EUROPEAN ACADEMIC RESEARCH
Vol. IV, Issue 1/ April 2016

Impact Factor: 3.4546 (UIF)
DRJI Value: 5.9 (B+)

ISSN 2286-4822
www.euacademic.org

On Artin cokernel of the Group($Q_{2m}\times C_5$) where $m=p_1.p_2$, g.c.d$(p_1,p_2) = 1$,p_1, $p_2>2$ and p_1, p_2 are primes numbers

Ass. Prof. NASERR RASOOL MAHMOOD
University of Kufa
Faculty of Education for Girls
Department of Mathematics
SALAH HSAAOUN JIHADI
University of Kufa
Faculty of Education for Girls
Department of Mathematics

Abstract:

The main purpose of This paper is to find Artin's character table $Ar(Q_{2m}\times C_5)$ when m is an 0dd number such that $m=p_1.p_2$,p_1, $p_2>2$ where g.c.d$(p_1,p_2) = 1$ and p_1, p_2 are primes numbers; where Q_{2m} is denoted to Quaternion group of order 4m, time is said to have only one dimension and space to have three dimension ,the mathematical quaternion partakes of both these elements ; in technical language it may be said to be "time plus space" ,or "space plus time" and in this sense it has , or at least involves a reference to four dimensions ,and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols.,1882,1885,1889)) ,andC_5 is Cyclic group of order 5.In 1962, C. W. Curits & I. Reiner studied Representation Theory of finite groups ,In 1976 ,I. M. Isaacs studied Charactrs Theory of finite groups, In 1982 , M. S. Kirdar studied The Factor Group of the Z-Valued class function modulo the group of the Generalized Characters, In 1995,N.R.Mahmood studies The Cyclic Decomposition of the factor Group cf(Q_{2m},Z)/ $\overline{R}$(G) (Q_{2m}), In 2002 ,K-Sekiguchi studies Extensions and the Irreducibilies of the Induced

Characters of Cyclic P-Group , In 2008, A.H.Abdul-Munem studied Artin Cokernel of The Quaternion group Q_{2m} when m is an Odd number, In 2006, A.S. Abed found the Artin characters table of dihedral group D_n when n is an odd number.

Key words: odd number, prime number, Quaternion group, and Cyclic group

1. INTRODUCTION:

Representation Theory is a branch of mathematics that studies abstract algebra structures by Representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication in which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication, Moreover, representation and character theory provide applications ,not only in other branches of mathematics but also in physics and chemistry.

Fore a finite group **G** ,The factor group $\overline{R}$**(G) /T(G)** is called the Artin cokernel of **G** denoted **AC**(**G**), $\overline{R}(G)$ denoted the a belian group generated by Z-valued characters of G under the operation of point wise addition, **T(G**) is a sub group of $\overline{R}$ **(G)** which is generated by Artin's characters.

2-PRELIMINARS: (3,1) :[1]

The Generalized Quaternion Group Q_{2m}: For each positive integer m≥2 ,The generalized Quaternion Group Q_{2m} of order 4m with two generators x and y satisfies Q_{2m} ={x^h y^k,0≤h≤2m-

1,k=0,1}Which has the following properties {$x^{2m}=y^4=I, yx^my^{-1}=x^{-m}$}.Let G be a finite group ,all the characters of group G induced from a principal character of cyclic subgroup of G are called Artin characters of G . Artin characters of the finite group can be displayed in a table called Artin characters table of G which is denoted by Ar(G); The first row is Γ-conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G(CL_\alpha)|$ and other rows contains the values of Artin characters.

Theorem: (3,2): [2]

The general form of Artin characters table of Cp^s When p is a prime number and s is a positive integer number is given by :-

Ar(Cp^s)=

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	⋯	$[x]$
$\lvert CL_\alpha \rvert$	1	1	1	1	⋯	1
$\lvert C_{p^s}(CL_\alpha)\rvert$	p^s	p^s	p^s	p^s	⋯	p^s
φ'_1	p^s	0	0	0	⋯	0
φ'_2	p^{s-1}	p^{s-1}	0	0	⋯	0
φ'_3	p^{s-2}	p^{s-2}	p^{s-2}	0	⋯	0
⋮	⋮	⋮	⋮	⋮	⋮	
φ'_s	p^1	p^1	p^1	p^1	⋯	0
φ'_{s+1}	1	1	1	1	⋯	1

Table (3,1)

Example : (3,3):-

We can write Artin characters tables of the groups $C_{p_1}$and C_{p_2} ,p_1, p_2>2 and p_1, p_2 are primes numbers

Ar(C_{p_2})=

Γ- classes	[1]	[x]
$\|cL_\alpha\|$	1	1
$\|Cc_5(CL\alpha)\|$	p_1	p_1
φ'_1	p_1	0
φ'_2	1	1

Table(3,2)

Ar(C_{p_1})=

Γ- classes	[1]	[x]
$\|cL_\alpha\|$	1	1
$\|Cc_5(CL\alpha)\|$	p_2	p_2
φ'_1	p_2	0
φ'_2	1	1

Table(3,3)

Corollary :(3,4):[2]

Let $m = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots\cdots p_n^{\alpha n}$ where g.c.d(p_i, p_j)=1, if i ≠ j and p_i's are primes numbers, and α_n any positive integers, then; $Ar(C_m) = Ar\left(C_{p_1^{\alpha 1}}\right) \otimes Ar\left(C_{p_2^{\alpha 2}}\right) \otimes \cdots\cdots \otimes Ar\left(C_{p_n^{\alpha n}}\right)$.

