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Further Study on Luo Shu by Linear Algebra

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Abstract:

This is a follow-up article to the authors' previous publication in 2015 by treating Luo Shu, the foundation of Chinese Compass School of Feng Shui, as a matrix and expanding it to 72 variations that cover applications in both northern and southern hemispheres. With the help of three mathematical techniques of linear algebra, i.e. determinants, eigenvalues and eigenvectors, it can be proved that the 72 variations form one complete system for application, which is unique out of the random combinations of 362,880 matrices. It was concluded that the original Luo Shu was merely treated as a magic square in recreational mathematics in the western world while the Chinese ancestors in Feng Shui study adopted and expanded it into a mathematical platform as claimed by the authors. It is a comprehensive, unique and standalone system for use, with a specific meaning in terms of the "Three Big Trigrams", an important concept to

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Feng Shui masters. It is hoped that by using such a discovery, Feng Shui study based on Luo Shu could be more logical and mathematical.

Key words: Luo shu, magic square, linear algebra, matrix, feng shui, compass school.

LUO SHU AS A MAGIC SQUARE

The authors (So et al 2015) reported that the name of Luo Shu appeared in *Lun Yu* (*Conversations and Discourses of Confucius*) written in the 5th century BCE and then *Yi Jing* (*Book of Changes*) but there was no detail about the diagram of Luo Shu. In *Da Dai Liji* (*Record of Rites by the Elder Dai*) written in 80 CE, the arrangement of numbers from "1" to "9" in three sets was depicted, which resembled Luo Shu closely. The first printed version of Luo Shu appeared in the Song Dynasty, written by the famous Daoist Chen Tuan in the 10th century CE. The 1977 archaeological discovery at a tomb of the Western Han Dynasty built around 2nd century BCE at Anhui, China revealed that the diagram of Luo Shu had been used on an astronomical instrument well before 80 CE. Various modern western authors, including Joseph Needham, asserted that the earliest magic square known outside China was given by Theon of Smyrna, a Neo-Pythagorean, at about 130 CE (Cammann 1960). But Cammann immediately pointed out that the so-called magic square of Theon was not a magic square in any sense, both squares being shown in Figure 1.

1	4	7		4	9	2
2	5	8		3	5	7
3	6	9		8	1	6

Figure 1 Theon of Smyrna Square **Luo Shu**

This is called the theory of "Duality of Lo Shu" but it was denied by Swallows (2013) by quoting Cammann's comment as well as Ahrens' opinion saying that the Theon square had nothing to do with magic squares and therefore should deserve no place in the history of magic squares (Ahrens 1917). The next question is what is a perfect magic square.

A perfect magic square is a N x N square with N^2 different numbers placed in all cells. The sum of all numbers along any row or column or diagonal must be equal to a constant which is N times the number of the central cell while N is an odd number. The well known algebraic formula suggested by Édouard Lucas as quoted by Swallows (1997) is shown in Figure 2.

$c-b$	$c+a+b$	$c-a$
$c-a+b$	c	$c+a-b$
$c+a$	$c-a-b$	$c+b$

Figure 2 Lucas's formula for a Magic Square

Here, a, b and c are all natural numbers and the nine number should also be different natural numbers, not bigger than nine. In this case, the following inequalities must be valid. The numbers at opposing corners must sum up to $N^2 + 1$.

Let $a > b$, then $c > a + b$; $10 > c + a + b$; $a \neq b \neq a - b \neq a + b \Rightarrow 2b \neq a$. Under these constraints, there is only one valid combination, i.e. $c = 5$, $a = 3$ and $b = 1$, resulting in the standard Luo Shu shown in Figure 1, because $2c$ must be equal to $3^2+1 = 10$. Swallows (1997) even went further to turn the three variables into complex numbers, such as **a**, **b** and **c** shown in Figure 3, for a more general form of magic

c-b = -7+0**i**	**c+a+b** =8+10**i**	**c-a** =2-4**i**
c-a+b =10-2**i**	**c** =1+2**i**	**c+a-b** =-8+6**i**
c+a =0+8**i**	**c-a-b** =-6-6**i**	**c+b** =9+4**i**

Figure 3 An Example of Complex Magic Square suggested by Swallows (1997)

squares but the discussion of which is beyond the scope of this article.

In this case, the sum of all three variables along any row, or column, or diagonal is equal to 3+6**i** where $\mathbf{i}=\sqrt{-1}$, indicating that this complex square is a perfect magic square, but in a complex sense.

LUO SHU – BEYOND A MAGIC SQUARE

Cammann (1960) stated that "*To sum up, we have seen that the Chinese probably did indeed invent the magic square of three, centuries before anyone else; but they were apparently so hypnotized by this early solution, and by all the cosmic and magical properties ascribed to it, that they continued to try to adapt the L(u)o Shu principle to the solving of higher squares as well. From an evolutionary point of view, this was a blind alley; and it prevented them from going on to find quicker and more efficient methods*." and "*The Chinese certainly had plenty of inventive genius, as illustrated by their numerous significant discoveries; but they were too often satisfied by preliminary results based on an early discovered method, without trying to improve upon this*." The authors of this article are of the opinion that Cammann did answer the query by himself. The ancient Chinese invented Luo Shu not for recreation, but for divination. And this concept was also shared by Ellis (2001) stating that "*In Old China, through a reductivist system of squares within squares, it seems that the magic square of three had*

I	*E*	*G*
H	*A*	*C*
D	*F*	*B*

Figure 4 Trajectory of the Original Luo

D	*H*	*I*
F	*A=x*	*E*
B	*C*	*G*

Figure 5 Matrix of FS(x+, 90^o)

been used to order their world on earth from the macrocosm of 'The Celestial Numbers of the Nine Halls' to the microcosm of their immediate environment. So much so, that the L(u)o Shu became the basic theory of the Flying Stars School of Feng Shui. Furthermore, the Eight Types of Houses Theory makes use of the kwa (gua), or the eight Chinese trigrams, to diagnose a home."

