} \sum_{i=1}^n \emptyset(hi) & \text{if } hi \in H \cap cl(g) \\ 0 & \text{if } H \cap cl(g) = \emptyset \end{cases}$$

$H = \langle(q, s) \rangle$

$$1: H_{13} = \langle(I, s) \rangle \quad (a) \quad \text{if } g = (I, 1) \quad \text{then } \emptyset_{13}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \\ \frac{24 \cdot 2p_1 p_2}{2|C<x>(I)|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C<x>(I)|} \cdot 1 = 3 \cdot \emptyset_j(I).$$

$$(b) \quad \text{if } g = (I, s) \quad \text{then } \emptyset_{13}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8 \cdot 2p_1 p_2}{2|C<x>(I)|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C<x>(I)|} \cdot 1 = \emptyset_j(q).$$

Since  $H \cap cl(g) = \{(I, 1), (I, s)\}$  otherwise  $= 0$

$$2: H_{23} = \langle(x^{2p_1 p_2}, s) \rangle \quad (a) \quad \text{if } g = (I, 1) \quad \text{then } \emptyset_{23}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{2|C<x>(x^{2p_1 p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{2p_1 p_2})|} \cdot 1 \\ = 3 \cdot \emptyset_j(I).$$

$$(b) \quad \text{if } g = (x^{2p_1 p_2}, 1) \quad \text{then } \emptyset_{23}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{2|C<x>(x^{2p_1 p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{2p_1 p_2})|} \cdot 1 = 3 \cdot \emptyset_j(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g = (I, s) \quad \text{then } \emptyset_{23}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8 \cdot 2p_1 p_2}{2|C<x>(x^{2p_1 p_2})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{2p_1 p_2})|} \cdot 1 = \emptyset_j(q).$$

$$(d) \quad \text{if } g=(x^{2p_1 p_2}, s) \quad \text{then} \quad \emptyset_{23}(g)= \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1 p_2}{2|C< x>(x^{2p_1 p_2})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C< x>(x^{2p_1 p_2})|} \cdot 1 = \emptyset_j(q).$$

Since  $H \cap \text{cl}(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (I, s), (x^{2p_1 p_2}, s)\}$  And  $\emptyset(g) = \emptyset((g^{-1})) = 1$  otherwise  $= 0$ .

$$3:- H_{33} = \langle (x^{p_1 p_2}, s) \rangle \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \quad \emptyset_{33}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \\ \frac{24.2p_1 p_2}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = 3 \cdot \emptyset_j(I).$$

$$(b) \quad \text{if } g=(x^{2p_1 p_2}, 1) \quad \text{then} \quad \emptyset_{33}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1 p_2}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = 3 \cdot \emptyset_j(x^{2p_1 p_2}).$$

$$(c) \quad \text{if } g=(x^{p_1 p_2}, 1) \quad \text{then} \quad \emptyset_{33}(g) = \\ \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) +$$

$$\emptyset((g^{-1})) = \frac{12.2p_1 p_2}{2|C< x>(x^{p_1 p_2})|} (1+1) = \frac{3|CQ4p(q)|}{2|C< x>(x^{p_1 p_2})|} \cdot 2 = 3 \cdot \emptyset_j(x^{p_1 p_2}).$$

$$(d) \quad \text{if } g=(I, s) \quad \text{then} \quad \emptyset_{33}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1 p_2}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = \emptyset_j(q).$$

$$(e) \quad \text{if } g=(x^{2p_1 p_2}, s) \quad \text{then} \quad \emptyset_{33}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1 p_2}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C< x>(x^{p_1 p_2})|} \cdot 1 = \emptyset_j(q).$$

$$(f) \quad \text{if } g=(x^{p_1 p_2}, s) \quad \text{then} \quad \emptyset_{33}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \\ \emptyset((g^{-1})) = \frac{4.2p_1 p_2}{2|C< x>(x^{p_1 p_2})|} (1+1) = \frac{|CQ4p(q)|}{2|C< x>(x^{p_1 p_2})|} \cdot 2 = \emptyset_j(q).$$

Since  $H \cap \text{cl}(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (x^{p_1 p_2}, 1), (I, s), (x^{2p_1 p_2}, s), (x^{p_1 p_2}, s)\}$  And  $\emptyset(g) = \emptyset((g^{-1})) = 1$  otherwise  $= 0$ .

$$4:- H_{43} = \langle (x^{4p_2}, s) \rangle \quad (a) \quad \text{if } g=(I, 1) \quad \text{then} \quad \emptyset_{43}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \\ \frac{24.2p_1 p_2}{2|C< x>(x^{4p_2})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C< x>(x^{4p_2})|} \cdot 1 = 3 \cdot \emptyset_j(I).$$

$$(b) \quad \text{if } g=(x^{4p_2}, 1) \quad \text{then} \quad \emptyset_{43}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \\ \emptyset((g^{-1})) = \frac{12.2p_1 p_2}{2|C< x>(x^{4p_2})|} (1+1) = \frac{3|CQ4p(q)|}{2|C< x>(x^{4p_2})|} \cdot 2 = 3 \emptyset_j(x^{4p_2}).$$

$$(c) \quad \text{if } g=(I, s) \quad \text{then} \quad \emptyset_{43}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1 p_2}{2|C< x>(x^{4p_2})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C< x>(x^{4p_2})|} \cdot 1 = \emptyset_j(q).$$

$$(d) \quad \text{if } g=(x^{4p^2}, s) \quad \text{then } \emptyset_{43}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{4p^2})|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{4p^2})|}.2 = \emptyset_j(q).$$

Since  $H \cap \text{cl}(g) = \{(I, 1), (x^{4p^2}, 1), (I, s), (x^{4p^2}, s)\}$  And  $\emptyset(g) = \emptyset((g^{-1}) = 1$  otherwise  $= 0$ .