<u>Example (3.5)</u>:-

Consider the cyclic group $C_{2\,.\,p_1.p_2}$,p_1, $p_2 > 2$,Where g.c.d(p_1,p_2) = 1 and p1, p2 are primes numbers. To find Artin characters table for it, we use corollary (3,4) as the following: Ar($C_{2\,.\,p_1.p_2}$)=Ar(C_2)⊗Ar(C_{p_1})⊗Ar(C_{p_2}) , by using theorem (3.2) to find Ar(C_2) is given as follows :

Ar(C_2)=

Γ- classes	[1]	[x]
$\|cL_\alpha\|$	1	1
$\|Cc_2(CL\alpha)\|$	2	2
φ'_1	2	0
φ'_2	1	1

Table(3,4)

Ar($C_2 . p_1.p_2$)=

Γ- classes	[1]	$[x^2]$	$[x^{2p_1}]$	$[x^{2p_2}]$	$[x^{p_1.p_2}]$	$[x^{p_1}]$	$[x^{p_2}]$	$[x]$
$\|CL_\alpha\|$	1	2	2	2	1	2	2	2
$\|C_{C_2 .p_1.p_2}(CL_\alpha)\|$	$2p_1p_2$	$p_1.p_2$	$p_1.p_2$	$p_1.p_2$	$2p_1p_2$	$p_1.p_2$	$p_1.p_2$	$p_1.p_2$
φ_1'	$2p_1p_2$	0	0	0	0	0	0	0
φ_2'	2	2	2	2	0	0	0	0
φ_3'	$2p_1$	0	$2p_1$	0	0	0	0	0
φ_4'	$2p_2$	0	0	$2p_2$	0	0	0	0
φ_5'	P_1p_2	0	0	0	P_1p_2	0	0	0
φ_6'	P_1	0	P_1	0	0	P_1	0	0
φ_7'	P_2	0	0	P_2	0	0	P_2	0
φ_8'	1	1	1	1	1	1	1	1

Table (3,5)

Theorem(3.6):[1]

The Artin characters table of the Quaternion group Q_{2m} when m is an odd number is given as follows :

Ar(Q_{2m}) =

	Γ-Classes of C_{2m}								
Γ-Classes	x^{2r}				x^{2r+1}				$[y]$
$\|CL_\alpha\|$	1	2	…	2	1	2	…	2	2m
$\|C_{Q_{2m}}(CL_\alpha)\|$	4m	2m	…	2m	4m	2m	…	2m	2
Φ_1	2.Ar(C_{2m})								0
Φ_2									0
:									:
Φ_l									0
Φ_{l+1}	m	0	…	0	m	0	…	0	1

Table (3.7)

where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m} , for all $1 \le j \le l+1$.

Example (3.6):-

To construct Ar($Q_2 . p_1.p_2$) ,$p_1, p_2 > 2$ Where g.c.d(p_1,p_2) = 1 and p_1, p_2 are primes numbers .by using theorem(3.5):

Ar($Q_2 . p_1.p_2$)=

Γ-Classes	[1]	$[x^2]$	$[x^{2p_1}]$	$[x^{2p_2}]$	$[x^{p_1.p_2}]$	$[x^{p_1}]$	$[x^{p_2}]$	$[x]$	$\|y\|$
| CL_α |	1	2	2	2	1	2	2	2	$2p_1p_2$
|$CQ_{2p_1p_2}(CL_\alpha)$|	$4p_1p_2$	$2p_1p_2$	$2p_1p_2$	$2p_1p_2$	$4p_1p_2$	$2p_1p_2$	$2p_1p_2$	$2p_1p_2$	2
Φ_1	$4p_1p_2$	0	0	0	0	0	0	0	0
Φ_2	4	4	4	4	0	0	0	0	0
Φ_3	$4p_1$	0	$4p_1$	0	0	0	0	0	0
Φ_4	$4p_2$	0	0	$4p_2$	0	0	0	0	0
Φ_5	$2p_1p_2$	0	0	0	$2p_1p_2$	0	0	0	0
Φ_6	$2p_1$	0	2p1	0	0	$2p_1$	0	0	0
Φ_7	$2p_2$	0	0	$2p_2$	0	0	$2p_1$	0	0
Φ_8	2	2	2	2	2	2	2	2	0
Φ_9	p_1p_2	0	0	0	p_1p_2	0	0	0	1

Table (3.8)

Theorem(3,8): [4]

Let H be a cyclic subgroup of G and $h_1, h_2, \ldots, h_m$ are chosen_representatives for the m-conjugate classes of H contained in CL(g) in G, then :

$$\phi'(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\displaystyle\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i \in H \cap CL(g) \\ 0 & if \quad H \cap CL(g) = \phi \end{cases}$$

Proposition(3,9).[3]

The number of all distinct Artin characters on group G is equal to the number of Γ-classes on G .Furthermore, Artin characters are constant on each Γ-classes .