The original or standard Luo Shu as depicted in Figure 1 is only one form which was known to people outside ancient China. But ancient Chinese further extended its application based on the trajectory as shown in Figure 4, which was unknown to any civilization outside ancient China. It is obvious that the original Luo Shu picks A=5, B=6, C=7, D=8, E=9, F=1, G=2, H=3 and I=4, thus completing the cycle. According to Chinese Feng Shui, A could take any integer from 1, 2, …, 9. Once A is fixed, values at the remaining 8 positions are known and this is called "forward flying". At the same time, a backward flying path also exists, i.e. A=5, B=4, C=3, D=2, E=1, F=9 … In this way, eighteen charts could be established. In the authors' previous article (So et al 2015), the authors denote a forward (+) or backward (-1) flying Luo Shu as a matrix with the symbol **FS**(x±, 0°). Here, x refers to the integer which is at position A, i.e. the center, and "+" means forward flying with x=5 depicting the original Luo Shu. Obviously, **FS**(x-, 0°) refers to the backward flying charts. 0° means no rotation of the matrix is manipulated. A 90° rotation applied to a matrix produces a new matrix as shown in Figure 5, showing **FS**(x+, 90°). The authors, in the previous article (So et al 2015), also pointed out **FS**(x+, 180°)=**FS**(x-, 0°). Therefore, there is no need to discriminate between forward flying and backward flying, the two sets being clearly denoted by **FS**(x, 0°) and **FS**(x, 180°). These eighteen standard charts are frequently used in different schools of Compass Feng Shui, which will be discussed later. In limited schools of Qi Mun Dun Jia, 18 more charts are used but not popular. Therefore, totally, there are thirty six standard charts available for use, namely **FS**(x, D°), x = 1, 2, 3, ….9, and *D(egree)* = 0, 90, 180 and 270.

LUO SHU APPLICATIONS IN FENG SHUI IN GENERAL

According to Walters (1995), if the Feng Shui of a site is beneficial based on the Form School, a well-defined Dragon (east), Bird (south), Tortoise (north) and Tiger (west) in the location will increase the beneficial aspects of those parts of the site that receive the location's Qi. If, on the other hand, the site is generally inauspicious, perhaps because there is a lack of Qi and the existence of abundant Sha (killing), certain unfavorable aspects of the site can be eradicated by the location's favorable position and orientation. The key word is "orientation". Stephan Feuchtwang said, "to be in the right place facing the right direction doing the right thing at the right time is a cross between being practically efficient and being ritually correct. It is being in tune with the universe." (cited in Skinners, 1982.) The Compass School concerns space and time. Space normally refers to the eight directions, namely north, northeast, east, southeast, south, southwest, west and northwest respectively, and time refers to a period of say 20 years, a year, a month or even a day.

There are many techniques within the Compass School while the most popular one in nowadays is called "Nine Palaces Flying Stars Method". It is hundred percent based on Luo Shu as depicted in Figure 1 and Figure 4. The original Luo Shu, **FS**(5+, 0^o), is further manipulated into different charts. According to the Flying Stars method, every property has good and bad "stars". The technique of the Flying Stars method is to make sure that the beneficial stars are located in places such as an office, bedroom, lounge or dining room, while the bad ones are in locations such as bathrooms, toilets, cloakrooms, closets or utility rooms. Stars here refer to the nine numbers, actually eight trigrams plus the number "5" with no trigram, e.g.

"1" for trigram Kan ☵,

"2" for trigram "Kun" ☷ ,

"3" for trigram Zhen ☳,

"9" for trigram Li ☲ etc.

The Flying Stars method basically manipulates up to three or four charts to determine whether a building in general or a particular corner of that building is auspicious or not. The first chart is of course the standard Luo Shu chart, i.e. **FS**(5+, 0^{o}), also called the Earth chart. Since the Earth chart is always the same, it is usually not depicted in the Flying Stars method. The second chart, called the Time chart or Heaven chart, is derived from the 20-year period within which the building is constructed. For example, **FS**(6+, 0^{o}) is used for a house constructed within the period from February, 1964 to February, 1984; **FS**(7+, 0^{o}) is used for a house constructed within the period from February, 1984 to February, 2004; **FS**(8+, 0^{o}) is used for a house constructed within the period from February, 2004 to February, 2024 and so on. The third and fourth charts are derived from the facing direction, thus called Facing chart, and the sitting direction of the building, thus called Sitting chart or Mountain chart. By superposing these three charts together, the following Flying Stars combined chart can be obtained, as shown in Figure 6. Here, "T" denotes Time; "M" denotes Mountain or Sitting; "F" denotes Facing. The discussion on how to draw the Facing and Mountain charts is beyond the scope of this paper. An example is given here for a building facing 75^{o} (i.e. east-north-east with north given by 0^{o}) and built within the 20-year period from 1984 to 2004, shown in Figure 7. Using our mathematical notation, the complete Flying Stars chart of this building is the superposition of **FS**(7+, 0^{o}), **FS**(5+, 0^{o}) and **FS**(9+, 0^{o}). The Flying Stars charts are mainly built upon the 18 variations of the standard Luo Shu, i.e. **FS**(x+, 0^{o}) or **FS**(x-, 0^{o}) which is also equal to **FS**(x+, 180^{o}). In the authors' previous article (So et al 2015), the situations of "The Three Big Trigrams" and "Combined to Ten" were discussed. Traditionally, "The Three Big Trigrams" refer to a combination of the three groups, "1, 4, 7", "2, 5, 8" or "3, 6, 9" while the "Combined to Ten" refers to a combination of the four groups, "1, 9", "2, 8", 3, 7" or "4, 6". The definition of "Combined to Ten" has not been controversial but that of "The Three Big Trigrams" has been subject to argument.

M F T	M F T	M F T
M F T	M F T	M F T
M F T	M F T	M F T

Figure 6 A Complete Flying Stars chart

8 4 6	4 9 2	6 2 4
7 3 5	9 5 7	2 7 9
3 8 1	5 1 3	1 6 8

Figure 7 Flying Stars chart of a house facing 70° built within the period from Feb, 1984 to Feb, 2004

LUO SHU APPLICATIONS IN SOUTHERN HEMISPHERE

Since the 70's of the last century, there has been an increasing Chinese immigration to Australia and New Zealand. Because of this, the practice of Feng Shui has therefore become more popular in the southern hemisphere. Many Feng Shui masters, however, began to discover that problems or inaccuracies arose when conventional Feng Shui rules of the northern hemisphere were directly applied in southern locations. Obviously, such considerations were not included in any classical Feng Shui literature because all the ancient Feng Shui masters had cultivated their craft in Mainland China which is in the northern hemisphere. Thus, a new theory and a new method was proposed by these masters.