$$5: H_{53} = \langle (x^{2p^2}, s) \rangle \quad (a) \quad \text{if } g=(I, 1) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{2p^2})|}.1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.1 = 3.\emptyset_j(I).$$

$$(b) \quad \text{if } g=(x^{2p^1p^2}, 1) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{2p^2})|}.1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.1 = 3.\emptyset_j(x^{2p^1p^2}).$$

$$(c) \quad \text{if } g=(x^{4p^2}, 1) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{4p^2})|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{4p^2})|}.2 = 3.\emptyset_j(x^{4p^2}).$$

$$(d) \quad \text{if } g=(x^{2p^2}, 1) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{2p^2})|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.2 = 3.\emptyset_j(x^{2p^2}).$$

$$(e) \quad \text{if } g=(I, s) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{2p^2})|}.1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.1 = \emptyset_j(q).$$

$$(f) \quad \text{if } g=(x^{2p^1p^2}, s) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{2p^2})|}.1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.1 = \emptyset_j(q).$$

$$(g) \quad \text{if } g=(x^{4p^2}, s) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{2p^2})|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.2 = \emptyset_j(q).$$

$$(h) \quad \text{if } g=(x^{2p^2}, s) \quad \text{then } \emptyset_{53}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{2p^2})|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{2p^2})|}.2 = \emptyset_j(q).$$

Since

$$H \cap \text{cl}(g) =$$

$\{(I, 1), (x^{2p^1p^2}, 1), (x^{4p^2}, 1), (x^{2p^2}, 1), (I, s), (x^{2p^1p^2}, s), (x^{4p^2}, s), (x^{2p^2}, s)\}$  And  $\emptyset(g) = \emptyset((g^{-1}) = 1$  otherwise  $= 0$ .

$$6: H_{63} = \langle (x^{p^2}, s) \rangle \quad (a) \quad \text{if } g=(I, 1) \quad \text{then } \emptyset_{63}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{p^2})|}.1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{p^2})|}.1 = 3.\emptyset_j(I).$$

- (b) if  $g=(x^{2p_1p_2}, 1)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p_1p_2}{2|C< x>(x^{p_2})|}.1=\frac{6|CQ4p(q)|}{2|C< x>(x^{p_2})|}.1=3.\emptyset_j(x^{2p_1p_2}).$
- (c) if  $g=(x^{p_1p_2}, 1)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=3.\emptyset_j(x^{p_1p_2}).$
- (d) if  $g=(x^{4p_2}, 1)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=3.\emptyset_j(x^{4p_2}).$
- (e) if  $g=(x^{2p_2}, 1)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=3.\emptyset_j(x^{2p_2}).$
- (f) if  $g=(x^{p_2}, 1)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{3|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=3.\emptyset_j(x^{2p}).$
- (g) if  $g=(I, s)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p_1p_2}{2|C< x>(x^{p_2})|}.1=\frac{2|CQ4p(q)|}{2|C< x>(x^{p_2})|}.1=\emptyset_j(q).$
- (h) if  $g=(x^{2p_1p_2}, s)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p_1p_2}{2|C< x>(x^{p_2})|}.1=\frac{2|CQ4p(q)|}{2|C< x>(x^{p_2})|}.1=\emptyset_j(q).$
- (i) if  $g=(x^{p_1p_2}, s)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=\emptyset_j(q).$
- (j) if  $g=(x^{4p_2}, s)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=\emptyset_j(q).$
- (k) if  $g=(x^{2p_2}, s)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=\emptyset_j(q).$
- (l) if  $g=(x^{p_2}, s)$  then  $\emptyset_{63}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+\emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C< x>(x^{p_2})|}(1+1)=\frac{|CQ4p(q)|}{2|C< x>(x^{p_2})|}.2=\emptyset_j(q).$

Since

$$H \cap cl(g) = \{(I, 1), (x^{2p_1p_2}, 1), (x^{p_1p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{p_2}, 1), (I, s), (x^{2p_1p_2}, s), (x^{p_1p_2}, s), (x^{4p_2}, s), (x^{2p_2}, s), (x^{p_2}, s)\}$$

And  $\emptyset(g) = \emptyset((g^{-1}) = 1$  otherwise  $= 0$ .

$$7:- H_{73} = \langle (x^{4p_1}, s) \rangle; \text{ (a) if } g = (I, 1) \text{ then } \emptyset_{73}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{4p_1})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{4p_1})|} \cdot 1 = 3 \cdot \emptyset_j(I).$$

$$\text{(b) if } g = (x^{4p_1}, 1) \text{ then } \emptyset_{73}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{4p_1})|} (1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{4p_1})|} \cdot 2 = 3 \cdot \emptyset_j(x^{4p_1})$$

$$\text{(c) if } g = (I, s) \text{ then } \emptyset_{73}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{4p_1})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{4p_1})|} \cdot 1 = \emptyset_j(q).$$

$$\text{(d) if } g = (x^{4p_1}, s) \text{ then } \emptyset_{73}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{4p_1})|} (1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{4p_1})|} \cdot 2 = \emptyset_j(q).$$

Since  $H \cap \text{cl}(g) = \{(I, 1), (x^{4p_1}, 1), (I, s), (x^{4p_1}, s)\}$

and  $\emptyset(g) = \emptyset((g^{-1}) = 1$  otherwise  $= 0$

$$8:- H_{83} = \langle (x^{2p_1}, s) \rangle; \text{ (a) if } g = (I, 1) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{2p_1})|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{2p_1})|} \cdot 1 = 3 \cdot \emptyset_j(I).$$

$$\text{(b) if } g = (x^{2p_1p_2}, 1) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{2p_1})|} \cdot 1 = \frac{6|CQ4p|}{2|C<x>(x^{2p_1})|} \cdot 1 = 3 \cdot \emptyset_j(x^{2p_1p_2}).$$

$$\text{(c) if } g = (x^{4p_1}, 1) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{2p_1})|} (1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{2p_1})|} \cdot 2 = 3 \cdot \emptyset_j(x^{4p_1})$$

$$\text{(d) if } g = (x^{2p_1}, 1) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{2p_1})|} (1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{2p_1})|} \cdot 2 = 3 \cdot \emptyset_j(x^{2p_1})$$

$$\text{(e) if } g = (I, s) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{2p_1})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{2p_1})|} \cdot 1 = \emptyset_j(q).$$

$$\text{(f) if } g = (x^{2p_1p_2}, s) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{2p_1})|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{2p_1})|} \cdot 1 = \emptyset_j(q).$$

$$\text{(g) if } g = (x^{4p_1}, s) \text{ then } \emptyset_{83}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{2p_1})|} (1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{2p_1})|} \cdot 2 = \emptyset_j(q).$$

$$(h) \quad \text{if } g=(x^{2p_1}, s) \quad \text{then } \emptyset_{83}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{2p_1})|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{2p_1})|}.2 = \emptyset_j(q).$$