3. THE MAIN RESULTS:

In this section we give the general form of Artin's characters table of the group ($Q_{2m}\times C_5$), When m=$p_1.p_2$, p_1, p_2> 2 Where g. c. d(p_1,p_2) = 1 and p_1, p_2 are primes numbers. The group($Q_{2m}\times C_5$) is the direct product group of the quaternion groupQ_{2m} of order **4m** and the cyclic group C_5 of order 5, then the order of The group($Q_{2m}\times C_5$) is **20m.**

Example: (4.1):-

Let m=15=3.5, $p_1 = 3$,$p_2 = 5$, then ($Q_{2m}\times C_5$)= ($Q_{30}\times C_5$)= ($Q_{2.3.5}\times C_5$)={ (1,I), (1,z), (1,z^2) ,(1,z^3) ,(1,z^4) (x,I), (x,z), (x,z^2), (x,z^3) ,(x,z^4) (x^2,I), (x^2,z), (x^2,z^2), (x^2,z^3), (x^2,z^4), ... ,(x^{29},I), (x^{29},z), (x^{29},z^2), (x^{29},z^3), (x^{29},z^4), (y,I), (y,z), (y,z^2), (y,z^3), (y,z^4), (xy,I), (xy,z), (xy,z^2), (xy,z^3), (xy,z^4), (x^2y,I), (x^2y, z), (x^2y, z^2), (x^2y ,z^3), (x^2y ,z^4),..., (x^{29}y ,I), (x^{29}y ,z), (x^{29}y ,z^2), (x^{29}y ,z^3), (x^{29}y ,z^4)},

to find Artin's characters for this group, there are **18**cyclic subgroups, which are : <1,I>,<x^2,I>,<x^6,I>,<x^{10},I>,<x^{15},I>,<x^3,I>,<x^5,I><x,I>,<y,I>,<1,z>, <x^2,z>,<x^6,z>,<x^{10},z>,<x^{15},z>, <x^3,z>,<x^5,z> <x,z>,<y,z>,then there are **18** Γ-Classes , we have **18** distinct Artin's characters,Let g∈($Q_{30}\times C_5$),g=(q,I)or g=(q,z),q∈Q_{10}, I, z∈$C_5$and let φ the principal character of H , Φ_j Artin characters of Q_{10},1 ≤ j ≤ 9 , then by using theorem (3,8):

$$\Phi_j(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i \in H\cap CL(g) \\ 0 & if \quad H\cap CL(g)=\phi \end{cases}$$

Case (I): If H is a cyclic subgroup of ($Q_{2m}\times$ {I})then:

H_1=<1,I> ,If g=(1,I)

$\Phi_{(1,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}\varphi((I,1)) = \frac{300}{1}.1 = 300 = 5.60 = 5.\Phi_1(1)$, since H ∩ CL(g)={ (1,I)} and φ (g)=1

Otherwise $\Phi_{(1,1)}$ (g)= 0 since H ∩ CL(g) = ϕ

H_2=<x^2,I> ,If g=(1,I)

$\Phi_{(2,1)}(g)=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}\varphi(g) = \frac{300}{15}.1 = 20 = 5.4 = 5.\Phi_2(1)$ since H ∩ CL(g)={ (1,I)} and φ (g)=1

If g=(x^2,I), $\Phi_{(2,1)}((x^2,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{15}(1+1) = 20 = 5.4 = 5.\ \Phi_2\ (x^2)$ since H ∩ CL(g)={ (g,g^{-1} } and φ (g)= φ (g^{-1})=1

If $g=(x^6,I),\Phi_{(2,1)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{15}(1+1)=20=5.4=5.\ \Phi_2\ (x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{10},I),\Phi_{(2,1)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{15}(1+1)=20=5.4=5.\ \Phi_2\ (x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(2,1)}(g)=0$ since $H\cap CL(g)=\phi$

$H_3=\langle x^6,I\rangle$, If $g=(1,I),\Phi_{(3,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{5}.1=60=5.12=5.\Phi_3\ (1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^6,I),\Phi_{(3,1)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{5}(1+1)=60=5.12=5.\Phi_3\ (x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(3,1)}(g)=0$ since $H\cap CL(g)=\phi$

$H_4=\langle x^{10},I\rangle$, If $g=(1,I),\Phi_{(4,1)}((1,I))=\frac{|C_{30\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{3}.1=100=5.20=5.\ \Phi_4\ (1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^{10},I),\Phi_{(4,1)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{3}(1+1)=100=5.20=5.\ \Phi_4\ (x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(4,1)}(g)=0$ since $H\cap CL(g)=\phi$

$H_5=\langle x^{15},I\rangle$,If $g=(1,I),\Phi_{(5,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{2}.1=150=5.30=5.\ \Phi_5\ (1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^{15},I),\Phi_{(5,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{2}.1=150=5.30=5.\ \Phi_5\ (x^{15})$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

Otherwise $\Phi_{(5,1)}(g)=0$ since $H\cap CL(g)=\phi$

$H_6=\langle x^3,I\rangle$If $g=(1,I),\Phi_{(6,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{10}.1=30=5.6=5.\ \Phi_6\ (1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^{15},I),\Phi_{(6,1)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{10}.1=30=5.6=5.\ \Phi_6\ (x^{15})$ since $H\cap CL(g)=\{(x^{15},I)\}$ and $\varphi(g)=1$

If $g=(x^6,I),\Phi_{(6,1)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{10}(1+1)=30=5.6=5.\Phi_6\ (x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^3,I),\Phi_{(6,1)}((x^3,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{10}(1+1)=30=5.6=5.\Phi_6(x^3)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(6,1)}(g)=0$ since $H\cap CL(g)=\phi$

H_7=<x^5,I>,If g=(1,I)$\Phi_{(7,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{6}.1=50=5.10=5.\Phi_7(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{10},I),$\Phi_{(7,1)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{6}(1+1)=50=5.10=5.\Phi_7(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{15},I),$\Phi_{(7,1)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{6}.1=50=5.10=5.\Phi_7(x^{15})$ since H $\cap$ CL(g)={ (x^{15},I)} and φ (g)=1