It is common knowledge that the effect of night and day is due to the rotation of the Earth, and it takes a year for the Earth to orbit the sun in more or less a circle. Also, it is generally understood that the Earth's rotation is actually tilted, creating the effect of seasonal changes. Because of the tilted rotation, one hemisphere will be in the sun's light for longer than the other hemisphere will be, creating the effect of summer in one hemisphere (such as China) while locations in the other hemisphere (such as Australia) experience winter. As the Earth progresses round the sun, the time of facing the sun of both hemispheres gradually changes until the time becomes equal. This point is the equinox, where day and night are of equal length at any place on the earth. Thereafter the situation is reversed, with Australia basking in sunshine while China becomes plunged into winter.

Classical Feng Shui masters maintain that the sun is the energy source of the Earth. The influences of the eight trigrams were configured for China, a temperate area in the northern hemisphere with its energy profile opposite to that of Australia. So, some Feng Shui masters proposed the "mirror approach." The concept is that no matter in the northern or southern hemisphere, people experience the same direction of movement of the sun on a daily basis,

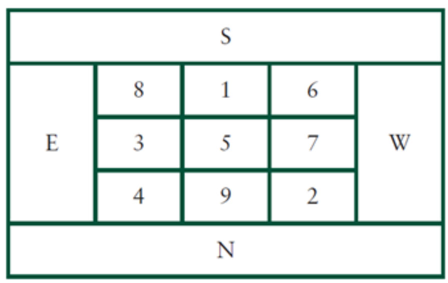

Figure 8(a) Luo Shu for the Southern Hemisphere

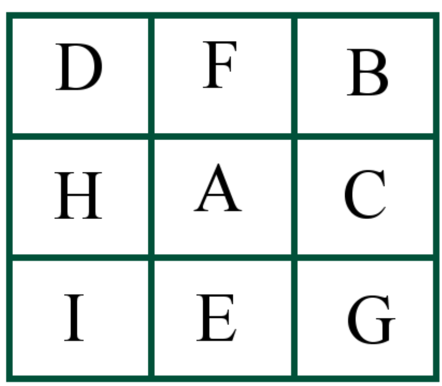

Figure 8(b) Trajectory of Forward Flying in the Souther Hemisphere

i.e., from east to west. On the other hand, they experience a converse direction of gradual movement on a seasonal basis. Therefore, while the east and west of the Eight Trigrams remain unchanged for both hemispheres, those of the north and south are inverted. This is called the "Mirror Postulate," as if observers in Australia or New Zealand were to look at the classic Chinese arrangement of the Eight Trigrams through a mirror. This approach was also supported by two western scholars (Brown, 1997; Essen, 2000) who suggested that in the southern hemisphere, the north and south directions of the Feng Shui model should be reversed while keeping that of the east and west directions unchanged. Because of this, thirty six new charts are formulated, denoted as **SFS**(x+, D^o), where x runs from 1 to 9 ad D could be one of 0, 90, 180 or 270. The rotation, D^o, is also clockwise as in the northern hemisphere. For example, the standard Luo Shu now becomes **SFS**(5+, 0^o) in the southern hemisphere as shown in Figure 8(a) while the trajectory is shown in Figure 8(b). Here, "S" denotes south; "E" denotes east; "N" denotes north and "W" denotes west.

UNIQUENESS OF LUO SHU

Up to this stage, 72 charts have been generated from the standard Luo Shu, i.e. thirty six **FS**' for the northern hemisphere, i.e. **FS**(x+, D^o) for x = 1 to 9 and D = 0, 90, 180 or 270, and thirty **SFS**' for the southern hemisphere, i.e. **SFS** (x+, D^o) for x = 1 to 9 and D = 0, 90, 180 or 270. In this section, we start to check whether these seventy two charts were merely randomly picked by ancestors of China who developed Feng Shui. It is straight forward that if nine integers, running from 1 to 9, are randomly filled into a 3 x 3 grid, there are 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 362,880 combinations. In the previous article (So et al 2015), the authors found that the thirty six **FS**' of the northern hemisphere possess some features. First, the absolute values of their determinants are either 36, 144 or 360, which can be mathematically proved in a later section. Second, as stated above, **FS**(x-, D^o) = **FS**(x+, $D^o\pm180^o$). Therefore, from herein onward, the positive sign after x can be neglected. It is assumed positive without the sign. These two features are used to assess all the 362,880 matrices and totally seventy two plus one hundred and twelve matrices have been identified. Half of the seventy two matrices belong to **FS'** and half to **SFS'**. Third, matrices are finally checked for similarity using the approach mentioned in the previous article (So et al 2015). The criteria to judge that two matrices are similar are:

i) their determinants must be identical both in value and in sign; and

ii) their eigenvalues must be identical; and

iii) their eigenvectors must look similar, i.e. in the form of "$\pm a \pm b$ **i**".

Nine observations have been concluded for the seventy two matrices, each being illustrated by one example only, due to the limitation of page length. All examples are with the same format of presentation, shown in Figure 9. The first three columns show the matrix. Here, it is **FS**(5, 0^o), the original Luo Shu. The subsequent two columns show the three eigenvectors of the 1st eigenvalue; the next two columns show the three eigenvectors of the 2nd eigenvalue; the next two columns show the three eigenvectors of the 3rd eigenvalue. The two columns that follow show the three eigenvalues of the matrix and the last column shows the determinant. A new matrix design was proposed in authors' previous article (So et al 2015), i.e. **FS**(x-5, D^o) or **SFS**(x-5, D^o)which is not used by Feng Shui masters. This new **FS** or **SFS**, with all the nine original elements [1, 2, 3, 4, 5, 6, 7, 8, 9] replaced by [1-5 = -4, -3, -2, -1, 0, 1, 2, 3, 4] respectively has merely been established for further analysis by linear algebra while they have no meaning in Feng Shui. An immediate impression is that this new set of **FS'** or **SFS'** looks more symmetrical.