Since

$$H \cap \text{cl}(g) = \{(I, 1), (x^{2p_1p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (I, s), (x^{2p_1p_2}, s), (x^{4p_1}, s), (x^{2p_1}, s)\} \text{ And } \emptyset(g) = \emptyset((g^{-1})) = 1 \text{ otherwise } = 0$$

$$9: H_{93} = \langle(x^{p_1}, s) \rangle; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{p_1})|}.1 = \frac{6|CQ4p|}{2|C<x>(x^{p_1})|}.1 = 3.\emptyset_j(I).$$

$$(b) \quad \text{if } g=(x^{2p_1p_2}, 1) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24.2p_1p_2}{2|C<x>(x^{p_1})|}.1 = \frac{6|CQ4p(q)|}{2|C<x>(x^{p_1})|}.1 = 3.\emptyset_j(x^{2p_1p_2}).$$

$$(c) \quad \text{if } g=(x^{p_1p_2}, 1) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{p_1})|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{p_1})|}.2 = 3.\emptyset_j(x^{p_1p_2}).$$

$$(d) \quad \text{if } g=(x^{4p_1}, 1) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{p_1})|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{p_1})|}.2 = 3.\emptyset_j(x^{4p_1})$$

$$(e) \quad \text{if } g=(x^{2p_1}, 1) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{p_1})|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{p_1})|}.2 = 3.\emptyset_j(x^{2p_1})$$

$$(f) \quad \text{if } g=(x^{p_1}, 1) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1p_2}{2|C<x>(x^{p_1})|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x^{p_1})|}.2 = 3.\emptyset_j(x^{p_1})$$

$$(g) \quad \text{if } g=(I, s) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{2p_1})|}.1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{p_1})|}.1 = \emptyset_j(q).$$

$$(h) \quad \text{if } g=(x^{2p_1p_2}, s) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8.2p_1p_2}{2|C<x>(x^{p_1})|}.1 = \frac{2|CQ4p(q)|}{2|C<x>(x^{p_1})|}.1 = \emptyset_j(q).$$

$$(i) \quad \text{if } g=(x^{4p_1}, s) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{p_1})|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{p_1})|}.2 = \emptyset_j(q).$$

$$(j) \quad \text{if } g=(x^{2p_1}, s) \quad \text{then } \emptyset_{93}(g)= \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1p_2}{2|C<x>(x^{p_1})|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^{p_1})|}.2 = \emptyset_j(q).$$

$$(k) \quad \text{if } g=(x^{p_1}, s) \quad \text{then } \emptyset_{93}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4 \cdot 2p_1 p_2}{2|C< x>(x^{p_1})|}(1+1) = \frac{|CQ4p(q)|}{2|C< x>(x^{p_1})|} \cdot 2 = \emptyset_j(q).$$

Since

$$H \cap \text{cl}(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (x^{p_1}, 1), (I, s), (x^{2p_1 p_2}, s), (x^{4p_1}, s), (x^{2p_1}, s), (x^{p_1}, s)\}$$

And  $\emptyset(g) = \emptyset((g^{-1}) = 1 \text{ otherwise } = 0.$

$$10:- H_{103} = <(x^4, s)>; \quad (a) \quad \text{if } g=(I, 1) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{24 \cdot 2p_1 p_2}{2|C< x>(x^4)|} \cdot 1 = \frac{6|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 1 = 3 \cdot \emptyset_j(I).$$

$$(b) \quad \text{if } g=(x^{4p_2}, 1) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{2|C< x>(x^4)|}(1+1) = \frac{3|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 2 = 3 \cdot \emptyset_j(x^{4p_2}).$$

$$(c) \quad \text{if } g=(x^{4p_1}, 1) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{2|C< x>(x^4)|}(1+1) = \frac{3|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 2 = 3 \cdot \emptyset_j(x^{4p_1})$$

$$(d) \quad \text{if } g=(x^4, 1) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12 \cdot 2p_1 p_2}{2|C< x>(x^4)|}(1+1) = \frac{3|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 2 = 3 \cdot \emptyset_j(x^4)$$

$$(e) \quad \text{if } g=(I, s) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8 \cdot 2p_1 p_2}{2|C< x>(x^4)|} \cdot 1 = \frac{2|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 1 = \emptyset_j(q).$$

$$(f) \quad \text{if } g=(x^{4p_2}, s) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4 \cdot 2p_1 p_2}{2|C< x>(x^4)|}(1+1) = \frac{|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 2 = \emptyset_j(q).$$

$$(g) \quad \text{if } g=(x^{4p_1}, s) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4 \cdot 2p_1 p_2}{2|C< x>(x^4)|}(1+1) = \frac{|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 2 = \emptyset_j(q).$$

$$(h) \quad \text{if } g=(x^4, s) \quad \text{then } \emptyset_{103}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4 \cdot 2p_1 p_2}{2|C< x>(x^4)|}(1+1) = \frac{|CQ4p(q)|}{2|C< x>(x^4)|} \cdot 2 = \emptyset_j(q).$$