If g=(x^5,I),$\Phi_{(7,1)}((x^5,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{6}(1+1)=50=5.10=5.\Phi_7(x^5)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(7,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_8=<x,I>,If g=(1,I),$\Phi_{(8,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{30}.1=10=5.2=5.\Phi_8(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^2,I),$\Phi_{(8,1)}((x^2,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}(1+1)=10=5.2=5.\Phi_8(x^2)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^6,I),$\Phi_{(8,1)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}(1+1)=10=5.2=5.\Phi_8(x^6)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{10},I),$\Phi_{(8,1)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}(1+1)=10=5.2=5.\Phi_8(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^3,I),$\Phi_{(8,1)}((x^3,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}(1+1)=10=5.2=5.\Phi_8(x^3)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^5,I),$\Phi_{(8,1)}((x^5,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}(1+1)=10=5.2=5.\Phi_8(x^5)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{15},I) ,$\Phi_{(8,1)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{30}.1=10=5.2=5.\Phi_8(x^{15})$ since H $\cap$ CL(g)={ (x^{15},I)} and φ (g)=1

If g=(x,I),$\Phi_{(8,1)}((x,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}(1+1)=10=5.2=5.\Phi_8(x)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(8,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_9= <y,I>,If g=(1,I),$\Phi_{(9,1)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{4}.1=75=5.15=5.\Phi_9(1)$ since H $\cap$ CL(g)={(1,I)} and φ (g)=1

If g=(x^{15},I),$\Phi_{(9,1)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{4}.1=75=5.15=5.\Phi_9(x^{15})$ since $H\cap CL(g)=\{(x^{15},I)\}$ and $\varphi(g)=1$

If g=(y,I),$\Phi_{(9,1)}((y,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{10}{4}(1+1)=5=5.1=5.\Phi_9(y)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(9,1)}(g)=0$ since $H\cap CL(g)=\phi$

<u>Case (II)</u>: If H is a cyclic subgroup of ($Q_{2m}\times\{z\}$)then:

H_1=<1,z> ,If g=(1,I), $\Phi_{(1,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{5}.1=60=\Phi_1(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If g=(1,z),$\Phi_{(1,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{5}.1=60=\Phi_1(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$

Otherwise $\Phi_{(1,2)}(g)=0$ since $H\cap CL(g)=\phi$

H_2=<x^2,z>,If g=(1,I),$\Phi_{(2,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{75}.1=4=\Phi_2(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If g=(1,z),$\Phi_{(2,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{75}.1=4=\Phi_2(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$

If g=(x^2,I) ,$\Phi_{(2,2)}((x^2,I))=(\varphi(g)\ \frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{75}(1+1)=4=\Phi_2(x^2)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^2,z) ,$\Phi_{(2,2)}((x^2,z))=(\varphi(g)\ \frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{75}(1+1)=4=\Phi_2(x^2)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,I) ,$\Phi_{(2,2)}((x^6,I))=(\varphi(g)\ \frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{75}(1+1)=4=\Phi_2(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,z) ,$\Phi_{(2,2)}((x^6,z))=(\varphi(g)\ \frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{75}(1+1)=4=\Phi_2(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{10},I) ,$\Phi_{(2,2)}((x^{10},I))=(\varphi(g)\ \frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{75}(1+1)=4=\Phi_2(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{10},z) ,$\Phi_{(2,2)}((x^{10},z))=(\varphi(g)\ \frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{150}{75}(1+1)=4=\Phi_2(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(2,2)}(g)=0$ since $H\cap CL(g)=\phi$

H_3=<x^6,z>,If g=(1,I),$\Phi_{(3,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{25}.1=12=\Phi_3(1)$ since H $\cap$ CL(g)={(1,I)} and φ(g)=1

If g=(1,z) ,$\Phi_{(3,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{25}.1=12=\Phi_3(1)$ since H $\cap$ CL(g)={(1,z)} and φ(g)=1

If g=(x^6,I) ,$\Phi_{(3,2)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{25}.(1+1)=12=\Phi_3(x^6)$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,z),$\Phi_{(3,2)}((x^6,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{25}.(1+1)=12=\Phi_3(x^6)$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(3,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_4=<x^{10},z>,If g=(1,I),$\Phi_{(4,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{15}.1=20=\Phi_4(1)$ since H $\cap$ CL(g)={(1,I)} and φ(g)=1

If g=(1,z),$\Phi_{(4,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{15}.1=20=\Phi_4(1)$ since H $\cap$ CL(g)={(1,z)} and φ(g)=1

If g=(x^{10},I),$\Phi_{(4,2)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{15}.(1+1)=20=\Phi_4(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{10},z),$\Phi_{(4,2)}((x^{10},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{15}.(1+1)=20=\Phi_4(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(4,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_5=<x^{15},z>,If g=(1,I),$\Phi_{(5,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{10}.1=30=\Phi_5(1)$ since H $\cap$ CL(g)={(1,I)} and φ(g)=1

If g=(1,z),$\Phi_{(5,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{10}.1=30=\Phi_5(1)$ since H $\cap$ CL(g)={(1,z)} and φ(g)=1

If g=(x^{15},I),$\Phi_{(5,2)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{10}.1=30=\Phi_5(x^{15})$ since H $\cap$ CL(g)={(x^{15},I)} and φ(g)=1

If g=(x^{15},z),$\Phi_{(5,2)}((x^{15},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{10}.1=30=\Phi_5(x^{15})$ since H $\cap$ CL(g)={(x^{15},z) and φ(g)=1