FS or **SFS**			**1st Eigenvectors**		**2nd Eigenvectors**		**3rd Eigenvectors**		**3 Eigenvalues**		**Det**
4	9	2	0.5774	0	0.6236	0	0.6236	0	15	0	
3	5	7	0.5774	0	-0.1782	0.4364	-0.1782	-0.4364	0	4.8990	360
8	1	6	0.5774	0	-0.4454	-0.4364	-0.4454	0.4364	0	-4.8990	

Figure 9 Format of Matrix Presentation

Observation 1: All **FS**(x, D^{o}) are similar in groups for x = "1, 4, 7" or "2, 5, 8" or "3, 6, 9" , i.e. Three Big Trigrams, with D = 90 or 270, Figure 10 being referred to.

Observation 2: No **FS**(x, D^{o}) is similar in groups for x = "1, 4, 7" or "2, 5, 8" or "3, 6, 9" , i.e. Three Big Trigrams, with D = 0 or 180, except same values of determinants, Figure 11 being referred to.

Observation 3: All **SFS**(x, D^{o}) are similar in groups for x = "1, 4, 7" or "2, 5, 8" or "3, 6, 9" i.e. Three Big Trigrams, with D = 0 or 180, Figure 12 being referred to.

Observation 4: No **SFS**(x, D^{o}) is similar in groups for x = "1, 4, 7" or "2, 5, 8" or "3, 6, 9" , i.e. Three Big Trigrams, with D = 90 or 270, except same values of determinants, Figure 13 being referred to.

Observation 5: No similarity for all **FS**(x, D^{o}) and **SFS**(x, D^{o}) in groups for x = "1, 9" or "3, 7" or "4, 6" , i.e. Combined to Ten, with D = 0, 90, 180 of 270 has been found.

Observation 6: All **FS**(x-5, D^{o}) are similar in groups for x = "1, 4, 7" or "2, 5, 8" or "3, 6, 9" , i.e. Three Big Trigrams, with D = 90 or 270, Figure 14 being referred to.

Observation 7: All **FS**(x-5, D^{o}) are similar in groups for x = "1, 9" or "2, 8" or "3, 7" or "4, 6" , i.e. Combined to Ten, with D = 0 or 180, same absolute values of determinants only, Figure 15 being referred to.

Observation 8: All **SFS**(x-5, D^o) are similar in groups for x = "1, 4, 7" or "2, 5, 8" or "3, 6, 9" , i.e. Three Big Trigrams, with D = 0 or 180, Figure 16 being referred to.

Observation 9: All **SFS**(x-5, D^o) are similar in groups for x = "1, 9" or "2, 8" or "3, 7" or "4, 6" , i.e. Combined to Ten, with D = 90 or 270, same absolute values of determinants only, Figure 17 being referred to.

Regarding the remaining 112 candidates that do not belong to the family of **FS** or **SFS** but suit the first two criteria are checked for similarity. Unfortunately, no similarity has been found and this is the tenth observation. Moreover, they are not symmetrical regarding the distribution density from the central integer's point of view. Out of the 112 candidates, there is no matrix with either "1" or "5" at the central palace. Sixteen candidates are with "2" at the central palace, sixteen with "3" at the central palace, sixteen with "4" at the central palace, twenty four with "6" at the central palace, sixteen with "7" at the central palace, sixteen with "8" at the central palace and eight with "9" at the central palace, totally 112.

LIMITED VALUES OF LUO SHU DETERMINANTS

It was stated before that the first criterion for shortlisting matrices of interest out of the 362,880 combinations is a determinant with absolute value equal to 36, 144 or 360. Here is the mathematical illustration to show why the thirty six **FS**' and the thirty six **SFS**' could possess such characteristics. Let's define TBT (Three Big Trigram Constant) = 1x4x7+2x5x8+3x6x9 = 270.

For the northern hemisphere and for both D = 0 or 180, it can be shown that

FS(1+,0^o)=**FS**(1+,180^o)=**FS**(4+,0^o)=**FS**(4+,180^o)=**FS**(7+,0^o)=**FS**(7+,180^o)

= 9x$\underline{1}$x2+3x$\underline{4}$x5+6x$\underline{7}$x8 – TBT = 144;

FS(2+,0^o)=**FS**(2+,180^o)=**FS**(5+,0^o)=**FS**(5+,180^o)=**FS**(8+,0^o)=**FS**(8+,180^o)

= 1x2x3+4x5x6+7x8x9 – TBT = 360;
FS(3+,0°)=**FS**(3+,180°)=**FS**(6+,0°)=**FS**(6+,180°)=**FS**(9+,0°)=**FS**(9+,180°)
= 2x3x4+5x6x7+8x9x1 – TBT = 36.

For the northern hemisphere and for both *D* = 90 or 270, it can be shown that

FS(1+,90°)=**FS**(1+,270°)=**FS**(4+,90°)=**FS**(4+,270°)=**FS**(7+,90°)=**FS**(7+,270°)
= TBT – 9x1x2-3x4x5-6x7x8 = -144;
FS(2+,90°)=**FS**(2+,270°)=**FS**(5+,90°)=**FS**(5+,270°)=**FS**(8+,90°)=**FS**(8+,270°)
= TBT – 1x2x3-4x5x6-7x8x9 = -360;
FS(3+,90°)=**FS**(3+,270°)=**FS**(6+,90°)=**FS**(6+,270°)=**FS**(9+,90°)=**FS**(9+,270°)
=TBT – 2x3x4-5x6x7-8x9x1 = -36.

For the southern hemisphere and for both *D* = 0 or 180, it can be shown that

SFS(1+,0°)=**SFS**(1+,180°)=**SFS**(4+,0°)=**SFS**(4+,180°)=**SFS**(7+,0°)=**SFS**(7+,180°)
= TBT - 9x1x2-3x4x5-6x7x8 = -144;
SFS(2+,0°)=**SFS**(2+,180°)=**SFS**(5+,0°)=**SFS**(5+,180°)=**SFS**(8+,0°)=**SFS**(8+,180°)
= TBT - 1x2x3+4x5x6+7x8x9 = -360;
SFS(3+,0°)=**SFS**(3+,180°)=**SFS**(6+,0°)=**SFS**(6+,180°)=**SFS**(9+,0°)=**SFS**(9+,180°)
= 2x3x4+5x6x7+8x9x1 – TBT = -36.

