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On Artin cokernel of The Group($Q_{2m}\times C_5$) Where m=$p_1^{s_1}.p_2^{s_2}$,$p_{1,}p_2 > 2$, g.c.d$(p_{1,}p_2) = 1$, $p_{1,}p_2$ are primes numbers and $s_{1,}s_2$ are a positive integers numbers

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Abstract:

The main purpose of This paper is to find Artin,s character table Ar($Q_{2m}\times C_5$)when m is odd number such that m= $p_1^{s_1}.p_2^{s_2}, g.c.d(p_{1,}p_2) = 1, p_{1,}p_2 > 2$ *, $p_{1,}p_2$* ***are primes numbers and $s_{1,}s_2$ are a positive integers numbers*** *;where Q_{2m} is denoted to Quaternion group of order 4m, time is said to have only one dimension and space to have three dimension ,the mathematical quaternion partakes of both these elements ; in technical language it may be said to be "time plus space" ,or "space plus time" and in this sense it has , or at least involves a reference to four dimensions ,and how the one of time of space the three might in the chain of symbols girdled "- William Rowan Hamilton (Quoted in Robert Percival Graves "Life of sir William Rowan Hamilton" (3 vols.,1882,1885,1889)) ,andC$_5$ is Cyclic group of order 5.In 1962, C. W. Curits & I. Reiner studied Representation Theory of finite groups, in 1976, I. M. Isaacs studied Characters Theory of finite groups, In 1982, M. S. Kirdar studied The Factor Group of the Z-Valued class function modulo the group of the Generalized Characters, In 1995, N. R. Mahmood studies The Cyclic*

Decomposition of the factor Group cf(Q_{2m},Z)/ $\overline{R}$ (G) (Q_{2m}), In 2002 ,K-Sekiguchi studies Extensions and the Irreducibilies of the Induced Characters of Cyclic P-Group , In 2008, A. H. Abdul-Munem studied Artin Cokernel of The Quaternion group Q_{2m} when m is an Odd number, In 2006, A.S. Abed found the Artin characters table of dihedral group D^n when n is an odd number.

Key words: odd number, prime number, Quaternion group, *positive integers numbers* and Cyclic group

1. INTRODUCTION:

Representation Theory is a branch of mathematics that studies abstract algebra structures by Representing their elements as linear transformations of vector spaces, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in items of matrix addition and matrix multiplication ,In which elements of a group are represented by invertible matrices in such a way that the group operation is matrix multiplication, Moreover, representation and character theory provide applications ,no only in other branches of mathematics but also in physics and chemistry.

Fore a finite group **G** ,The factor group $\overline{R}$ **(G) /T(G)** is called the Artin cokernel of **G** denoted **AC(G)**, deno $\overline{R}(G)$ ted the a belian group generated by Z-valued characters of G under the operation of pointwise addition, **T(G)** is asubgroup of $\overline{R}$ **(G)**which is generated by Artin's characters.

2-Preliminars:[1]:(3,1)

The Generalized Quaternion Group Q_{2m}, For each positive integer m≥2 ,The generalized Quaternion Group Q_{2m} of order

4m with two generators x and y satisfies $Q_{2m}=\{x^h y^k, 0\leq h\leq 2m-1, k=0,1\}$ Which has the following properties $\{x^{2m}=y^4=I, yx^m y^{-1}=x^{-m}\}$.

Let G be a finite group, all the characters of group G induced from a principal character of cyclic subgroup of G are called Artin characters of G. Artin characters of the finite group can be displayed in a table called Artin characters table of G which is denoted by Ar(G); The first row is Γ-conjugate classes; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G(CL_\alpha)|$ and other rows contains the values of Artin characters.

Theorem: [2] :(3,2)

The general form of Artin characters table of Cp^s When p is a prime number and s is a positive integer number is given by :-

$Ar(Cp^s)$

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	$\cdots$	$[x]$
$\vert CL_\alpha\vert$	1	1	1	1	$\cdots$	1
$\vert C_{p^s}(CL_\alpha)\vert$	p^s	p^s	p^s	p^s	$\cdots$	p^s
φ'_1	p^s	0	0	0	$\cdots$	0
φ'_2	p^{s-1}	p^{s-1}	0	0	$\cdots$	0
φ'_3	p^{s-2}	p^{s-2}	p^{s-2}	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
φ'_s	p^1	p^1	p^1	p^1	$\cdots$	0
φ'_{s+1}	1	1	1	1	$\cdots$	1

Table (3,1)

Example : (3,3):-

We can write Artin characters table of the group C_5

Ar(C_5)=

Γ- classes	[1]	$[x]$
$\|cL\alpha\|$	1	1
$\|C_{C_5}(CL\alpha)\|$	5	5
φ'_1	5	0
φ'_2	1	1

Table (3,2)

Corollary : (3,4) :[2] :

Let $m = p_1^{\alpha 1} \cdot p_2^{\alpha 2} \cdots\cdots p_n^{\alpha n}$ where g.c.d(p_i, p_j)=1, if i ≠ j and p_i 's are primes numbers, and α_n any positive integers, then; $Ar(C_m) = Ar(C_{P_1^{\alpha 1}}) \otimes Ar(C_{P_2^{\alpha 2}}) \otimes \cdots\cdots \otimes Ar(C_{P_n^{\alpha n}})$ —

Example (2.3.12):

Consider the cyclic group C_{18} .To find Artin characters table for it, we use corollary (3,4)_as the following Ar(C_{90})= Ar($C_{2.3^25}$)= Ar(C_2)⊗Ar(C_{3^2}) Ar(C_5) by using theorem (3.2) to find Ar(C_2) and Ar(C_{3^2}) are as follows :

Ar(C_2)=

Γ- cΓ- classes	[1]	$[x]$
$\|CL\alpha\|$	1	1
$\|C_{C_2}(CL\alpha)\|$	2	2
φ_1'	2	0
φ_2'	1	1

Table(3,4)

Ar(C_{3^2})=

Γ- classes	[1]	$[x^3]$	$[x]$
$\|CL\alpha\|$	1	1	1
$\|C_{C_{3^2}(CL\alpha)}\|$	3^2	3^2	3^2
φ_1'	3^2	0	0
φ_2'	3	3	0
φ_3'	1	1	1

Table(3,5)