Since

$$H \cap \text{cl}(g) = \{(I, 1), (x^{4p_2}, 1), (x^{4p_1}, 1), (x^4, 1), (I, s), (x^{4p_2}, s), (x^{4p_1}, s), (x^4, s)\}$$

and  $\emptyset(g) = \emptyset((g^{-1}) = 1 \text{ otherwise } = 0.$

$$\begin{aligned}
 & 11:-H_{113}=\langle(x^2,s)\rangle; \quad (\text{a}) \text{ if } g=(I,1) \text{ then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}\emptyset(g)= \\
 & \frac{24.2p_1p_2}{2|C<x>(x^2)|}.1=\frac{6|CQ4p(q)|}{2|C<x>(x^2)|}.1=3.\emptyset_j(I). \\
 & (\text{b}) \quad \text{if } g=(x^{2p_1p_2},1) \quad \text{then } \emptyset_{113}(g)= \\
 & \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{24.2p_1p_2}{2|C<x>(x^2)|}.1=\frac{6|CQ4p(q)|}{2|C<x>(x^2)|}.1=3.\emptyset_j(x^{2p_1p_2}). \\
 & (\text{c}) \quad \text{if } g=(x^{4p_2},1) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^{4p_2}). \\
 & (\text{d}) \quad \text{if } g=(x^{2p_2},1) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^{2p_2}). \\
 & (\text{e}) \quad \text{if } g=(x^{4p_1},1) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^{4p_1}) \\
 & (\text{f}) \quad \text{if } g=(x^{2p_1},1) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^{2p_1}) \\
 & (\text{g}) \quad \text{if } g=(x^4,1) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^4) \\
 & (\text{h}) \quad \text{if } g=(x^2,1) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{12.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{3|CQ4p(q)|}{2|C<x>(x^2)|}.2=3.\emptyset_j(x^2) \\
 & (\text{i}) \quad \text{if } g=(I,s) \quad \text{then } \emptyset_{113}(g)= \\
 & \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p_1p_2}{2|C<x>(x^2)|}.1=\frac{2|CQ4p(q)|}{2|C<x>(x^2)|}.1=\emptyset_j(q). \\
 & (\text{j}) \quad \text{if } g=(x^{2p_1p_2},s) \quad \text{then } \emptyset_{113}(g)= \\
 & \frac{|CG(g)|}{|CH(g)|}\emptyset(g)=\frac{8.2p_1p_2}{2|C<x>(x^2)|}.1=\frac{2|CQ4p(q)|}{2|C<x>(x^2)|}.1=\emptyset_j(q). \\
 & (\text{k}) \quad \text{if } g=(x^{4p_2},s) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x^2)|}.2=\emptyset_j(q). \\
 & (\text{l}) \quad \text{if } g=(x^{2p_2},s) \quad \text{then } \emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g)+ \\
 & \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x^2)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x^2)|}.2=\emptyset_j(q).
 \end{aligned}$$

- (m) if  $g = (x^{4p_1}, s)$  then  $\emptyset_{113}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1 p_2}{2|C<x>(x^2)|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^2)|}.2 = \emptyset_j(q).$
- (n) if  $g = (x^{2p_1}, s)$  then  $\emptyset_{113}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1 p_2}{2|C<x>(x^2)|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^2)|}.2 = \emptyset_j(q).$
- (o) if  $g = (x^4, s)$  then  $\emptyset_{113}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1 p_2}{2|C<x>(x^2)|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^2)|}.2 = \emptyset_j(q).$
- (p) if  $g = (x^2, s)$  then  $\emptyset_{113}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{4.2p_1 p_2}{2|C<x>(x^2)|}(1+1) = \frac{|CQ4p(q)|}{2|C<x>(x^2)|}.2 = \emptyset_j(q).$

Since

$$H \cap \text{cl}(g) = \{(I, 1), (x^{2p_1 p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (x^4, 1), (x^2, 1), (I, s), (x^{2p_1 p_2}, s), (x^{4p_2}, s), (x^{2p_2}, s), (x^{4p_1}, s), (x^{2p_1}, s), (x^4, s), (x^2, s)\}$$

and  
 $\emptyset(g) = \emptyset((g^{-1}) = 1 \text{ otherwise } = 0.$

- 12:-  $H_{123} = \langle (x, s) \rangle$ ; (a) if  $g = (I, 1)$  then  $\emptyset_{123}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p_1 p_2}{2|C<x>(x)|}.1 = \frac{6|CQ4p(q)|}{2|C<x>(x)|}.1 = 3.\emptyset_j(I).$
- (b) if  $g = (x^{2p_1 p_2}, 1)$  then  $\emptyset_{123}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p_1 p_2}{2|C<x>(x)|}.1 = \frac{6|CQ4p(q)|}{2|C<x>(x)|}.1 = 3.\emptyset_j(x^{2p_1 p_2}).$
- (c) if  $g = (x^{p_1 p_2}, 1)$  then  $\emptyset_{123}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{2|C<x>(x)|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x)|}.2 = 3.\emptyset_j(x^{p_1 p_2}).$
- (d) if  $g = (x^{4p_2}, 1)$  then  $\emptyset_{123}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{2|C<x>(x)|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x)|}.2 = 3.\emptyset_j(x^{4p_2}).$
- (e) if  $g = (x^{2p_2}, 1)$  then  $\emptyset_{123}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{2|C<x>(x)|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x)|}.2 = 3.\emptyset_j(x^{2p_2}).$
- (f) if  $g = (x^{p_2}, 1)$  then  $\emptyset_{123}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{12.2p_1 p_2}{2|C<x>(x)|}(1+1) = \frac{3|CQ4p(q)|}{2|C<x>(x)|}.2 = 3.\emptyset_j(x^{2p}).$