For the southern hemisphere and for both *D* = 90 or 270, it can be shown that

SFS(1+,90°)=S**FS**(1+,270°)=S**FS**(4+,90°)=**SFS**(4+,270°)=**SFS**(7+,90°)=**SFS**(7+,270°)
= 9x1x2+3x4x5+6x7x8 - TBT = 144;
SFS(2+,90°)=**SFS**(2+,270°)=**SFS**(5+,90°)=**SFS**(5+,270°)=**SFS**(8+,90°)=**SFS**(8+,270°)
= 1x2x3+4x5x6+7x8x9 = 360;
SFS(3+,90°)=**SFS**(3+,270°)=**SFS**(6+,90°)=**SFS**(6+,270°)=**SFS**(9+,90°)=**SFS**(9+,270°)
= 2x3x4+5x6x7+8x9x1 = 36.

Based on such patterns, readers may easily notice that the flying patterns of the 72 variations of the original Luo Shu, itself inclusive, are actually taking care of the calculation of determinants.

CONCLUSION

The nine observations mentioned in this article actually summarize all observations mentioned in the previous article (So et al 2015) with the addition of those used in the northern hemisphere. By using the two criteria, i.e. absolute value of determinants equal to 36, 144 or 360, and rotation of a matrix by 180^o switching a matrix between forward and backward flying, 72 plus 112 matrices have been shortlisted. The 72 candidates have been found to belong to the **FS** (northern hemisphere) or the **SFS** (southern hemisphere) family while lots of similarity have been observed. The remaining 112 matrices have no similarity among themselves at all. That implies the 72 candidates, 36 plus 36, were not randomly selected by the Chinese ancestors in Feng Shui study. They are unique and they autonomously form a complete system or tool for possibly diagnosing the environment. In particular, the specific meaning in terms of the "Three Big Trigrams", i.e. "1, 4, 7", "2, 5, 8" and "3, 6, 9" has been an important concept to Feng Shui study.

Another characteristic, "Combined to Ten", was also illustrated by observation (7) and (9) in a previous section of this article when x-5 is used instead of x. The "Combined to Ten" concept is also important, and has been less controversial among Feng Shui masters during the past century. But as stated in authors' previous article, similarities in **FS**(x-5, D^o) or **SFS**(x-5, D^o) are much more than that in the normal matrices without subtraction by five, and therefore these matrices are only generated for analysis, not recommended for use in Feng Shui.

At this moment, the authors have not yet obtained any evidence that such 72 matrices could tell whether a site is auspicious or not as claimed by Feng Shui masters, which should be further studied by either the authors or scholars who have read these two articles with interest. At least, we are able to show that a unique platform is available for us to manipulate.

As mentioned in the introduction, the original Luo Shu was not uniquely found in ancient China. It was independently suggested in the western world as a magic square. However, throughout the millennia, the magic square has not been varied or expanded to do anything in the western world while up to now, it can only be treated as an element in recreational mathematics. However, in China, pioneers in Feng Shui study did expand such magic square to up to 36 variations and then further 36 by modern Feng Shui masters living in the southern hemisphere. As a matter of fact, most modern Feng Shui practitioners are now using 18 **FS**' and 18 **SFS**' only where D = 0 or 180. It is hoped that up to this stage, the Luo Shu mathematical foundation has been established and researchers could use it to study its application in divination including the Nine Palace Flying Stars Method and Qi Mun Dun Jia Method etc. in a more mathematical and logical manner.

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FS(1, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
4	8	9	-0.7582	0.0000	-0.6295	0.0000	-0.6044	0.0000	14.2394	0.0000	
6	1	5	-0.5011	0.0000	0.7715	0.0000	-0.5335	0.0000	-4.4911	0.0000	-144
2	3	7	-0.4171	0.0000	-0.0919	0.0000	0.5917	0.0000	2.2517	0.0000	

FS(4, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
7	2	3	0.4171	0.0000	0.5917	0.0000	-0.0919	0.0000	14.2394	0.0000	
9	4	8	0.7582	0.0000	-0.6044	0.0000	-0.6295	0.0000	2.2517	0.0000	-144
5	6	1	0.5011	0.0000	-0.5335	0.0000	0.7715	0.0000	-4.4911	0.0000	

FS(7, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
1	5	6	-0.5011	0.0000	-0.7715	0.0000	0.5335	0.0000	14.2394	0.0000	
3	7	2	-0.4171	0.0000	0.0919	0.0000	-0.5917	0.0000	-4.4911	0.0000	-144
8	9	4	-0.7582	0.0000	0.6295	0.0000	0.6044	0.0000	2.2517	0.0000	

FS(1, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
7	3	2	0.4171	0.0000	0.5917	0.0000	0.0919	0.0000	14.2394	0.0000	
5	1	6	0.5011	0.0000	-0.5335	0.0000	-0.7715	0.0000	2.2517	0.0000	-144
9	8	4	0.7582	0.0000	-0.6044	0.0000	0.6295	0.0000	-4.4911	0.0000	

FS(4, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
1	6	5	-0.5011	0.0000	-0.7715	0.0000	-0.5335	0.0000	14.2394	0.0000	
8	4	9	-0.7582	0.0000	0.6295	0.0000	-0.6044	0.0000	-4.4911	0.0000	-144
3	2	7	-0.4171	0.0000	0.0919	0.0000	0.5917	0.0000	2.2517	0.0000	

FS(7, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
4	9	8	-0.7582	0.0000	-0.6295	0.0000	0.6044	0.0000	14.2394	0.0000	
2	7	3	-0.4171	0.0000	-0.0919	0.0000	-0.5917	0.0000	-4.4911	0.0000	-144
6	5	1	-0.5011	0.0000	0.7715	0.0000	0.5335	0.0000	2.2517	0.0000	