Ar(C_{90})=

Γ-classes	[1]	$[x^{30}]$	$[x^{10}]$	$[x^{18}]$	$[x^{6}]$	$[x^{2}]$	$[x^{45}]$	$[x^{15}]$	$[x^{5}]$	$[x^{9}]$	$[x^{3}]$	$[x]$
$\lvert CL\alpha \rvert$	1	2	2	2	2	2	1	2	2	2	2	2
$\lvert Cc_{90}(CL\alpha) \rvert$	90	45	45	45	45	45	90	45	45	45	45	45
$\varphi_1^{'}$	90	0	0	0	0	0	0	0	0	0	0	0
$\varphi_2^{'}$	30	30	0	0	0	0	0	0	0	0	0	0
$\varphi_3^{'}$	10	10	10	0	0	0	0	0	0	0	0	0
$\varphi_4^{'}$	18	0	0	18	0	0	0	0	0	0	0	0
$\varphi_5^{'}$	6	6	0	6	6	0	0	0	0	0	0	0
$\varphi_6^{'}$	2	2	2	2	2	2	0	0	0	0	0	0
$\varphi_7^{'}$	45	0	0	0	0	0	45	0	0	0	0	0
$\varphi_8^{'}$	15	15	0	0	0	0	15	15	0	0	0	0
$\varphi_9^{'}$	5	5	5	0	0	0	5	5	5	0	0	0
$\varphi_{10}^{'}$	9	0	0	9	0	0	9	0	0	9	0	0
$\varphi_{11}^{'}$	3	3	0	3	3	0	3	3	0	3	3	0
$\varphi_{12}^{'}$	1	1	1	1	1	1	1	1	1	1	1	1

Table(3,6)

Theorem:(3.5):[1]:

The Artin characters table of the Quaternion group Q_{2m} when m is an odd number is given as follows :

Ar(Q_{2m}) =

	Γ-Classes of C_{2m}								
Γ-Classes	x^{2r}				x^{2r+1}				$[y]$
$\lvert CL_\alpha \rvert$	1	2	…	2	1	2	…	2	2m
$\lvert C^{Q_{2m}}(CL_\alpha) \rvert$	4m	2m	…	2m	4m	2m	…	2m	2
Φ_1	2.Ar(C_{2m})								0
Φ_2									0
:									:
Φ_l									0
Φ_{l+1}	m	0	…	0	m	0	…	0	1

Table (3.7)

where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m} , for all $1 \le j \le l+1$.

Example (2.3.14):

To construct Ar(Q_{30}) by using theorem(3.5):-

Ar(Q_{90})=

Γ-classes	[1]	$[x^{30}]$	$[x^{10}]$	$[x^{18}]$	$[x^6]$	$[x^2]$	$[x^{45}]$	$[x^{15}]$	$[x^5]$	$[x^9]$	$[x^3]$	$[x]$	$[Y]$
$\|CL\alpha\|$	1	2	2	2	2	2	1	2	2	2	2	2	90
$\|Cc_{90}(CL\alpha)\|$	180	90	90	90	90	90	180	90	90	90	90	90	2
Φ_1													0
Φ_2													0
Φ_3													0
Φ_4													0
Φ_5													0
Φ_6							2.Ar(C_{90})						0
Φ_7													0
Φ_8													0
Φ_9													0
Φ_{10}													0
Φ_{11}													0
Φ_{12}													0
Φ_{13}	45	0	0	0	0	0	45	0	0	0	0	0	1

Γ-classes	[1]	$[x^{30}]$	$[x^{10}]$	$[x^{18}]$	$[x^6]$	$[x^2]$	$[x^{45}]$	$[x^{15}]$	$[x^5]$	$[x^9]$	$[x^3]$	$[x]$	$[y]$
$\|CL\alpha\|$	1	2	2	2	2	2	1	2	2	2	2	2	90
$\|Cc_{90}(CL\alpha)\|$	180	90	90	90	90	90	180	90	90	90	90	90	2
Φ_1	180	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	60	60	0	0	0	0	0	0	0	0	0	0	0
Φ_3	20	20	20	0	0	0	0	0	0	0	0	0	0
Φ_4	36	0	0	36	0	0	0	0	0	0	0	0	0
Φ_5	12	12	0	12	12	0	0	0	0	0	0	0	0
Φ_6	`4	4	4	4	4	4	0	0	0	0	0	0	0
Φ_7	90	0	0	0	0	0	90	0	0	0	0	0	0
Φ_8	30	30	0	0	0	0	30	30	0	0	0	0	0
Φ_9	10	10	10	0	0	0	10	10	10	0	0	0	0
Φ_{10}	18	0	0	18	0	0	18	0	0	18	0	0	0
Φ_{11}	6	6	0	6	6	0	6	6	0	6	6	0	0
Φ_{12}	2	2	2	2	2	2	2	2	2	2	2	2	0
Φ_{13}	45	0	0	0	0	0	45	0	0	0	0	0	1

Table (3.8)

Theorem(3,8): [4]

Let H be a cyclic subgroup of G and $h_1, h_2, \ldots, h_m$ are chosen representatives for the m-conjugate classes of H contained in CL(g) in G, then :

$$\phi'(g)=\begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\displaystyle\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i \in H\cap CL(g) \\ 0 & if \quad H\cap CL(g)=\phi \end{cases}$$

Proposition(3,9).[3]

The number of all distinct Artin characters on group G is equal to the number of Γ-classes on G .Furthermore, Artin characters are constant on each Γ-classes .

3. THE MAIN RESULTS:

In this section we give the general form of Artin's characters table of the group ($Q_{2m}\times C_5$), When m= $\boldsymbol{p_1^{s_1}.p_2^{s_2}, g.c.d(p_1,p_2)=}$ $\mathbf{1}$,$\boldsymbol{p_1,p_2>2}$, $\boldsymbol{p_1,p_2}$ ***are primes numbers and*** $\boldsymbol{s_1,s_2}$ ***are a positive integers numbers***. The group($Q_{2m}\times C_5$) is the direct product group of the quaternion groupQ_{2m} of order **4m** and the cyclic group C_5 of order 5, then the order of The group($Q_{2m}\times C_5$) is**20m**

Example (4.1):

Let m=3^2.5 then ($Q_{2m}\times C_5$)= ($Q_{2.45}\times C_5$)= ($Q_{2.3^2.5}\times C_5$)={ (1,I), (1,z), (1,z^2) ,(1,z^3) ,(1,z^4) (x,I), (x,z), (x,z^2), (x,z^3) ,(x,z^4) (x^2,I), (x^2,z), (x^2,z^2), (x^2,z^3), (x^2,z^4), ... ,(x^{89},I), (x^{89},z), (x^{89},z^2), (x^{89},z^3), (x^{89},z^4), (y,I), (y,z), (y,z^2), (y,z^3), (y,z^4), (xy,I), (xy,z), (xy,z^2), (xy,z^3), (xy,z^4), (x^2y,I), (x^2y, z), (x^2y, z^2), (x^2y ,z^3), (x^2y ,z^4),..., (x^{89}y ,I), (x^{89}y ,z), (x^{89}y ,z^2), (x^{89}y ,z^3), (x^{89}y ,z^4) }.to find Artin's characters for this group, there are **26**cyclic subgroups, which are:

<1,I>,<x^{30},I>,<x^{10},I>,<x^{18},I>,<x^6,I>,<x^2,I>,<x^{45},I>,<x^{15},I>,<x^5,I>,<x^9,I>,<x^3,I>,<x,I>,<y,I>,<1,z>,<x^{30},z>,<x^{10},z>,<x^{18},z>,<x^6,z>,<x^2,z>,<x^{54},z>,<x^{15},z>,<x^5,z>, <x^9,z>,<x^3,z>,<x,z>,<y,z>,then there are **26** Γ-Classes , we have **26** distinct Artin's characters,.Let g∈(Q_{10}×C_5),g=(q,I)or g=(q,z),q∈Q_{10},z∈$C_5$and let φ the principal character of H , Φ_j Artin characters of Q_{10}, 1 ≤j ≤ *13* , then by using theorem (3,8):

$$\Phi_j(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i \in H \cap CL(g) \\ 0 & if \quad H \cap CL(g) = \phi \end{cases}$$

Case (I): If H is a cyclic subgroup of (Q_{2m}× {I}),then:

H_1=<1,I> ,If g=(1,I)

$\Phi_{(1,1)}$((1,I))=$\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ $\varphi((1,I))$ = $\frac{900}{1}.1 = 900 = 5.180 = 5.\Phi_1(1)$, since H ∩ CL(g)={ (1,I)} and φ (g)=1

Otherwise $\Phi_{(1,1)}$ (g)= 0 since H ∩ CL(g) = ϕ

H_2=<x^{30},I> ,If g=(1,I) ,$\Phi_{(2,1)}$(g)=$\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ φ (g) = $\frac{900}{3}.1 = 300 = 5.60 = 5.\Phi_2(1)$ since H ∩ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{30},I) , $\Phi_{(2,1)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$($\varphi$ (g) +φ (g^{-1})=$\frac{450}{3}(1+1) = 300 = 5.60 = 5.\Phi_2$ (x^{30}) since H ∩ CL(g)={ (g,g^{-1} } and φ (g)= φ (g^{-1})=1 ,Otherwise $\Phi_{(2,1)}$ (g)= 0 since H ∩ CL(g) = ϕ

H_3= <x^{10},I>, If g=(1,I) ,$\Phi_{(3,1)}$((1,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ (φ (g))=$\frac{900}{9}$.1=100=5.20=5.Φ_3 (1) since H ∩ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{10},I), $\Phi_{(3,1)}$((x^{10},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ (φ (g)+ φ (g^{-1}))=$\frac{450}{9}(1+1) = 100 = 5.20 = 5.\Phi_3$ (x^{10})since H ∩ CL(g)={ g,g^{-1}} and φ (g)=φ (g^1)=1

If g=(x^{30},I), $\Phi_{(3,1)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ (φ (g)+ φ (g^{-1}))=$\frac{450}{9}(1+1) = 100 = 5.20 = 5.\Phi_3$ (x^{30})

since H ∩ CL(g)={ g,g^{-1}} and φ (g)=φ (g^{-1})=1, Otherwise $\Phi_{(3,1)}$ (g)= 0 since H ∩ CL(g) = ϕ

H_4= <x^{18},I>, If g=(1,I) ,$\Phi_{(4,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{5}.1=180=5.36=5.\Phi_4(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{18},I), $\Phi_{(4,1)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{5}(1+1)=180=5.36=5.\Phi_4(x^{18})$

since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$,Otherwise $\Phi_{(4,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_5= <x^6,I>, If g=(1,I) ,$\Phi_{(5,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{15}.1=60=5.12=5.\Phi_5(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(5,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{15}(1+1)=60=5.12=5.\Phi_5(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},I), $\Phi_{(5,1)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{15}(1+1)=60=5.12=5.\Phi_5(x^{18})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,I), $\Phi_{(5,1)}((x^6,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{15}(1+1)=60=5.12=5.\Phi_5(x^6)$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$ Otherwise $\Phi_{(5,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_6= <x^2,I>, If g=(1,I) , $\Phi_{(6,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{45}.1=20=5.4=5.\Phi_6(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(6,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{45}(1+1)=20=5.4=5.\Phi_6(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$ If g=(x^{10},I), $\Phi_{(6,1)}((x^{10},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{45}(1+1)=20=5.4=5.\Phi_6(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},I), $\Phi_{(6,1)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{45}(1+1)=20=5.4=5.\Phi_6(x^{18})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,I), $\Phi_{(6,1)}((x^6,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{45}(1+1)=20=5.4=5.\Phi_6(x^6)$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^2,I), $\Phi_{(6,1)}((x^2,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{45}(1+1)=20=5.4=5.\Phi_6(x^2)$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$,Otherwise $\Phi_{(6,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

$H_7=<x^{45},I>$, If g=(1,I) ,$\Phi_{(7,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{2}.1=450=5.90=5.\Phi_7(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{45},I) , $\Phi_{(7,1)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{2}.1=450=5.90=5.\Phi_7(x^{45})$ since H $\cap$ CL(g)={ (x^{45},I)} and φ (g)=1

Otherwise $\Phi_{(7,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

$H_8=<x,I>$, If g=(1,I) ,$\Phi_{(8,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{6}.1=150=5.30=5.\Phi_8(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(8,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{6}(1+1)=150=5.30=5.\Phi_8(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{45},I) , $\Phi_{(8,1)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{2}.1=150=5.30=5.\Phi_8(x^{45})$ since H $\cap$ CL(g)={ (x^{45},I)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(8,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{6}(1+1)=150=5.30=5.\Phi_8(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(8,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

$H_9=<x^5,I>$, If g=(1,I) ,$\Phi_{(9,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{18}.1=50=5.10=5.\Phi_9(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(9,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{18}(1+1)=50=5.10=5.\Phi_9(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{10},I), $\Phi_{(9,1)}((x^{10},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{18}(1+1)=50=5.10=5.\Phi_9(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{45},I) , $\Phi_{(9,1)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{18}.1=50=5.10=5.\Phi_9(x^{45})$ since H $\cap$ CL(g)={ (x^{45},I)} and φ (g)=1

If g=(x^{15},I), $\Phi_{(9,1)}((x^{15},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{18}(1+1)=50=5.10=5.\Phi_9(x^{15})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^5,I), $\Phi_{(9,1)}((x^5,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{18}(1+1)=50=5.10=5.\Phi_9(x^5)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(9,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

$H_{10}=$ $<x^9,I>$, If $g=(1,I)$,$\Phi_{(10,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{10}.1=90=5.18=5.\Phi_{10}(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^{18},I)$, $\Phi_{(10,1)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{10}(1+1)=90=5.18=5.\Phi_{10}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{45},I)$, $\Phi_{(10,1)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{10}.1=90=5.18=5.\Phi_{10}(x^{45})$ since $H\cap CL(g)=\{(x^{45},I)\}$ and $\varphi(g)=1$

If $g=(x^9,I)$, $\Phi_{(10,1)}((x^9,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{10}(1+1)=90=5.18=5.\Phi_{10}(x^9)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(10,1)}(g)=0$ since $H\cap CL(g)=\phi$