(g)	if	$g=(x^{4p_1}, 1)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{12 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{3 CQ4p(q) }{2 C<x>(x) } \cdot 2 = 3 \cdot \emptyset_j(x^{4p_1})$		
(h)	if	$g=(x^{2p_1}, 1)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{12 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{3 CQ4p(q) }{2 C<x>(x) } \cdot 2 = 3 \cdot \emptyset_j(x^{2p_1})$		
(i)	if	$g=(x^{p_1}, 1)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{12 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{3 CQ4p(q) }{2 C<x>(x) } \cdot 2 = 3 \cdot \emptyset_j(x^{p_1})$		
(j)	if	$g=(x^4, 1)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{12 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{3 CQ4p(q) }{2 C<x>(x) } \cdot 2 = 3 \cdot \emptyset_j(x^4)$		
(k)	if	$g=(x^2, 1)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{12 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{3 CQ4p(q) }{2 C<x>(x) } \cdot 2 = 3 \cdot \emptyset_j(x^2)$		
(l)	if	$g=(x, 1)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{12 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{3 CQ4p(q) }{2 C<x>(x) } \cdot 2 = 3 \cdot \emptyset_j(x)$		
(m)	if	$g=(I, s)$	then	$\emptyset_{123}(g) =$
		$\frac{ CG(g) }{ CH(g) } \emptyset(g) = \frac{8 \cdot 2p_1 p_2}{2 C<x>(x) } \cdot 1 = \frac{2 CQ4p(q) }{2 C<x>(x) } \cdot 1 = \emptyset_j(q).$		
(n)	if	$g=(x^{2p_1 p_2}, s)$	then	$\emptyset_{123}(g) =$
		$\frac{ CG(g) }{ CH(g) } \emptyset(g) = \frac{8 \cdot 2p_1 p_2}{2 C<x>(x) } \cdot 1 = \frac{2 CQ4p(q) }{2 C<x>(x) } \cdot 1 = \emptyset_j(q).$		
(o)	if	$g=(x^{p_1 p_2}, s)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{4 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{ CQ4p(q) }{2 C<x>(x) } \cdot 2 = \emptyset_j(q).$		
(p)	if	$g=(x^{4p_2}, s)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{4 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{ CQ4p(q) }{2 C<x>(x) } \cdot 2 = \emptyset_j(q).$		
(q)	if	$g=(x^{2p_2}, s)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{4 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{ CQ4p(q) }{2 C<x>(x) } \cdot 2 = \emptyset_j(q).$		
(r)	if	$g=(x^{p_2}, s)$	then	$\emptyset_{123}(g) = \frac{ CG(g) }{ CH(g) }(\emptyset(g) +$
		$\emptyset(g^{-1}) = \frac{4 \cdot 2p_1 p_2}{2 C<x>(x) }(1+1) = \frac{ CQ4p(q) }{2 C<x>(x) } \cdot 2 = \emptyset_j(q).$		

- (s) if  $g=(x^{4p_1}, s)$  then  $\emptyset_{123}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x)|}.2=\emptyset_j(q).$
- (t) if  $g=(x^{2p_1}, s)$  then  $\emptyset_{123}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x)|}.2=\emptyset_j(q).$
- (u) if  $g=(x^{p_1}, s)$  then  $\emptyset_{123}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x)|}.2=\emptyset_j(q).$
- (v) if  $g=(x^4, s)$  then  $\emptyset_{123}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x)|}.2=\emptyset_j(q).$
- (w) if  $g=(x^2, s)$  then  $\emptyset_{123}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x)|}.2=\emptyset_j(q).$
- (x) if  $g=(x, s)$  then  $\emptyset_{113}(g)=\frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1}))=\frac{4.2p_1p_2}{2|C<x>(x)|}(1+1)=\frac{|CQ4p(q)|}{2|C<x>(x)|}.2=\emptyset_j(q).$

Since

$H \cap cl(g) = \{(I, 1), (x^{2p_1p_2}, 1), (x^{4p_2}, 1), (x^{2p_2}, 1), (x^{p_2}, 1), (x^{4p_1}, 1), (x^{2p_1}, 1), (x^{p_1}, 1),$

$(x^4, 1), (x^2, 1), (x, 1),$

$(I, s), (x^{2p_1p_2}, s), (x^{4p_2}, s), (x^{2p_2}, s), (x^{p_2}, s), (x^{4p_1}, s), (x^{2p_1}, s), (x^{p_1}, s), (x^4, s), (x^2, s), (x, s)\}$  and  $\emptyset(g) = \emptyset((g^{-1}) = 1 \text{ otherwise } = 0.$

13:-  $H_{133} = \langle(y, s) \rangle$ ; (a) if  $g = (I, 1)$  then  $\emptyset_{133}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p_1p_2}{8}.1 = 3.\emptyset_{i+1}(I).$

(b) if  $g = (y^2, 1) = (x^{2p_1p_2}, 1)$  then  $\emptyset_{133}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{24.2p_1p_2}{8}.1 = 3.\emptyset_{i+1}(x^{2p_1p_2}).$

(c) if  $g = (y, 1)$  or  $(y^3, 1)$  then  $\emptyset_{133}(g) = \frac{|CG(g)|}{|CH(g)|}(\emptyset(g) + \emptyset(g^{-1})) = \frac{24}{8}(1+1) = 6.$

(d) if  $g = (I, s)$  then  $\emptyset_{133}(g) = \frac{|CG(g)|}{|CH(g)|}\emptyset(g) = \frac{8.2p_1p_2}{8}.1 = \emptyset_{i+1}(q).$

$$(e) \quad \text{if } g=(y^2,s)=(x^{2p_1p_2},s) \quad \text{then} \quad \emptyset_{133}(g)= \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8 \cdot 2p_1p_2}{8} \cdot 1 = \emptyset_{i+1}(q).$$

$$(f) \quad \text{if } g=(y,s) \text{ or } (y^3,s) \quad \text{then} \emptyset_{133}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) \\ = \frac{8}{8} (1+1) = 2.$$

Since  $H \cap cl(g) = \{(I, 1), (y^2, 1), (y, 1), (I, s), (y^2, s), (y, s)\}$  And  $\emptyset(g) = \emptyset((g^{-1})) = 1$  otherwise = 0.