Figure 10 Examples of Observation 1

FS(1, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
9	5	7	0.7626	0.0000	-0.2797	-0.2552	-0.2797	0.2552	16.0968	0.0000	
8	1	3	0.4884	0.0000	-0.3185	0.4419	-0.3185	-0.4419	-2.0484	2.1794	144
4	6	2	0.4242	0.0000	0.7483	0.0000	0.7483	0.0000	-2.0484	-2.1794	

FS(4, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	8	1	0.4089	0.0000	0.7281	0.0000	0.7281	0.0000	14.7059	0.0000	
2	4	6	0.5031	0.0000	-0.3909	0.3149	-0.3909	-0.3149	-1.3529	2.8216	144
7	9	5	0.7614	0.0000	-0.0420	-0.4648	-0.0420	0.4648	-1.3529	-2.8216	

FS(7, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
6	2	4	-0.4004	0.0000	-0.7264	0.0000	-0.0384	0.0000	14.4047	0.0000	
5	7	9	-0.8042	0.0000	-0.4304	0.0000	-0.8794	0.0000	4.2345	0.0000	144
1	3	8	-0.4392	0.0000	0.5358	0.0000	0.4746	0.0000	2.3608	0.0000	

FS(1, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
2	6	4	0.4242	0.0000	0.7483	0.0000	0.7483	0.0000	16.0968	0.0000	
3	1	8	0.4884	0.0000	-0.3185	0.4419	-0.3185	-0.4419	-2.0484	2.1794	144
7	5	9	0.7626	0.0000	-0.2797	-0.2552	-0.2797	0.2552	-2.0484	-2.1794	

FS(4, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
5	9	7	0.7614	0.0000	-0.0420	-0.4648	-0.0420	0.4648	14.7059	0.0000	
6	4	2	0.5031	0.0000	-0.3909	0.3149	-0.3909	-0.3149	-1.3529	2.8216	144
1	8	3	0.4089	0.0000	0.7281	0.0000	0.7281	0.0000	-1.3529	-2.8216	

FS(7, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
8	3	1	-0.4392	0.0000	-0.4746	0.0000	-0.5358	0.0000	14.4047	0.0000	
9	7	5	-0.8042	0.0000	0.8794	0.0000	0.4304	0.0000	2.3608	0.0000	144
4	2	6	-0.4004	0.0000	0.0384	0.0000	0.7264	0.0000	4.2345	0.0000	

Figure 11 Examples of Observation 2

SFS(3, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
6	8	4	0.6466	0.0000	0.9678	0.0000	0.6117	0.0000	14.5246	0.0000	
1	3	5	0.3501	0.0000	-0.1220	0.0000	-0.6946	0.0000	4.0825	0.0000	-36
2	7	9	0.6777	0.0000	-0.2200	0.0000	0.3787	0.0000	-0.6071	0.0000	

SFS(6, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
9	2	7	-0.6777	0.0000	-0.2200	0.0000	-0.3787	0.0000	14.5246	0.0000	
4	6	8	-0.6466	0.0000	0.9678	0.0000	-0.6117	0.0000	4.0825	0.0000	-36
5	1	3	-0.3501	0.0000	-0.1220	0.0000	0.6946	0.0000	-0.6071	0.0000	

SFS(9, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	5	1	-0.3501	0.0000	-0.6946	0.0000	-0.1220	0.0000	14.5246	0.0000	
7	9	2	-0.6777	0.0000	0.3787	0.0000	-0.2200	0.0000	-0.6071	0.0000	-36
8	4	6	-0.6466	0.0000	0.6117	0.0000	0.9678	0.0000	4.0825	0.0000	

SFS(3, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
9	7	2	-0.6777	0.0000	-0.2200	0.0000	0.3787	0.0000	14.5246	0.0000	
5	3	1	-0.3501	0.0000	-0.1220	0.0000	-0.6946	0.0000	4.0825	0.0000	-36
4	8	6	-0.6466	0.0000	0.9678	0.0000	0.6117	0.0000	-0.6071	0.0000	

SFS(6, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	1	5	-0.3501	0.0000	-0.6946	0.0000	0.1220	0.0000	14.5246	0.0000	
8	6	4	-0.6466	0.0000	0.6117	0.0000	-0.9678	0.0000	-0.6071	0.0000	-36
7	2	9	-0.6777	0.0000	0.3787	0.0000	0.2200	0.0000	4.0825	0.0000	

SFS(9, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
6	4	8	0.6466	0.0000	0.9678	0.0000	-0.6117	0.0000	14.5246	0.0000	
2	9	7	0.6777	0.0000	-0.2200	0.0000	-0.3787	0.0000	4.0825	0.0000	-36
1	5	3	0.3501	0.0000	-0.1220	0.0000	0.6946	0.0000	-0.6071	0.0000	

Figure 12 Examples of Observation 3

SFS(1, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
2	3	7	0.4727	0.0000	0.0334	0.4732	0.0334	-0.4732	16.0968	0.0000	144
6	1	5	0.4406	0.0000	0.6988	0.0000	0.6988	0.0000	-2.0484	2.1794	
4	8	9	0.7631	0.0000	-0.4662	-0.2633	-0.4662	0.2633	-2.0484	-2.1794	

SFS(4, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
5	6	1	0.5063	0.0000	0.4689	0.2946	0.4689	-0.2946	14.7059	0.0000	144
9	4	8	0.7473	0.0000	-0.6157	0.0000	-0.6157	0.0000	-1.3529	2.8216	
7	2	3	0.4304	0.0000	-0.1155	-0.5486	-0.1155	0.5486	-1.3529	-2.8216	

SFS(7, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
8	9	4	-0.8289	0.0000	-0.4497	0.0000	-0.3745	0.0000	14.4047	0.0000	144
3	7	2	-0.4319	0.0000	0.5827	0.0000	-0.2412	0.0000	2.3608	0.0000	
1	5	6	-0.3555	0.0000	-0.6769	0.0000	0.8953	0.0000	4.2345	0.0000	

SFS(1, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
9	8	4	0.7631	0.0000	-0.4662	-0.2633	-0.4662	0.2633	16.0968	0.0000	144
5	1	6	0.4406	0.0000	0.6988	0.0000	0.6988	0.0000	-2.0484	2.1794	
7	3	2	0.4727	0.0000	0.0334	0.4732	0.0334	-0.4732	-2.0484	-2.1794	