$H_{11}=$ $<x^3,I>$, If $g=(1,I)$,$\Phi_{(11,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{30}.1=30=5.6=5.\Phi_{11}(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^{30},I)$, $\Phi_{(11,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=5.6=5.\Phi_{11}(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},I)$, $\Phi_{(11,1)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=5.6=5.\Phi_{11}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^6,I)$, $\Phi_{(11,1)}((x^6,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=5.6=5.\Phi_{11}(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{45},I)$, $\Phi_{(11,1)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{30}.1=30=5.6=5.\Phi_{11}(x^{45})$ since $H\cap CL(g)=\{(x^{45},I)\}$ and $\varphi(g)=1$

If $g=(x^{15},I)$, $\Phi_{(11,1)}((x^{15},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=5.6=5.\Phi_{11}(x^{15})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^9,I)$, $\Phi_{(11,1)}((x^9,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=5.6=5.\Phi_{11}(x^9)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^3,I)$, $\Phi_{(11,1)}((x^3,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=5.6=5.\Phi_{11}(x^3)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(11,1)}(g)=0$ since $H\cap CL(g)=\phi$

H_{12}= <x,I>, If g=(1,I) ,$\Phi_{(12,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{90}.1=10=5.2=5.\Phi_{12}(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(x^{30},I)$, $\Phi_{(12,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{10},I)$, $\Phi_{(12,1)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},I)$, $\Phi_{(12,1)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{6},I)$, $\Phi_{(12,1)}((x^{6},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{6})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{2},I)$, $\Phi_{(12,1)}((x^{2},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{2})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{45},I)$, $\Phi_{(12,1)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{90}(1)=10=5.2=5.\Phi_{12}(x^{45})$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(x^{15},I)$, $\Phi_{(12,1)}((x^{15},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{15})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{5},I)$, $\Phi_{(12,1)}((x^{5},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{5})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{9},I)$, $\Phi_{(12,1)}((x^{9},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{9})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{3},I)$, $\Phi_{(12,1)}((x^{3},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x^{3})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x,I)$, $\Phi_{(12,1)}((x,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=5.2=5.\Phi_{12}(x)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(12,1)}(g)=0$ since $H\cap CL(g)=\phi$

H_{13}= <y,I>,If g=(1,I), $\Phi_{(13,1)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{4}.1=225=5.45=5.\Phi_{13}(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If g=(x^{45},I), $\Phi_{(13,1)}$((x^{45} ,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g))=$\frac{900}{4}$.1=225=5.45=5.Φ_{13} (x^{45}) since H $\cap$ CL(g)={(x^{45} , I)} and φ (g)=1

If g=(y,I), $\Phi_{(13,1)}$((y,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g) + φ (g^{-1})=$\frac{10}{4}(1+1)=5=5.1=$ 5.Φ_{13} (y) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(13,1)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

<u>Case (II)</u>: If H is a cyclic subgroup of ($Q_{2m}\times$ {z}),then:

H_1=<1,z>,If g=(1,I), $\Phi_{(1,2)}$((1,I))=$\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ $\varphi\big((1\ ,I)\big)=\frac{900}{5}.1=180=\Phi_1(1)$, since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) , $\Phi_{(1,2)}$((1,z))=$\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ $\varphi\big((1\ ,I)\big)=\frac{900}{5}.1=180=\Phi_1(1)$, since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

Otherwise $\Phi_{(13,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_2=<x^{30},z> ,If g=(1,I) ,$\Phi_{(2,2)}$(g)=$\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ φ (g) $=\frac{900}{15}.1=60=\Phi_2(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(2,2)}$(g)=$\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$ φ (g) $=\frac{900}{15}.1=60=\Phi_2(1)$ since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

If g=(x^{30},I) , $\Phi_{(2,2)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g) + φ (g^{-1})=$\frac{450}{15}(1+1)=60=\Phi_2$ (x^{30}) since H $\cap$ CL(g)={ (g,g^{-1} } and φ (g)= φ (g^{-1})=1

If g=(x^{30},z) , $\Phi_{(2,2)}$((x^{30},z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g) + φ (g^{-1})=$\frac{450}{15}(1+1)=60=\Phi_2$ (x^{30}) since H $\cap$ CL(g)={ (g,g^{-1} } and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(2,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_3= <x^{10},z>, If g=(1,I) ,$\Phi_{(3,2)}$((1,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g))=$\frac{900}{45}$.1=20=Φ_3 (1) since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(3,2)}$((1,z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g))=$\frac{900}{45}$.1=20=Φ_3 (1) since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(3,2)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g)+ φ (g^{-1}))=$\frac{450}{45}(1+1)=20=\Phi_3$ (x^{30}) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{30},z), $\Phi_{(3,2)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}$(φ (g)+ φ (g^{-1}))=$\frac{450}{45}(1+1)=20=\Phi_3$ (x^{30}) since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{10},I), $\Phi_{(3,2)}((x^{10},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{45}(1+1)=20=\Phi_3(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^1)=1$

If g=(x^{10},z), $\Phi_{(3,2)}((x^{10},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{9}(1+1)=20=\Phi_3(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^1)=1$

Otherwise $\Phi_{(3,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_4= <x^{18},z>, If g=(1,I) ,$\Phi_{(4,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{25}.1=36=\Phi_4(1)$ since H $\cap$ CL(g)={ (1,I)} and $\varphi(g)=1$

If g=(1,z) ,$\Phi_{(4,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{25}.1=36=\Phi_4(1)$ since H $\cap$ CL(g)={ (1,z)} and $\varphi(g)=1$

If g=(x^{18},I), $\Phi_{(4,2)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{25}(1+1)=36=\Phi_4(x^{18})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},z), $\Phi_{(4,2)}((x^{18},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{25}(1+1)=36=\Phi_4(x^{18})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(4,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_5= <x^6,z>, If g=(1,I) ,$\Phi_{(5,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{75}.1=12=\Phi_5(1)$ since H $\cap$ CL(g)={ (1,I)} and $\varphi(g)=1$

If g=(1,z) ,$\Phi_{(5,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{75}.1=12=\Phi_5(1)$ since H $\cap$ CL(g)={ (1,z)} and $\varphi(g)=1$

If g=(x^{30},I), $\Phi_{(5,2)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{75}(1+1)=12=\Phi_5(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{30},z), $\Phi_{(5,2)}((x^{30},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{75}(1+1)=12=\Phi_5(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},I), $\Phi_{(5,2)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{75}(1+1)=60=\Phi_5(x^{18})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},z), $\Phi_{(5,2)}((x^{18},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{75}(1+1)=60=\Phi_5(x^{18})$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,I), $\Phi_{(5,2)}((x^6,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{75}(1+1)=12=\Phi_5(x^6)$ since H $\cap$ CL(g)={ g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,z), $\Phi_{(5,2)}((x^6,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{75}(1+1)=12=\Phi_5(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(5,2)}(g)=0$ since $H\cap CL(g)=\phi$

$H_6=<x^2,z>$, If g=(1,I) , $\Phi_{(6,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{225}.1=4=\Phi_6(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If g=(1,z) , $\Phi_{(6,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{225}.1=4=\Phi_6(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$