Otherwise $\Phi_{(5,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_6=<x^3,z>,If g=(1,I),$\Phi_{(6,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{50}.1=6=\Phi_6(1)$ since H $\cap$ CL(g)={(1,I)} and φ(g)=1

If g=(1,z) ,$\Phi_{(6,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{50}.1=6=\Phi_6$ (1) since H $\cap$ CL(g)={(1,z)} and φ (g)=1

If g=(x^6,I), $\Phi_{(6,2)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{50}.(1+1)=6=\Phi_6$ (x^6) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^6,z),$\Phi_{(6,2)}((x^6,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{50}.(1+1)=6=\Phi_6$ (x^6) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{15},I) ,$\Phi_{(6,2)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{50}.1=6=\Phi_6$ (x^{15}) since H $\cap$ CL(g)={(x^{15},I)} and φ (g)=1

If g=(x^{15},z) ,$\Phi_{(6,2)}((x^{15},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{50}.1=6=\Phi_6$ (x^{15}) since H $\cap$ CL(g)={(x^{15},z)} and φ (g)=1

If g=(x^3,I),$\Phi_{(6,2)}((x^3,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{50}.(1+1)=6=\Phi_6$ (x^3) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^3,z),$\Phi_{(6,2)}((x^3,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{50}.(1+1)=6=\Phi_6$ (x^3) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(6,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_7=<x^5,z>,If g=(1,I),$\Phi_{(7,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{30}.1=10=\Phi_7$ (1) since H $\cap$ CL(g)={(1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(7,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{30}.1=10=\Phi_7$ (1) since H $\cap$ CL(g)={(1,z)} and φ (g)=1

If g=(x^{10},I) ,$\Phi_{(7,2)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}.(1+1)=10=\Phi_7$ (x^{10}) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{10},z),$\Phi_{(7,2)}((x^{10},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}.(1+1)=10=\Phi_7$ (x^{10}) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{15},I) ,$\Phi_{(7,2)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{30}.1=10=\Phi_7$ (x^{15}) since H $\cap$ CL(g)={(x^{15},I)} and φ (g)=1

If g=(x^{15},z) ,$\Phi_{(7,2)}((x^{15},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{50}.1=10=\Phi_7$ (x^{15}) since H $\cap$ CL(g)={(x^{15},z)} and φ (g)=1

If g=(x^5,I),$\Phi_{(7,2)}((x^5,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}.(1+1)=10=\Phi_7$ (x^5) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x⁵,z), $\Phi_{(7,2)}((x^5,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{150}{30}.(1+1)=10=\Phi_7(x^5)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(7,2)}(g)=0$ since $H\cap CL(g)=\phi$

$H_8=\langle x,z\rangle$, If g=(1,I), $\Phi_{(8,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{150}.1=2=\Phi_8(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If g=(1,z), $\Phi_{(8,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{150}.1=2=\Phi_8(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$

If $g=(x^2,I)$, $\Phi_{(8,2)}((x^2,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^2)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^2,z)$, $\Phi_{(8,2)}((x^2,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^2)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^6,I)$, $\Phi_{(8,2)}((x^6,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^6,z)$, $\Phi_{(8,2)}((x^6,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{10},I)$, $\Phi_{(8,2)}((x^{10},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{10},z)$, $\Phi_{(8,2)}((x^{10},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{15},I)$, $\Phi_{(8,2)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{150}(1)=2=\Phi_8(x^{15})$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(x^{15},z)$, $\Phi_{(8,2)}((x^{15},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{150}(1)=2=\Phi_8(x^{15})$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(x^3,I)$, $\Phi_{(8,2)}((x^3,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^3)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^3,z)$, $\Phi_{(8,2)}((x^3,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^3)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^5,I)$, $\Phi_{(8,2)}((x^5,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^5)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^5,z),$\Phi_{(8,2)}((x^5,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x^5)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ(g)= φ(g^{-1})=1

If g=(x,I),$\Phi_{(8,2)}((x,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ(g)= φ(g^{-1})=1

If g=(x,z),$\Phi_{(8,2)}((x,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{300}{150}(1+1)=2=\Phi_8(x)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ(g)= φ(g^{-1})=1

Otherwise $\Phi_{(8,2)}$(g)= 0 since H $\cap$ CL(g) = ϕ

H_9=<y,z>,If g=(1,I),$\Phi_{(9,2)}((1,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{20}.1=15=\Phi_9(1)$ since H $\cap$ CL(g)={(1,I)} and φ(g)=1

If g=(1,z),$\Phi_{(9,2)}((1,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{20}.1=15=\Phi_9(1)$ since H $\cap$ CL(g)={(1,z)} and φ(g)=1

If g=(x^{15},I),$\Phi_{(9,2)}((x^{15},I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{300}{20}(1)=15=\Phi_9(x^{15})$ since H $\cap$ CL(g)={ g} and φ(g)=1

If g=(x^{15},z),$\Phi_{(9,2)}((x^{15},z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}\varphi(g)=\frac{300}{20}(1)=15=\Phi_9(x^{15})$ since H $\cap$ CL(g)={ g} and φ(g)= 1

If g=(y,I),$\Phi_{(9,2)}((y,I))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{10}{20}(1+1)=1=\Phi_9(y)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ(g)= φ(g^{-1})=1

If g=(y,z),$\Phi_{(9,2)}((y,z))=\frac{|C_{Q_{30}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{10}{20}(1+1)=1=\Phi_9(y)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ(g)= φ(g^{-1})=1