SFS(4, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	2	7	-0.4304	0.0000	0.1155	0.5486	0.1155	-0.5486	14.7059	0.0000	144
8	4	9	-0.7473	0.0000	0.6157	0.0000	0.6157	0.0000	-1.3529	2.8216	
1	6	5	-0.5063	0.0000	-0.4689	-0.2946	-0.4689	0.2946	-1.3529	-2.8216	

SFS(7, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
6	5	1	-0.3555	0.0000	-0.8953	0.0000	0.6769	0.0000	14.4047	0.0000	144
2	7	3	-0.4319	0.0000	0.2412	0.0000	-0.5827	0.0000	4.2345	0.0000	
4	9	8	-0.8289	0.0000	0.3745	0.0000	0.4497	0.0000	2.3608	0.0000	

Figure 13 Examples of Observation 4

FS(-3, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
0	4	-4	0.6667	0.0000	0.5774	0.0000	-0.6667	0.0000	4.8990	0.0000	
2	-3	1	0.0749	0.0000	0.5774	0.0000	0.7416	0.0000	0.0000	0.0000	0
-2	-1	3	-0.7416	0.0000	0.5774	0.0000	-0.0749	0.0000	-4.8990	0.0000	

FS(0, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	-2	-1	0.0749	0.0000	0.7416	0.0000	0.5774	0.0000	-4.8990	0.0000	
-4	0	4	0.6667	0.0000	-0.6667	0.0000	0.5774	0.0000	4.8990	0.0000	0
1	2	-3	-0.7416	0.0000	-0.0749	0.0000	0.5774	0.0000	0.0000	0.0000	

FS(3, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-3	1	2	-0.0749	0.0000	0.7416	0.0000	0.5774	0.0000	4.8990	0.0000	
-1	3	-2	0.7416	0.0000	-0.0749	0.0000	0.5774	0.0000	-4.8990	0.0000	0
4	-4	0	-0.6667	0.0000	-0.6667	0.0000	0.5774	0.0000	0.0000	0.0000	

FS(-3, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	-1	-2	0.0749	0.0000	-0.7416	0.0000	0.5774	0.0000	-4.8990	0.0000	
1	-3	2	-0.7416	0.0000	0.0749	0.0000	0.5774	0.0000	4.8990	0.0000	0
-4	4	0	0.6667	0.0000	0.6667	0.0000	0.5774	0.0000	0.0000	0.0000	

FS(0, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-3	2	1	-0.0749	0.0000	-0.7416	0.0000	0.5774	0.0000	4.8990	0.0000	
4	0	-4	-0.6667	0.0000	0.6667	0.0000	0.5774	0.0000	-4.8990	0.0000	0
-1	-2	3	0.7416	0.0000	0.0749	0.0000	0.5774	0.0000	0.0000	0.0000	

FS(3, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
0	-4	4	-0.6667	0.0000	-0.5774	0.0000	-0.6667	0.0000	-4.8990	0.0000	
-2	3	-1	-0.0749	0.0000	-0.5774	0.0000	0.7416	0.0000	0.0000	0.0000	0
2	1	-3	0.7416	0.0000	-0.5774	0.0000	-0.0749	0.0000	4.8990	0.0000	

Figure 14 Examples of Observation 6

SFS(-4, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-1	1	-3	-0.1914	0.5549	-0.1914	-0.5549	0.1769	0.0000	0.8781	3.2530	
3	-4	-2	-0.1176	0.4197	-0.1176	-0.4197	-0.9786	0.0000	0.8781	-3.2530	-54
4	0	2	0.6823	0.0000	0.6823	0.0000	-0.1047	0.0000	-4.7563	0.0000	

SFS(4, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-2	0	-4	0.6823	0.0000	0.6823	0.0000	-0.1047	0.0000	-0.8781	3.2530	
2	4	-3	-0.1176	-0.4197	-0.1176	0.4197	-0.9786	0.0000	-0.8781	-3.2530	54
3	-1	1	-0.1914	-0.5549	-0.1914	0.5549	0.1769	0.0000	4.7563	0.0000	

SFS(-1, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
2	4	0	0.6823	0.0000	0.6823	0.0000	0.1047	0.0000	0.8781	3.2530	
-3	-1	1	-0.1914	0.5549	-0.1914	-0.5549	-0.1769	0.0000	0.8781	-3.2530	-54
-2	3	-4	-0.1176	0.4197	-0.1176	-0.4197	0.9786	0.0000	-4.7563	0.0000	

SFS(1, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
4	-3	2	0.9786	0.0000	0.1176	0.4197	0.1176	-0.4197	4.7563	0.0000	
-1	1	3	-0.1769	0.0000	0.1914	0.5549	0.1914	-0.5549	-0.8781	3.2530	54
0	-4	-2	0.1047	0.0000	-0.6823	0.0000	-0.6823	0.0000	-0.8781	-3.2530	

SFS(-4, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
2	0	4	0.6823	0.0000	0.6823	0.0000	-0.1047	0.0000	0.8781	3.2530	
-2	-4	3	-0.1176	0.4197	-0.1176	-0.4197	-0.9786	0.0000	0.8781	-3.2530	-54
-3	1	-1	-0.1914	0.5549	-0.1914	-0.5549	0.1769	0.0000	-4.7563	0.0000	

SFS(4, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
1	-1	3	-0.1914	-0.5549	-0.1914	0.5549	0.1769	0.0000	-0.8781	3.2530	
-3	4	2	-0.1176	-0.4197	-0.1176	0.4197	-0.9786	0.0000	-0.8781	-3.2530	54
-4	0	-2	0.6823	0.0000	0.6823	0.0000	-0.1047	0.0000	4.7563	0.0000	

SFS(-1, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-4	3	-2	0.9786	0.0000	0.1176	-0.4197	0.1176	0.4197	-4.7563	0.0000	
1	-1	-3	-0.1769	0.0000	0.1914	-0.5549	0.1914	0.5549	0.8781	3.2530	-54
0	4	2	0.1047	0.0000	-0.6823	0.0000	-0.6823	0.0000	0.8781	-3.2530	