If g=(x^{30},I), $\Phi_{(6,2)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{30},z), $\Phi_{(6,2)}((x^{30},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{10},I), $\Phi_{(6,2)}((x^{10},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{10},z), $\Phi_{(6,2)}((x^{10},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},I), $\Phi_{(6,2)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{18},z), $\Phi_{(6,2)}((x^{18},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,I), $\Phi_{(6,2)}((x^6,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,z), $\Phi_{(6,2)}((x^6,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^2,I), $\Phi_{(6,2)}((x^2,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^2)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^2,z), $\Phi_{(6,2)}((x^2,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{225}(1+1)=4=\Phi_6(x^2)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(6,2)}(g)=0$ since $H\cap CL(g)=\phi$

H_7= <x^{45},z>, If g=(1,I) ,$\Phi_{(7,2)}$((1,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{10}.1=90=\Phi_7(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(7,2)}$((1,z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{10}.1=90=\Phi_7(1)$ since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

If g=(x^{45},I) , $\Phi_{(7,2)}$((x^{45},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{10}.1=90=\Phi_7(x^{45})$ since H $\cap$ CL(g)={ (x^{45},I)} and φ (g)=1

If g=(x^{45},z) , $\Phi_{(7,2)}$((x^{45},z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{10}.1=90=\Phi_7(x^{45})$ since H $\cap$ CL(g)={ (x^{45},z)} and φ (g)=1

Otherwise $\Phi_{(7,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_8= <x^{15},z>, If g=(1,I) ,$\Phi_{(8,2)}$((1,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{30}.1=30=\Phi_8(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(8,2)}$((1,z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{30}.1=30=\Phi_8(1)$ since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(8,2)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=\Phi_8(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{30},z), $\Phi_{(8,2)}$((x^{30},z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=\Phi_8(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{45},I) , $\Phi_{(8,2)}$((x^{45},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{30}.1=30=\Phi_8(x^{45})$ since H $\cap$ CL(g)={ (x^{45},I)} and φ (g)=1

If g=(x^{45},z) , $\Phi_{(8,2)}$((x^{45},z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{30}.1=30=\Phi_8(x^{45})$ since H $\cap$ CL(g)={ (x^{45},z)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(8,2)}$((x^{30},I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=\Phi_8(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{30},z), $\Phi_{(8,2)}$((x^{30},z))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{30}(1+1)=30=\Phi_8(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(8,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_9= <x^5,z>, If g=(1,I) ,$\Phi_{(9,2)}$((1,I))= $\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{90}.1=10\Phi_9(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(9,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{90}.1=10\Phi_9(1)$ since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

If g=(x^{30},I), $\Phi_{(9,2)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{30},z), $\Phi_{(9,2)}((x^{30},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^{30})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{10},I), $\Phi_{(9,2)}((x^{10},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{10},z), $\Phi_{(9,2)}((x^{10},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^{10})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{45},I) , $\Phi_{(9,2)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{90}.1=10=\Phi_9(x^{45})$ since H $\cap$ CL(g)={ (x^{45},I)} and φ (g)=1

If g=(x^{45},z) , $\Phi_{(9,2)}((x^{45},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{90}.1=10=\Phi_9(x^{45})$ since H $\cap$ CL(g)={ (x^{45},z)} and φ (g)=1

If g=(x^{15},I), $\Phi_{(9,2)}((x^{15},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^{15})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^{15},z), $\Phi_{(9,2)}((x^{15},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^{15})$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^5,I), $\Phi_{(9,2)}((x^5,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^5)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^5,z), $\Phi_{(9,2)}((x^5,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{90}(1+1)=10=\Phi_9(x^5)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(9,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_{10}= <x^9,z>, If g=(1,I) ,$\Phi_{(10,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{50}.1=18=\Phi_{10}(1)$ since H $\cap$ CL(g)={ (1,I)} and φ (g)=1

If g=(1,z) ,$\Phi_{(10,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{50}.1=18=\Phi_{10}(1)$ since H $\cap$ CL(g)={ (1,z)} and φ (g)=1

If $g=(x^{18},I)$, $\Phi_{(10,2)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{50}(1+1)=18=\Phi_{10}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},z)$, $\Phi_{(10,2)}((x^{18},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{50}(1+1)=18=\Phi_{10}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{45},I)$, $\Phi_{(10,2)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{50}.1=18=\Phi_{10}(x^{45})$ since $H\cap CL(g)=\{(x^{45},I)\}$ and $\varphi(g)=1$

If $g=(x^{45},z)$, $\Phi_{(10,2)}((x^{45},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{50}.1=18=\Phi_{10}(x^{45})$ since $H\cap CL(g)=\{(x^{45},z)\}$ and $\varphi(g)=1$

If $g=(x^9,I)$, $\Phi_{(10,2)}((x^9,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{50}(1+1)=18=\Phi_{10}(x^9)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^9,z)$, $\Phi_{(10,2)}((x^9,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{50}(1+1)=18=\Phi_{10}(x^9)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(10,2)}(g)=0$ since $H\cap CL(g)=\phi$

$H_{11}=\langle x^3,z\rangle$, If $g=(1,I)$, $\Phi_{(11,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{150}.1=6=\Phi_{11}(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If $g=(1,z)$, $\Phi_{(11,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{150}.1=6=\Phi_{11}(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$

If $g=(x^{30},I)$, $\Phi_{(11,2)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{30},z)$, $\Phi_{(11,2)}((x^{30},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},I)$, $\Phi_{(11,2)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},z)$, $\Phi_{(11,2)}((x^{18},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^6,I)$, $\Phi_{(11,2)}((x^6,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^6,z), $\Phi_{(11,2)}((x^6,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^6)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{45},I), $\Phi_{(11,2)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{150}.1=6=\Phi_{11}(x^{45})$ since $H\cap CL(g)=\{(x^{45},I)\}$ and $\varphi(g)=1$

If g=(x^{45},z), $\Phi_{(11,2)}((x^{45},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{150}.1=6=\Phi_{11}(x^{45})$ since $H\cap CL(g)=\{(x^{45},z)\}$ and $\varphi(g)=1$

If g=(x^{15},I), $\Phi_{(11,2)}((x^{15},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^{15})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{15},z), $\Phi_{(11,2)}((x^{15},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^{15})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^9,I), $\Phi_{(11,2)}((x^9,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^9)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^9,z), $\Phi_{(11,2)}((x^9,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^9)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^3,I), $\Phi_{(11,2)}((x^3,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^3)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^3,z), $\Phi_{(11,2)}((x^3,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{150}(1+1)=6=\Phi_{11}(x^3)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(11,2)}(g)=0$ since $H\cap CL(g)=\phi$

$H_{12}=\langle x,I\rangle$, If g=(1,I), $\Phi_{(12,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{450}.1=2=\Phi_{12}(1)$ since $H\cap CL(g)=\{(1,I)\}$ and $\varphi(g)=1$