Otherwise $\Phi_{(9,2)}$(g)= 0 since H $\cap$ CL(g) = ϕ

Then, the Artin characters table of ($Q_{30}\times C_5$) is given in the following Table:

Ar($Q_{30}\times C_5$)=

Γ-Classes	[1,I]	[x^2,I]	[x^6,I]	[x^{10},I]	[x^{15},I]	[x^3,I]	[x^5,I]	[x,I]	[y,I]	[1,z]	[x^2,z]	[x^6,z]	[x^{10},z]	[x^{15},z]	[x^3,z]	[x^5,z]	[x,z]	[y,z]
$\|CL_\alpha\|$	1	2	2	2	1	2	2	2	30	1	2	2	2	1	2	2	2	30
$\|C_{Q_{30}\times C_5}(CL_\alpha)\|$	300	150	150	150	300	150	150	150	10	300	150	150	150	300	150	150	150	10
$\Phi_{(1,1)}$	300	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	20	20	20	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	60	0	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	100	0	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	150	0	0	0	150	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	30	0	30	0	30	30	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	50	0	0	50	50	0	50	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	10	10	10	10	10	10	10	10	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	75	0	0	0	75	0	0	0	5	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	60	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	4	4	4	4	0	0	0	0	0	4	4	4	4	0	0	0	0	0
$\Phi_{(3,2)}$	12	0	12	0	0	0	0	0	0	12	0	12	0	0	0	0	0	0
$\Phi_{(4,2)}$	20	0	0	20	0	0	0	0	0	20	0	0	20	0	0	0	0	0
$\Phi_{(5,2)}$	30	0	0	0	30	0	0	0	0	30	0	0	0	30	0	0	0	0
$\Phi_{(6,2)}$	6	0	6	0	6	6	0	0	0	6	0	6	0	6	6	0	0	0
$\Phi_{(7,2)}$	10	0	0	10	10	0	10	0	0	10	0	0	10	10	0	10	0	0
$\Phi_{(8,2)}$	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	0
$\Phi_{(9,2)}$	15	0	0	0	15	0	0	0	1	15	0	0	0	15	0	0	0	1

Table (4,1)

Theorem (4,2):-

The Artin's character table of the group (Q_{2m}×C_5) where m=$p_1.p_2$, $p_1, p_2 > 2$ *and* p_1,p_2 *are* primes numbers; is given as follows:_

Ar(Q_{2m}×C_5)=

<table>
<tr><th></th><th colspan="9">Γ-Classes of Q_{2m}×{I}</th><th colspan="9">Γ-Classes of Q_{2m}×{z}</th></tr>
<tr><th>Γ-Classes</th><th>[1,I]</th><th>[x^2,I]</th><th>[x^{2p_1},I]</th><th>[x^{2p_2},I]</th><th>[$x^{p_1p_2}$,I]</th><th>[x^{p_1},I]</th><th>[x^{p_2},I]</th><th>[x,I]</th><th>[y,I]</th><th>[1,z]</th><th>[x^2,z]</th><th>[x^{2p_1},z]</th><th>[x^{2p_2},z]</th><th>[$x^{p_1p_2}$,z]</th><th>[x^{p_1},z]</th><th>[x^{p_2},z]</th><th>[x,z]</th><th>[y,z]</th></tr>
<tr><th>$|CL_\alpha|$</th><td>1</td><td>2</td><td>2</td><td>2</td><td>1</td><td>2</td><td>2</td><td>2</td><td>2m</td><td>1</td><td>2</td><td>2</td><td>2</td><td>1</td><td>2</td><td>2</td><td>2</td><td>2m</td></tr>
<tr><th>$|C_{Q_{2m}\times C_5}(CL_\alpha)|$</th><td>20m</td><td>10m</td><td>10m</td><td>10m</td><td>20m</td><td>10m</td><td>10m</td><td>10m</td><td>10</td><td>20m</td><td>10m</td><td>10m</td><td>10m</td><td>20m</td><td>10m</td><td>10m</td><td>10m</td><td>10</td></tr>
<tr><th>$\Phi_{(1,1)}$</th><td colspan="9" rowspan="9">$5Ar(Q_{2m})$</td><td colspan="9" rowspan="9">0</td></tr>
<tr><th>$\Phi_{(2,1)}$</th></tr>
<tr><th>$\Phi_{(3,1)}$</th></tr>
<tr><th>$\Phi_{(4,1)}$</th></tr>
<tr><th>$\Phi_{(5,1)}$</th></tr>
<tr><th>$\Phi_{(6,1)}$</th></tr>
<tr><th>$\Phi_{(7,1)}$</th></tr>
<tr><th>$\Phi_{(8,1)}$</th></tr>
<tr><th>$\Phi_{(9,1)}$</th></tr>
<tr><th>$\Phi_{(1,2)}$</th><td colspan="9" rowspan="9">$Ar(Q_{2m})$</td><td colspan="9" rowspan="9">$Ar(Q_{2m})$</td></tr>
<tr><th>$\Phi_{(2,2)}$</th></tr>
<tr><th>$\Phi_{(3,2)}$</th></tr>
<tr><th>$\Phi_{(4,2)}$</th></tr>
<tr><th>$\Phi_{(5,2)}$</th></tr>
<tr><th>$\Phi_{(6,2)}$</th></tr>
<tr><th>$\Phi_{(7,2)}$</th></tr>
<tr><th>$\Phi_{(8,2)}$</th></tr>
<tr><th>$\Phi_{(9,2)}$</th></tr>
</table>

Table (4.2) Which is 18×18 matrix square.