SFS(1, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-2	-4	0	0.6823	0.0000	0.6823	0.0000	0.1047	0.0000	-0.8781	3.2530	
3	1	-1	-0.1914	-0.5549	-0.1914	0.5549	-0.1769	0.0000	-0.8781	-3.2530	54
2	-3	4	-0.1176	-0.4197	-0.1176	0.4197	0.9786	0.0000	4.7563	0.0000	

Figure 15 Examples of Observation 7

SFS(-3, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	1	-4	-0.8131	0.0000	0.5774	0.0000	0.3416	0.0000	4.8990	0.0000	
-1	-3	4	0.3416	0.0000	0.5774	0.0000	-0.8131	0.0000	0.0000	0.0000	0
-2	2	0	0.4714	0.0000	0.5774	0.0000	0.4714	0.0000	-4.8990	0.0000	

SFS(0, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	-4	1	-0.8131	0.0000	0.5774	0.0000	-0.3416	0.0000	4.8990	0.0000	
-2	0	2	0.4714	0.0000	0.5774	0.0000	-0.4714	0.0000	0.0000	0.0000	0
-1	4	-3	0.3416	0.0000	0.5774	0.0000	0.8131	0.0000	-4.8990	0.0000	

SFS(3, 0°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-3	-1	4	0.8131	0.0000	-0.5774	0.0000	0.3416	0.0000	-4.8990	0.0000	
1	3	-4	-0.3416	0.0000	-0.5774	0.0000	-0.8131	0.0000	0.0000	0.0000	0
2	-2	0	-0.4714	0.0000	-0.5774	0.0000	0.4714	0.0000	4.8990	0.0000	

SFS(-3, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	1	-4	-0.8131	0.0000	0.5774	0.0000	0.3416	0.0000	4.8990	0.0000	
-1	-3	4	0.3416	0.0000	0.5774	0.0000	-0.8131	0.0000	0.0000	0.0000	0
-2	2	0	0.4714	0.0000	0.5774	0.0000	0.4714	0.0000	-4.8990	0.0000	

SFS(0, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-3	4	-1	0.8131	0.0000	-0.5774	0.0000	-0.3416	0.0000	-4.8990	0.0000	
2	0	-2	-0.4714	0.0000	-0.5774	0.0000	-0.4714	0.0000	0.0000	0.0000	0
1	-4	3	-0.3416	0.0000	-0.5774	0.0000	0.8131	0.0000	4.8990	0.0000	

SFS(3, 180°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
0	-2	2	0.4714	0.0000	0.4714	0.0000	0.5774	0.0000	-4.8990	0.0000	
-4	3	1	0.3416	0.0000	-0.8131	0.0000	0.5774	0.0000	4.8990	0.0000	0
4	-1	-3	-0.8131	0.0000	0.3416	0.0000	0.5774	0.0000	0.0000	0.0000	

Figure 16 Examples of Observation 8

SFS(-3, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-2	-1	3	0.5774	0.0000	0.6667	0.0000	0.6667	0.0000	0.0000	0.0000	
2	-3	1	0.5774	0.0000	-0.0833	-0.3227	-0.0833	0.3227	-4.5000	1.9365	0
0	4	-4	0.5774	0.0000	-0.5833	0.3227	-0.5833	-0.3227	-4.5000	-1.9365	

SFS(3, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
4	-4	0	-0.5774	0.0000	0.6667	0.0000	0.6667	0.0000	0.0000	0.0000	
-1	3	-2	-0.5774	0.0000	-0.0833	-0.3227	-0.0833	0.3227	4.5000	1.9365	0
-3	1	2	-0.5774	0.0000	-0.5833	0.3227	-0.5833	-0.3227	4.5000	-1.9365	

SFS(-2, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-1	0	4	0.6813	0.0000	0.4789	0.3736	0.4789	-0.3736	-5.1283	0.0000	
3	-2	2	-0.2038	0.0000	0.6208	0.0000	0.6208	0.0000	-0.4358	3.2156	-54
1	-4	-3	-0.7031	0.0000	-0.2328	0.4377	-0.2328	-0.4377	-0.4358	-3.2156	

SFS(2, 90°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
3	4	-1	0.2328	0.4377	0.2328	-0.4377	-0.7031	0.0000	0.4358	3.2156	
-2	2	-3	-0.6208	0.0000	-0.6208	0.0000	-0.2038	0.0000	0.4358	-3.2156	54
-4	0	1	-0.4789	0.3736	-0.4789	-0.3736	0.6813	0.0000	5.1283	0.0000	

SFS(-3, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-4	4	0	-0.5774	0.0000	0.6667	0.0000	0.6667	0.0000	0.0000	0.0000	
1	-3	2	-0.5774	0.0000	-0.0833	0.3227	-0.0833	-0.3227	-4.5000	1.9365	0
3	-1	-2	-0.5774	0.0000	-0.5833	-0.3227	-0.5833	0.3227	-4.5000	-1.9365	

SFS(3, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
2	1	-3	0.5774	0.0000	0.6667	0.0000	0.6667	0.0000	0.0000	0.0000	
-2	3	-1	0.5774	0.0000	-0.0833	0.3227	-0.0833	-0.3227	4.5000	1.9365	0
0	-4	4	0.5774	0.0000	-0.5833	-0.3227	-0.5833	0.3227	4.5000	-1.9365	

SFS(-2, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
-3	-4	1	0.2328	-0.4377	0.2328	0.4377	-0.7031	0.0000	-0.4358	3.2156	
2	-2	3	-0.6208	0.0000	-0.6208	0.0000	-0.2038	0.0000	-0.4358	-3.2156	-54
4	0	-1	-0.4789	-0.3736	-0.4789	0.3736	0.6813	0.0000	-5.1283	0.0000	

SFS(2, 270°)			1st Eigenvectors		2nd Eigenvectors		3rd Eigenvectors		3 Eigenvalues		Det
1	0	-4	0.6813	0.0000	0.4789	-0.3736	0.4789	0.3736	5.1283	0.0000	
-3	2	-2	-0.2038	0.0000	0.6208	0.0000	0.6208	0.0000	0.4358	3.2156	54
-1	4	3	-0.7031	0.0000	-0.2328	-0.4377	-0.2328	0.4377	0.4358	-3.2156	

Figure 17 Examples of Observation 9