If g=(1,z), $\Phi_{(12,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{450}.1=2=\Phi_{12}(1)$ since $H\cap CL(g)=\{(1,z)\}$ and $\varphi(g)=1$

If g=(x^{30},I), $\Phi_{(12,2)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^{30},z), $\Phi_{(12,2)}((x^{30},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{30})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{10},I)$, $\Phi_{(12,2)}((x^{30},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{10},z)$, $\Phi_{(12,2)}((x^{30},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{10})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},I)$, $\Phi_{(12,2)}((x^{18},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{18},z)$, $\Phi_{(12,2)}((x^{18},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{18})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{6},I)$, $\Phi_{(12,2)}((x^{6},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{6})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{6},z)$, $\Phi_{(12,2)}((x^{6},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{6})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{2},I)$, $\Phi_{(12,2)}((x^{2},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{2})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{2},z)$, $\Phi_{(12,2)}((x^{2},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{2})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{45},I)$, $\Phi_{(12,2)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{450}(1)=2=\Phi_{12}(x^{45})$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(x^{45},z)$, $\Phi_{(12,2)}((x^{45},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{450}(1)=2=\Phi_{12}(x^{45})$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

If $g=(x^{15},I)$, $\Phi_{(12,2)}((x^{15},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{15})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{15},z)$, $\Phi_{(12,2)}((x^{15},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{15})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{5},I)$, $\Phi_{(12,2)}((x^{5},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{5})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^{5},z)$, $\Phi_{(12,2)}((x^{5},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^{5})$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

If g=(x^9,I), $\Phi_{(12,2)}((x^9,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^9)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^9,z), $\Phi_{(12,2)}((x^9,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^9)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^3,I), $\Phi_{(12,2)}((x^3,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^3)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x^3,z), $\Phi_{(12,2)}((x^3,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x^3)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x,I), $\Phi_{(12,2)}((x,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(x,z), $\Phi_{(12,2)}((x,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{450}{450}(1+1)=2=\Phi_{12}(x)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(12,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

H_{13}= <y,I>,If g=(1,I), $\Phi_{(13,2)}((1,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{20}.1=45=\Phi_{13}(1)$ since H $\cap$ CL(g)={(1,I)} and φ (g)=1

If g=(1,z), $\Phi_{(13,2)}((1,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{20}.1=45=\Phi_{13}(1)$ since H $\cap$ CL(g)={(1,z)} and φ (g)=1

If g=(x^{45},I), $\Phi_{(13,2)}((x^{45},I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{20}.1=45=\Phi_{13}(x^{45})$ since H $\cap$ CL(g)={(x^{45} , I)} and φ (g)=1

If g=(x^{45},z), $\Phi_{(13,2)}((x^{45},z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g))=\frac{900}{20}.1=45=\Phi_{13}(x^{45})$ since H $\cap$ CL(g)={(x^{45} , z)} and φ (g)=1

If g=(y,I), $\Phi_{(13,2)}((y,I))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{10}{20}(1+1)=1=\Phi_{13}(y)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

If g=(y,z), $\Phi_{(13,2)}((y,z))=\frac{|C_{Q_{90}\times C_5}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{10}{20}(1+1)=1=\Phi_{13}(y)$ since H $\cap$ CL(g)={ g,g^{-1}} and φ (g)= φ (g^{-1})=1

Otherwise $\Phi_{(13,2)}$ (g)= 0 since H $\cap$ CL(g) = ϕ

Then, the Artin characters table of (Q_{18} × C_5) is given in the following Table:

Ar(Φ_{90}×C_5)=

Γ-classes	[1,I]	[x^{30},I]	[x^{10},I]	[x^{18},I]	[x^{6},I]	[x^{2},I]	[x^{45},I]	[x^{15},I]	[x^{5},I]	[x^{9},I]	[x^{3},I]	[x,I]	[y,I]	[1,z]	[x^{30},z]	[x^{10},z]	[x^{18},z]	[x^{6},z]	[x^{2},z]	[x^{45},z]	[x^{15},z]	[x^{5},z]	[x^{9},z]	[x^{3},z]	[x,z]	[y,z]
$\|CL_\alpha\|$	1	2	2	2	2	2	1	2	2	2	2	2	90	1	2	2	2	2	2	1	2	2	2	2	2	90
$\|C_{Q_{90}\times C_5}(CL_\alpha)\|$	900	450	450	450	450	450	900	450	450	450	450	450	2	900	450	450	450	450	450	900	450	450	450	450	450	2
$\Phi_{(1,1)}$	900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	300	300	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	180	0	0	180	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	60	60	0	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	20	20	20	20	20	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	450	0	0	0	0	0	450	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	150	150	0	0	0	0	150	150	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	50	50	50	0	0	0	50	50	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,1)}$	90	0	0	90	0	0	90	0	0	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(11,1)}$	30	30	0	30	30	0	30	30	0	30	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(12,1)}$	102	10	10	10	10	10	10	10	10	10	10	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(13,1)}$	225	0	0	0	0	0	225	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	180	0	0	0	0	0	0	0	0	0	0	0	0	180	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	60	60	0	0	0	0	0	0	0	0	0	0	0	60	60	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	20	20	20	0	0	0	0	0	0	0	0	0	0	20	20	20	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	36	0	0	36	0	0	0	0	0	0	0	0	0	36	0	0	36	0	0	0	0	0	0	0	0	0
$\Phi_{(5,2)}$	12	12	0	12	12	0	0	0	0	0	0	0	0	12	12	0	12	12	0	0	0	0	0	0	0	0
$\Phi_{(6,2)}$	4	4	4	4	4	4	0	0	0	0	0	0	0	4	4	4	4	4	4	0	0	0	0	0	0	0
$\Phi_{(7,2)}$	90	0	0	0	0	0	90	0	0	0	0	0	0	90	0	0	0	0	0	90	0	0	0	0	0	0
$\Phi_{(8,2)}$	30	30	0	0	0	0	30	30	0	0	0	0	0	30	30	0	0	0	0	30	30	0	0	0	0	0
$\Phi_{(9,2)}$	10	10	10	0	0	0	10	10	10	0	0	0	0	10	10	10	0	0	0	10	10	10	0	0	0	0
$\Phi_{(10,2)}$	18	0	0	18	0	0	18	0	0	18	0	0	0	18	0	0	18	0	0	18	0	0	18	0	0	0
$\Phi_{(11,2)}$	6	6	0	6	6	0	6	6	0	6	6	0	0	6	6	0	6	6	0	6	6	0	6	6	0	0
$\Phi_{(12,2)}$	2	2	2	2	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	2	2	2	2	0
$\Phi_{(13,2)}$	45	0	0	0	0	0	45	0	0	0	0	0	1	45	0	0	0	0	0	45	0	0	0	0	0	1

Table (4.1)