Proof:-

Let g∈(Q_{2m}× C_5) ;g=(q,I) or g=(q,z) ,q∈ Q_{2m},I,z∈ C_5

***Case (I)**: If H is a cyclic subgroup of (Q_{2m}× {I}),then:*

1-H=$\langle (x,I) \rangle$ 2-H=$\langle (y,I) \rangle$

and φ the principal character of H , Φ_j Artin characters of Q_{2m}, , $1\le j\le l+1$ then by using theorem (3,8):

$$\Phi_j(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i\in H\cap CL(g) \\ 0 & if \quad H\cap CL(g)=\phi \end{cases}$$

(i) If g =(1,I) ,$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2m}\times C_5}(CL_\alpha)|}{|C_H(g)|}\varphi((I,1)) = \frac{20m}{|C_H((1,I))|}.1$ $=\frac{5\,|Q_{2m}(1)|}{|C_{<x>}(1)|}$=5 $\Phi_j(1)$ since H $\cap$ CL(1,I)={(1,I)}and φ (g)= 1

(ii) If g= (x^m,I),g$\in$H ,$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2m}\times C_5}(CL_\alpha)|}{|C_H(g)|}\varphi(g) = \frac{20m}{|C_H(x^m,1)|}.1=\frac{5|Q_{2m}(x^m)|}{|C_{<x>}(x^m)|}.1=5\Phi_j(x^m)$ since H $\cap$ CL(g) = {g}and φ (g)= 1

(iii) If g$\neq$ (x^m, I) and, g$\in$H ,$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2m}\times C_5}(CL_\alpha)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{10m}{|C_H(g)|}(1+1)=\frac{20m}{|C_H(g)|}=\frac{5\,|Q_{2m}(q)|}{|C_{<x>}(q)|}=5\Phi_j(q)$ since H$\cap$ CL(g)={g,g^{-1}} , g=(q,I), q$\in Q_{2m}$, q$\neq x^m$ and φ (g)=φ (g^{-1})=1

(iv) If g$\notin$H ,$\Phi_{(j,1)}(g)$= 0 = 5. 0 =5$\Phi_j(q)$ since H $\cap$ CL(g) = ϕ and q$\in Q_{2m}$

2-IF H = <(y,I) > ={(1,I),(y, I),(y^2,I),(y^3,I)}

(i) If g = (1,I) ,$\Phi^{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_5}(CL_\alpha)|}{|C_H(g)|}\varphi(g)=\frac{20m}{4}.1$=5.m=5$\Phi_j(1)$ since H $\cap$ CL(1,I) ={(1,I)} and φ (g)= 1

(ii) If g= (x^m,I) =(y^2,I) and g∈H ,$\Phi^{(l+1,1)}$(g)= $\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}$ φ (g))= $\frac{20m}{4}$.1=5.m=5$\Phi_{(i+1)}$ (x^m) since H∩CL(g)={g} and φ (g)= 1

(ii) If g≠ (x^m,I) and g∈H, i.e.{g =(y,I)or g =(y^3,I)} ,$\Phi^{(l+1,1)}$ (g)= $\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}$ (φ (g) +φ (g^{-1}) =$\frac{10}{4}(1+1)=\frac{20}{4}=5.1=$ $5\Phi_{(i,1)}(y)$ since H∩CL(g)={g,g^{-1}} and φ (g)=φ (g^{-1})=1 ,Otherwise $\Phi^{(l+1,1)}$(g)= 0 since H ∩ CL(g) = ϕ

<u>Case (II)</u>: If H is a cyclic subgroup of ($Q_{2m}\times$\{z\}) then:

1-H=$\langle (x,z)\rangle$ 2-H=$\langle (y,z)\rangle$

and φ the principal character of H , Φ_j Artin characters of Q_{2m}, , $1\le j\le l+1$ then by using theorem (3,8):

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\displaystyle\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i\in H\cap CL(g)\\ 0 & if \quad H\cap CL(g)=\phi\end{cases}$$

1-H=$\langle x,z\rangle$ ***(i) If g =(1,I),(1,z)*** ,$\Phi_{(j,2)}$(g)= $\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}$ φ (g) =$\frac{20m}{|C_H((1,I))|}$.1 =$\frac{5\,|Q_{2m}(1)|}{5\,|C_{<x>}(1)|}$ φ (1) = Φj(1) since H ∩ CL(g)={(1,I),(1,z)} and φ (g)= 1

(ii) g= (1,I),$(x^m,I)$$(x^m,z)$,(1,z) ; g∈H

If g=(1,I),(1,z) ,$\Phi_{(j,2)}$(g)= $\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}$ φ (g) = $\frac{20m}{|C_H((1,I))|}$.1 = $\frac{5\,|Q_{2m}(1)|}{5\,|C_{<x>}(1)|}$ φ (1) = $\Phi_j(1)$ since H ∩ CL(g) ={g} and φ (g)=1

If g=(x^m,I),(x^m,z) ,$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g)$ $=\frac{5|Q_{2m}|(x^m)|}{5\ |C_{<x>}(x^m)|}\varphi(x^m)=\Phi_j(x^m)$ since H ∩ CL(g) ={g} and φ(g)=1

(iii) If g≠ (x^m,I),(x^m,z) and, g∈H

$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}=(\varphi(g)+\varphi(g^{-1})=\frac{10}{|C_H(g)|}(1+1)=\frac{5\,|Q2m(q)|}{5\ |C_{<x>}(q)|}\varphi(q)=\Phi j(q)$ since H∩CL(g)={g,g^{-1}}, φ(g)=φ(g^{-1})=1 and g=(q,z),q∈ Q_{2m} ;$q\neq x^m$