$$14: H_{143} = \langle(xy, s) \rangle; \quad a) \quad \text{if } g=(I,1) \quad \text{then} \quad \emptyset_{143}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \\ \frac{24 \cdot 2p_1p_2}{8} \cdot 1 = 3 \cdot \emptyset_{i+2}(I).$$

$$(b) \quad \text{if } g=((xy)^2,1) = (x^{2p_1p_2},1) \quad \text{then} \quad \emptyset_{143}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \\ \frac{24 \cdot 2p_1p_2}{8} \cdot 1 = 3 \cdot \emptyset_{i+2}(x^{2p_1p_2}).$$

$$(c) \quad \text{if } g=(xy,1) \text{ or } ((xy)^3,1) \quad \text{then} \quad \emptyset_{143}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) = \\ \frac{24}{8} (1+1) = 6.$$

$$(d) \quad \text{if } g=(I,s) \quad \text{then} \quad \emptyset_{143}(g) = \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8 \cdot 2p_1p_2}{8} \cdot 1 = \emptyset_{i+2}(q).$$

$$(e) \quad \text{if } g=((xy)^2,s) = (x^{2p_1p_2},s) \quad \text{then} \quad \emptyset_{143}(g) = \\ \frac{|CG(g)|}{|CH(g)|} \emptyset(g) = \frac{8 \cdot 2p_1p_2}{8} \cdot 1 = \emptyset_{i+2}(q).$$

$$(f) \quad \text{if } g=(xy,s) \text{ or } ((xy)^3,s) \quad \text{then} \quad \emptyset_{143}(g) = \frac{|CG(g)|}{|CH(g)|} (\emptyset(g) + \emptyset(g^{-1})) \\ = \frac{8}{8} (1+1) = 2.$$

Since  $H \cap cl(g) = \{(I, 1), ((xy)^2, 1), (xy, 1), (I, s), ((xy)^3, s), (xy, s)\}$  And  $\emptyset(g) = \emptyset((g^{-1})) = 1$  otherwise = 0.

Note:  $(xy)^2=y^2$  since  $(xy)^2=xyxy=xyxy.y^2=y^2=xyx^3y^2=x(yxy^3)y^2=xx^{-1}y^2=y^2$ .

### Example:-(4.2)

Let  $p_1=3$  and  $p_2=7$ ;  $m=2p_1p_2=2 \cdot 3 \cdot 7=42$ , such that  $p_1, p_2$  are prime numbers,  $Q_{2m}=Q_{84}$ , To find Artin's character of the group ( $Q_{84} \times D_3$ ) the cyclic subgroup of  $Q_{84}$  which are  $\langle I \rangle, \langle x^{42} \rangle, \langle x^{21} \rangle, \langle x^{28} \rangle, \langle x^{14} \rangle, \langle x^7 \rangle, \langle x^{12} \rangle, \langle x^6 \rangle, \langle x^3 \rangle, \langle x^4 \rangle, \langle x^2 \rangle, \langle x \rangle, \langle y \rangle$ ,

$\langle xy \rangle$  and subgroup of  $D_3$  which are  $\{1, r, s\}$ .  
 The subgroup of  $(Q_{84} \times D_3)$  are  
 $\{1, x^4, x^8, x^{12}, x^{16}, x^{20}, x^{24}, x^{28}, x^{32}, x^{36}, x^{40}, x^{44}, x^{48}, x^{52}, x^{56}, x^{60}, x^{64}, x^{68}, x^{72}, x^{76}, x^{80}, x^{84}, x^{88}, x^{92}, x^{96}, x^{100}, x^{104}, x^{108}, x^{112}, x^{116}, x^{120}, x^{124}, x^{128}, x^{132}, x^{136}, x^{140}, x^{144}, x^{148}, x^{152}, x^{156}, x^{160}, x^{164}, x^{168}, x^{172}, x^{176}, x^{180}, x^{184}, x^{188}, x^{192}, x^{196}, x^{200}, x^{204}, x^{208}, x^{212}, x^{216}, x^{220}, x^{224}, x^{228}, x^{232}, x^{236}, x^{240}, x^{244}, x^{248}, x^{252}, x^{256}, x^{260}, x^{264}, x^{268}, x^{272}, x^{276}, x^{280}, x^{284}, x^{288}, x^{292}, x^{296}, x^{20}, x^{40}, x^{80}, x^{120}, x^{160}, x^{200}, x^{240}, x^{280}, x^{320}, x^{360}, x^{400}, x^{440}, x^{480}, x^{520}, x^{560}, x^{600}, x^{640}, x^{680}, x^{720}, x^{760}, x^{800}, x^{840}, x^{880}, x^{920}, x^{960}, x^{1000}, x^{1040}, x^{1080}, x^{1120}, x^{1160}, x^{1200}, x^{1240}, x^{1280}, x^{1320}, x^{1360}, x^{1400}, x^{1440}, x^{1480}, x^{1520}, x^{1560}, x^{1600}, x^{1640}, x^{1680}, x^{1720}, x^{1760}, x^{1800}, x^{1840}, x^{1880}, x^{1920}, x^{1960}, x^{2000}, x^{2040}, x^{2080}, x^{2120}, x^{2160}, x^{2200}, x^{2240}, x^{2280}, x^{2320}, x^{2360}, x^{2400}, x^{2440}, x^{2480}, x^{2520}, x^{2560}, x^{2600}, x^{2640}, x^{2680}, x^{2720}, x^{2760}, x^{2800}, x^{2840}, x^{2880}, x^{2920}, x^{2960}\}$

By using theorem:-

$$\emptyset_j(g) = \begin{cases} \frac{|CG(g)|}{|CH(g)|} & \text{if } hi \in H \cap cl(g) \\ 0 & \text{if } H \cap cl(g) = \emptyset \end{cases}$$