Theorem (4,2):-

The Artin's character table of the group (Q_{2m}×C_5) where m= ***$p_1^{s_1}.p_2^{s_2}$,p_1,p_2 > 2 , p_1,p_2 are primes numbers and s_1,s_2 are a positive integers numbers*** ; is given as follows:_

Ar(Q_{2m}x×C_5)=

<table>
<tr><td></td><td colspan="9">Γ-Classes of Q2m×{I}</td><td colspan="9">Γ-Classes of Q2m×{z}</td></tr>
<tr><td>Γ−Classes</td><td colspan="4">[x^{2r},I]</td><td colspan="4">[x^{2r+1},I]</td><td>[y,I]</td><td colspan="4">[x^{2r},z]</td><td colspan="4">[x^{2r+1},z]</td><td>[y,z]</td></tr>
<tr><td>$|CL_\alpha|$</td><td>1</td><td>2</td><td>…</td><td>2</td><td>1</td><td>2</td><td>…</td><td>2</td><td>2m</td><td>1</td><td>2</td><td>…</td><td>2</td><td>1</td><td>2</td><td>…</td><td>2</td><td>2m</td></tr>
<tr><td>$|C_{Q_{2m}\times C_5}(CL_\alpha)|$</td><td>20m</td><td>10m</td><td>…</td><td>10m</td><td>20m</td><td>10m</td><td>…</td><td>10m</td><td>10</td><td>20m</td><td>10m</td><td>…</td><td>10m</td><td>20m</td><td>10m</td><td>…</td><td>10m</td><td>10</td></tr>
<tr><td>$\Phi_{(1,1)}$</td><td colspan="9" rowspan="5">5 Ar(Q_{2m})</td><td colspan="9" rowspan="5">0</td></tr>
<tr><td>$\Phi_{(2,1)}$</td></tr>
<tr><td>⋮
⋮</td></tr>
<tr><td>$\Phi_{(l,1)}$</td></tr>
<tr><td>$\Phi_{(l+1,1)}$</td></tr>
<tr><td>$\Phi_{(1,1)}$</td><td colspan="9" rowspan="5">Ar(Q_{2m})</td><td colspan="9" rowspan="5">Ar(Q_{2m})</td></tr>
<tr><td>$\Phi_{(1,1)}$</td></tr>
<tr><td>⋮
⋮</td></tr>
<tr><td>$\Phi_{(l,2)}$</td></tr>
<tr><td>$\Phi_{(l+1,2)}$</td></tr>
</table>

Table (4.2)

Which is [4(s_1+1)(s_2+1)+2]×[4(s_1+1)(s_2+1)+2] matrix suquer.
where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m} , for all $1 \le j \le l+1$.

Proof:-

Let g$\in$($Q_{2m}\times C_5$) ;g=(q,I) or g=(q,z) ,q$\in$ Q_{2m},I,z$\in$ C_5

Case (I): If H is a cyclic subgroup of ($Q_{2m}\times$ {I}),then:

1-H=$\langle (x,I)\rangle$ 2-H=$\langle (y,I)\rangle$

and φ the principal character of H , Φ_j Artin characters of Q_{2m}, , then $1\le j\le l+1$ by using theorem (3,8):

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & \text{if} \quad h_i\in H\cap CL(g)\\[2ex] 0 & \text{if} \quad H\cap CL(g)=\phi\end{cases}$$

(i) If g =(1,I) ,$\Phi_{(j,1)}$(g)=$\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(\text{g})|}$ $\varphi((1\ ,I))$ = $\frac{20m}{|C_H\ ((I\ ,1)\ )\ |}.1$ =$\frac{5\ |Q_{2m}(1)\ |}{|C_{<\times>}(1)\ |}$=5 $\Phi_j(1)$ since H $\cap$ CL(1,I)={(1,I)}and φ (g)=1

(ii) If g= (x^m,I),g$\in$H

$\Phi_{(j,1)}$(g)=$\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(\text{g})|}$ φ (g) = $\frac{20m}{|C_H(\times^m\ ,1)|}.1$ = $\frac{5|Q_{2m}\ (x^m)|}{|C_{<\times>}(x^m)|}.1$ = 5 $\Phi_j(x^m)$ since H $\cap$ CL(g) = {g} and φ (g)=1

(iii) If g$\neq$ (x^m, I) and g$\in$H ,$\Phi_{(j,1)}$(g)=$\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(\text{g})|}$ (φ (g) +φ (g^{-1}) = $\frac{10m}{|C_H(\text{g})||}(1+1)$ = $\frac{20m}{|C_H(\text{g})|}$ = $\frac{5\ |Q_{2m}\ (q)|}{|\text{C<x>}(q)|}$= 5 $\Phi_j(q)$ since H$\cap$ CL(g)={g,g^{-1}} , g=(q,I), q$\in$ Q_{2m} , q$\neq x^m$ and φ (g)=φ (g^{-1})=1

(iv) If g$\notin$H , $\Phi_{(j,1)}$(g)= 0 = 5. 0 =5$\Phi_j(q)$ since H $\cap$ CL(g) = ϕ and q$\in$ Q_{2m}

2-IF H = <(y,I) > ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g = (1,I) ,$\Phi^{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g)=\frac{20m}{4}.1=5.m=5\Phi_j(1)$ since H $\cap$ CL(1,I) ={(1,I)} and φ(g)=1

(ii) If g= (x^m,I) =(y^2,I) and g∈H ,$\Phi^{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g))=\frac{20m}{4}.1=5.m=5\Phi_{(i+1)}(x^m)$ since H$\cap$CL(g)={g}and φ(g)=1

(ii) If g≠ (x^m,I) and g∈H , i.e.{g =(y,I)or g =(y^3,I)}
$\Phi^{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{10}{4}(1+1)=\frac{20}{4}=5.1=5\Phi_{(i,1)}(y)$since H$\cap$CL(g)={g,$g^{-1}$}and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi^{(l+1,1)}(g)=0$ since H $\cap$ CL(g) = ϕ

<u>Case (II)</u>:

If H is a cyclic subgroup of (Q_{2m}×{z}) then:

1-H=$\langle(x,z)\rangle$ 2-H=$\langle(y,z)\rangle$

and φ the principal character of H , Φ_j Artin characters of Q_{2m}, , then $1\le j\le l+1$ by using theorem (3,8):

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\displaystyle\sum_{i=1}^{m}\varphi(h_i) & if\quad h_i\in H\cap CL(g)\\ 0 & if\quad H\cap CL(g)=\phi\end{cases}$$

1-H=$\langle x,z\rangle$

(i) If g =(1,I),(1,z)

$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g) = \frac{20m}{|C_H((I,1))|}.1 = \frac{5\,|Q_{2m}(1)|}{5\,|C_{<x>}(1)|}\varphi(1) =$ $\Phi j(1)$ since H $\cap$ CL(g)={(1,I),(1,z)} and φ(g)=1