(iv) If g∉ H $\Phi_{(j,2)}$(g)= 0 = $\Phi_j(q)$ since H ∩ CL(g) = ϕ and $q\in$ Q_{2m}

2-IF H = <y,I > ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z) (1,z^2),(y,z^2),(y^2,z^2), (y^3,z^2) (1,z^3),(y,z^3),(y^2,z^3),(y^3,z^3) ,(1,z^4),(y,z^4),(y^2,z^4),(y^3,z^4)}

(i) If g = (1,I),(1,z) ,$\Phi^{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g)=\frac{20m}{20}.1=m=\Phi^{l+1}(g)$

(ii) If g=(y^2,I) = (x^m,I) ,(y^2,z) ,(y^2,z^2) , y^2 (,z^3) ,(y^2,z) and g∈H

$\Phi^{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g)=\frac{20m}{20}.1=m=\Phi^{l+1}(g)$ since H ∩ CL(g)={g}and φ(g) =1

(ii) If g≠ (x^m,I) and g∈H i.e g={ y,I),(y,z) ,(y,z^2) ,(y,z^3) ,(y,z^4)}or g =(y^3,I), (y^3,z), (y^3,z^2), (y^3,z^3), (y^3,z^4)}

$\Phi^{(l+1,2)}(g\ )=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}=(\varphi(g)+\varphi(g^{-1})=\frac{10}{20}(1+1)=1=\Phi_{(i,1)}(y)$ since H∩CL(g)={g,g^{-1}}and φ(g)=φ(g^{-1})=1

Otherwise $\Phi^{(l+1,2)}$(g)= 0 since H ∩ CL(g) = ϕ

Example (4.3):

To construct Ar(Q_{66}×C_5)=Ar($Q_{2.3.11}$×C_5),p_1=3, p_2 =11, we use theorem(3,5) as the following :-

Ar(Q_{66})=

Γ-Classes	[1]	$[x^2]$	$[x^6]$	$[x^{22}]$	$[x^{33}]$	$[x^3]$	$[x^{11}]$	$[x]$	$[y]$
| CL_α |	1	2	2	1	1	2	2	2	66
|$C_{Q_{66}}(CL_\alpha)$|	132	66	66	66	132	66	66	66	2
Φ_1	132	0	0	0	0	0	0	0	0
Φ_2	4	4	4	4	0	0	0	0	0
Φ_3	12	0	12	0	0	0	0	0	0
Φ_4	44	0	0	44	0	0	0	0	0
Φ_5	66	0	0	0	66	0	0	0	0
Φ_6	6	0	6	0	6	6	0	0	0
Φ_7	22	0	0	22	22	0	22	0	0
Φ_8	2	2	2	2	2	2	2	2	0
Φ_9	33	0	0	0	33	0	0	0	1

Table (4,3)

Then by using theorem (4,2) Artin characters table of the group (Q_{66}×C_5) is:-

Ar(Q_{66}×C_5)=

Γ-Classes	[1,I]	$[x^2,I]$	$[x^6,I]$	$[x^{22},I]$	$[x^{33},I]$	$[x^3,I]$	$[x^{11},I]$	[x,I]	[y,I]	[1,z]	$[x^2,z]$	$[x^6,z]$	$[x^{22},z]$	$[x^{33},z]$	$[x^3,z]$	$[x^{11},z]$	[x,z]	[y,z]
| CL_α |	1	2	2	2	1	2	2	2	66	1	2	2	2	1	2	2	2	66
|$C_{G_{66}\times C_5}(CL_\alpha)$|	660	330	330	330	660	330	330	330	2	660	330	330	330	660	330	330	330	2
$\Phi_{(1,1)}$	660	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	20	20	20	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	60	0	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	220	0	0	220	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	330	0	0	0	330	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	30	0	30	0	30	30	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	110	0	0	110	110	0	110	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	10	10	10	10	10	10	10	10	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	165	0	0	0	165	0	0	0	5	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	132	0	0	0	0	0	0	0	0	132	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	4	4	4	4	0	0	0	0	0	4	4	4	4	0	0	0	0	0
$\Phi_{(3,2)}$	12	0	12	0	0	0	0	0	0	12	0	12	0	0	0	0	0	0
$\Phi_{(4,2)}$	44	0	0	44	0	0	0	0	0	44	0	0	44	0	0	0	0	0
$\Phi_{(5,2)}$	66	0	0	0	66	0	0	0	0	66	0	0	0	66	0	0	0	0
$\Phi_{(6,2)}$	6	0	6	0	6	6	0	0	0	6	0	6	0	6	6	0	0	0
$\Phi_{(7,2)}$	22	0	0	22	22	0	22	0	0	22	0	0	22	22	0	22	0	0
$\Phi_{(8,2)}$	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	0
$\Phi_{(9,2)}$	33	0	0	0	33	0	0	0	1	33	0	0	0	33	0	0	0	1

Table (4,4)

REFERENCES

[1] A.H.Abdul-Munem" On Artin Cokernel Of The Quaternion group Q_{2m} When m is an Odd number", M.Sc., thesis, University of Kufa, 2008

[2] A.S. Abid, "Artin's Characters Table of Dihedral Group for Odd Number", M.Sc. thesis, University of Kufa , 2006

[3] C.Curits and I.Reiner," Methods of Representation Theory with Application to Finite Groups and Order", John willy& sons, New York, 1981.

[4] I. M.Isaacs, "On Character Theory of Finite Groups", Academic press, New York, 1976.