=Then  $Ar(Q_{84} \times D_3) = Ar(Q_{2^2 \cdot 3 \cdot 7} \times D_3) = Ar(Q_{2^2 \cdot 3 \cdot 7}) \otimes Ar(D_3)$

	Γ-classes of $Q_{84} \times \{1\}$											
	$[1,1]$	$[x^{42},1]$	$[x^{84},1]$	$[x^{126},1]$	$[x^{168},1]$	$[x^{210},1]$	$[x^{252},1]$	$[x^{294},1]$	$[x^{336},1]$	$[x^{378},1]$	$[x^{420},1]$	$[x^{462},1]$
$\Phi(1,1)$	1008	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,1)$	504	504	0	0	0	0	0	0	0	0	0	0
$\Phi(3,1)$	252	252	252	0	0	0	0	0	0	0	0	0
$\Phi(4,1)$	336	0	0	336	0	0	0	0	0	0	0	0
$\Phi(5,1)$	168	168	0	168	168	0	0	0	0	0	0	0
$\Phi(6,1)$	84	84	84	84	84	84	0	0	0	0	0	0
$\Phi(7,1)$	144	0	0	0	0	0	144	0	0	0	0	0
$\Phi(8,1)$	72	72	0	0	0	0	72	72	0	0	0	0
$\Phi(9,1)$	36	36	36	0	0	0	36	36	36	0	0	0
$\Phi(10,1)$	48	0	0	48	0	0	48	0	0	48	0	0
$\Phi(11,1)$	24	24	0	24	24	0	24	24	0	24	24	0
$\Phi(12,1)$	12	12	12	12	12	12	12	12	12	12	12	0
$\Phi(13,1)$	252	252	0	0	0	0	0	0	0	0	0	12
$\Phi(14,1)$	252	252	0	0	0	0	0	0	0	0	0	0

	Γ-classes of $Q_{84} \times \{r\}$											
	$[1,r]$	$[x^{42},r]$	$[x^{84},r]$	$[x^{126},r]$	$[x^{168},r]$	$[x^{210},r]$	$[x^{252},r]$	$[x^{294},r]$	$[x^{336},r]$	$[x^{378},r]$	$[x^{420},r]$	$[x^{462},r]$
$\Phi(1,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(3,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(4,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(5,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(6,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(7,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(8,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(9,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(10,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(11,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(12,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(13,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(14,1)$	0	0	0	0	0	0	0	0	0	0	0	0

Nesir Rasool Mahmood, Zinah Makki Kadhim- On Artin cokernel of The Group( $Q_{2m} \times D_3$ ) Where  $m= 2p_1p_2$  and  $p_1, p_2$  are prime numbers

	$\Gamma$ -classes of $Q_{2m} \times \{s\}$											
	[1,s]	[x <sup>2</sup> ,s]	[x <sup>3</sup> ,s]	[x <sup>4</sup> ,s]	[x <sup>5</sup> ,s]	[x <sup>6</sup> ,s]	[x <sup>7</sup> ,s]	[x <sup>8</sup> ,s]	[x <sup>9</sup> ,s]	[x <sup>10</sup> ,s]	[x <sup>11</sup> ,s]	[x <sup>12</sup> ,s]
$\Phi(1,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(3,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(4,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(5,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(6,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(7,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(8,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(9,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(10,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(11,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(12,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(13,1)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(14,1)$	0	0	0	0	0	0	0	0	0	0	0	0

	$\Gamma$ -classes of $Q_{2m} \times \{t\}$											
	[1,t]	[x <sup>2</sup> ,t]	[x <sup>3</sup> ,t]	[x <sup>4</sup> ,t]	[x <sup>5</sup> ,t]	[x <sup>6</sup> ,t]	[x <sup>7</sup> ,t]	[x <sup>8</sup> ,t]	[x <sup>9</sup> ,t]	[x <sup>10</sup> ,t]	[x <sup>11</sup> ,t]	[x <sup>12</sup> ,t]
$\Phi(1,2)$	336	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,2)$	168	168	0	0	0	0	0	0	0	0	0	0
$\Phi(3,2)$	84	84	0	0	0	0	0	0	0	0	0	0
$\Phi(4,2)$	112	0	0	112	0	0	0	0	0	0	0	0
$\Phi(5,2)$	56	56	0	56	56	0	0	0	0	0	0	0
$\Phi(6,2)$	28	28	28	28	28	0	0	0	0	0	0	0
$\Phi(7,2)$	48	0	0	0	0	0	48	0	0	0	0	0
$\Phi(8,2)$	24	24	0	0	0	0	24	24	0	0	0	0
$\Phi(9,2)$	12	12	12	0	0	0	12	12	12	0	0	0
$\Phi(10,2)$	16	0	0	16	0	0	16	0	0	16	0	0
$\Phi(11,2)$	8	8	0	8	8	0	8	8	0	8	8	0
$\Phi(12,2)$	4	4	4	4	4	4	4	4	4	4	4	0
$\Phi(13,2)$	84	84	0	0	0	0	0	0	0	0	0	4
$\Phi(14,2)$	84	84	0	0	0	0	0	0	0	0	0	4

	$\Gamma$ -classes of $Q_{2m} \times \{r\}$											
	[1,r]	[x <sup>2</sup> ,r]	[x <sup>3</sup> ,r]	[x <sup>4</sup> ,r]	[x <sup>5</sup> ,r]	[x <sup>6</sup> ,r]	[x <sup>7</sup> ,r]	[x <sup>8</sup> ,r]	[x <sup>9</sup> ,r]	[x <sup>10</sup> ,r]	[x <sup>11</sup> ,r]	[x <sup>12</sup> ,r]
$\Phi(1,2)$	336	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,2)$	168	168	0	0	0	0	0	0	0	0	0	0
$\Phi(3,2)$	84	84	84	0	0	0	0	0	0	0	0	0
$\Phi(4,2)$	112	0	0	112	0	0	0	0	0	0	0	0
$\Phi(5,2)$	56	56	0	56	56	0	0	0	0	0	0	0
$\Phi(6,2)$	28	28	28	28	28	0	0	0	0	0	0	0
$\Phi(7,2)$	48	0	0	0	0	0	48	0	0	0	0	0
$\Phi(8,2)$	24	24	0	0	0	0	24	24	0	0	0	0
$\Phi(9,2)$	12	12	12	0	0	0	12	12	12	0	0	0
$\Phi(10,2)$	16	0	0	16	0	0	16	0	0	16	0	0
$\Phi(11,2)$	8	8	0	8	8	0	8	8	0	8	8	0
$\Phi(12,2)$	4	4	4	4	4	4	4	4	4	4	4	0
$\Phi(13,2)$	84	84	0	0	0	0	0	0	0	0	0	4
$\Phi(14,2)$	84	84	0	0	0	0	0	0	0	0	0	4

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	$\Gamma$ -classes of $Q_{2m} \times \{1\}$											
	[1,1]	[ $x^{2^0}, 1$ ]	[ $x^{2^1}, 1$ ]	[ $x^{2^2}, 1$ ]	[ $x^{2^3}, 1$ ]	[ $x^{2^4}, 1$ ]	[ $x^{2^5}, 1$ ]	[ $x^{2^6}, 1$ ]	[ $x^{2^7}, 1$ ]	[ $x^{2^8}, 1$ ]	[ $x^{2^9}, 1$ ]	[ $x^{2^{10}}, 1$ ]
$\Phi(1,3)$	504	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,3)$	252	252	0	0	0	0	0	0	0	0	0	0
$\Phi(3,3)$	126	126	0	0	0	0	0	0	0	0	0	0
$\Phi(4,3)$	168	0	168	0	0	0	0	0	0	0	0	0
$\Phi(5,3)$	84	84	0	84	84	0	0	0	0	0	0	0
$\Phi(6,3)$	42	42	42	42	42	0	0	0	0	0	0	0
$\Phi(7,3)$	72	0	0	0	0	0	72	0	0	0	0	0
$\Phi(8,3)$	36	36	0	0	0	0	36	36	0	0	0	0
$\Phi(9,3)$	18	18	18	0	0	0	0	18	18	18	0	0
$\Phi(10,3)$	24	0	0	24	0	0	24	0	0	24	0	0
$\Phi(11,3)$	24	24	0	24	24	0	24	24	0	24	24	0
$\Phi(12,3)$	12	12	12	12	12	12	12	12	12	12	12	12
$\Phi(13,3)$	126	12	6	0	0	0	0	0	0	0	0	6
$\Phi(14,3)$	126	126	0	0	0	0	0	0	0	0	0	6

	$\Gamma$ -classes of $Q_{2m} \times \{s\}$											
	[1,s]	[ $x^{2^0}, s$ ]	[ $x^{2^1}, s$ ]	[ $x^{2^2}, s$ ]	[ $x^{2^3}, s$ ]	[ $x^{2^4}, s$ ]	[ $x^{2^5}, s$ ]	[ $x^{2^6}, s$ ]	[ $x^{2^7}, s$ ]	[ $x^{2^8}, s$ ]	[ $x^{2^9}, s$ ]	[ $x^{2^{10}}, s$ ]
$\Phi(1,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(3,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(4,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(5,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(6,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(7,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(8,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(9,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(10,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(11,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(12,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(13,2)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(14,2)$	0	0	0	0	0	0	0	0	0	0	0	0

	$\Gamma$ -classes of $Q_{2m} \times \{r\}$											
	[1,r]	[ $x^{2^0}, r$ ]	[ $x^{2^1}, r$ ]	[ $x^{2^2}, r$ ]	[ $x^{2^3}, r$ ]	[ $x^{2^4}, r$ ]	[ $x^{2^5}, r$ ]	[ $x^{2^6}, r$ ]	[ $x^{2^7}, r$ ]	[ $x^{2^8}, r$ ]	[ $x^{2^9}, r$ ]	[ $x^{2^{10}}, r$ ]
$\Phi(1,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(3,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(4,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(5,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(6,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(7,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(8,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(9,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(10,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(11,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(12,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(13,3)$	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(14,3)$	0	0	0	0	0	0	0	0	0	0	0	0

	$\Gamma$ -classes of $Q_{2m} \times \{s\}$												
	[1,s]	[ $x^{4^k},s$ ]	[ $x^{2^k},s$ ]	[ $x^{1^k},s$ ]	[ $x^{14},s$ ]	[ $x^8,s$ ]	[ $x^{12},s$ ]	[ $x^6,s$ ]	[ $x^9,s$ ]	[ $x^5,s$ ]	[ $x^3,s$ ]	[ $y,s$ ]	[ $xy,s$ ]
$\Phi(1,3)$	168	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi(2,3)$	84	84	0	0	0	0	0	0	0	0	0	0	0
$\Phi(3,3)$	42	42	42	0	0	0	0	0	0	0	0	0	0
$\Phi(4,3)$	56	0	0	56	0	0	0	0	0	0	0	0	0
$\Phi(5,3)$	28	28	0	28	28	0	0	0	0	0	0	0	0
$\Phi(6,3)$	42	42	42	42	42	42	0	0	0	0	0	0	0
$\Phi(7,3)$	24	0	0	0	0	0	24	0	0	0	0	0	0
$\Phi(8,3)$	12	12	0	0	0	0	12	12	0	0	0	0	0
$\Phi(9,3)$	6	6	6	0	0	0	6	6	6	0	0	0	0
$\Phi(10,3)$	8	0	0	8	0	0	8	0	0	8	0	0	0
$\Phi(11,3)$	4	4	0	4	4	0	4	4	0	4	4	0	0
$\Phi(12,3)$	2	2	2	2	2	2	2	2	2	2	2	2	0
$\Phi(13,3)$	42	42	0	0	0	0	0	0	0	0	0	0	2
$\Phi(14,3)$	42	42	0	0	0	0	0	0	0	0	0	0	2

## REFERENCES

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