(ii) g= (1,I),(x^m,I)(x^m,z) ,(1,z) ; g$\in$H

If g=(1,I),(1,z) ,

$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g) = \frac{20m}{|C_H((I,1))|}.1 = \frac{5\,|Q_{2m}(1)|}{5\,|C_{<x>}(1)|}\varphi(1) = \Phi_j(1)$ since H $\cap$ CL(g) ={g} and φ(g)=1

If g=(x^m,I),(x^m,z)

$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g) = \frac{5|\,Q_{2m}|(x^m)\,|}{5\,|C_{<x>}(x^m)\,|}\varphi(x^m) = \Phi_j(x^m)$ since H $\cap$ CL(g) ={g} and φ(g)=1

(iii) If g$\neq$ (x^m,I),(x^m, z) and, g$\in$H

$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|} = (\varphi(g) + \varphi(g^{-1}) = \frac{10}{|C_H(g)|}(1+1) = \frac{5\,|Q2m(q)|}{5\,|C_{<x>}(q)|}\varphi(q) = \Phi j(q)$ since H$\cap$CL(g)={g,g^{-1}}, φ(g)=φ(g^{-1})=1 and g=(q,z),q$\in$ Q_{2m} ;$q\neq x^m$ and φ(g)=φ(g^{-1})=1

(iv) If g$\notin$H , $\Phi_{(j,2)}(g)= 0 = \Phi_j(q)$ since H $\cap$ CL(g) = ϕ and $q\in$ Q_{2m}

2-IF H = <y,z > ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y, z),(y^2,z),(y^3,z) (1,z^2),(y, z^2),(y^2,z^2), (y^3,z^2) (1,z^3),(y, z^3), (y^2,z^3),(y^3,z^3) ,(1,z^4),(y, z^4),(y^2,z^4),(y^3,z^4)}

(i) If g = (1,I),(1,z)

$\Phi^{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\varphi(g) = \frac{20m}{20}.1 = m = \Phi^{l+1}(g)$ since H$\cap$ CL(g)={g} and φ(g) =1

(ii) If g=(y^2,I) = (x^m,I) ,(y^2,z) ,(y^2,z^2) ,(y^2,z^3) ,(y^2,z^4) and g $\in H$

$\Phi^{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}\ \varphi(g)=\frac{20m}{20}.1=m=\Phi^{l+1}(g)$ since H $\cap$ CL(g)={g}and φ (g) =1

(ii) If g$\neq$ (χ^m,I) and g$\in H$ i.e g={ (y,I),(y, z) ,(y, z^2) ,(y,z^3) ,(y,z^4)}or g =(y^3,I), (y^3,z), (y^3,z^2), (y^3,z^3), (y^3,z^4)}

$\Phi^{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_5(CL_\alpha)}|}{|C_H(g)|}=(\varphi(g)+\varphi(g^{-1})=\frac{10}{20}(1+1)=1=\Phi_{(i,1)}(y)$ since H$\cap$CL(g)={g,g^{-1}}and φ (g)=φ (g^{-1})=1

Otherwise $\Phi^{(l+1,2)}(g)=0$ since H $\cap$ CL(g) = ϕ

<u>Example (4.3)</u>:

To construct Ar($Q_{450}\times C_5$)=Ar($Q_{2.3^2 5^2}\times C_5$),p_1=3,$p_2=5$, we use theorem(3,5) as the following :-

Ar(Q_{450})=

Γ-Classes	[1]	[x^{150}]	[x^{50}]	[x^{90}]	[x^{30}]	[x^{10}]	[x^{18}]	[x^6]	[x^2]	[x^{225}]	[x^{75}]	[x^{25}]	[x^{45}]	[x^{15}]	[x^5]	[x^9]	[x^3]	[x]	[y]
$\|CL_\alpha\|$	1	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	450
$\|C_G(CL_\alpha)\|$	900	450	450	450	450	450	450	450	450	900	450	450	450	450	450	450	450	450	2
Φ_1	900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	300	300	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_3	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_4	180	0	0	180	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_5	60	60	0	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_6	20	20	20	20	20	20	0	0	0	0	0	0	0	0	0	0	0	0	0
Φ_7	36	0	0	36	0	0	36	0	0	0	0	0	0	0	0	0	0	0	0
Φ_8	12	12	0	12	12	0	12	12	0	0	0	0	0	0	0	0	0	0	0
Φ_9	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0
Φ_{10}	450	0	0	0	0	0	0	0	0	450	0	0	0	0	0	0	0	0	0
Φ_{11}	150	150	0	0	0	0	0	0	0	150	150	0	0	0	0	0	0	0	0
Φ_{12}	50	50	50	0	0	0	0	0	0	50	50	50	0	0	0	0	0	0	0
Φ_{13}	90	0	0	90	0	0	0	0	0	90	0	0	90	0	0	0	0	0	0
Φ_{14}	30	30	0	30	30	0	0	0	0	30	30	0	0	30	0	0	0	0	0
Φ_{15}	10	10	10	10	10	10	0	0	0	10	10	10	10	10	10	0	0	0	0
Φ_{16}	18	0	0	18	0	0	18	0	0	18	0	0	18	0	0	18	0	0	0
Φ_{17}	6	6	0	6	6	0	6	6	0	6	6	0	6	6	0	6	6	0	0
Φ_{18}	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
Φ_{19}	225	0	0	0	0	0	0	0	0	225	0	0	0	0	0	0	0	0	1

Table(4,3)

Then by using theorem (4,2) Artin characters table of the group ($Q_{98}\times C_5$) is:-

Ar($Q_{450}\times C_5$)

Γ-Classes	[1,1]	$[x^{150},1]$	$[x^{50},1]$	$[x^{90},1]$	$[x^{30},1]$	$[x^{10},1]$	$[x^{18},1]$	$[x^{6},1]$	$[x^{2},1]$	$[x^{225},1]$	$[x^{75},1]$	$[x^{25},1]$	$[x^{45},1]$	$[x^{15},1]$	$[x^{5},1]$	$[x^{9},1]$	$[x^{3},1]$	$[x,1]$	$[y,1]$
$\lvert CL_\alpha \rvert$	1	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2250
$\lvert C_G(CL_\alpha)\rvert$	4500	2250	2250	2250	2250	2250	2250	2250	2250	4500	2250	2250	2250	2250	2250	2250	2250	2250	2
$\Phi_{(1,1)}$	4500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	1500	1500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	500	500	500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	900	0	0	900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	300	300	0	300	300	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	180	0	0	180	0	0	180	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,1)}$	60	60	0	60	60	0	60	60	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,1)}$	20	20	20	20	20	20	20	20	20	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,1)}$	2250	0	0	0	0	0	0	0	0	2250	0	0	0	0	0	0	0	0	0
$\Phi_{(11,1)}$	750	750	0	0	0	0	0	0	0	750	750	0	0	0	0	0	0	0	0
$\Phi_{(12,1)}$	250	250	250	0	0	0	0	0	0	250	250	250	0	0	0	0	0	0	0
$\Phi_{(13,1)}$	450	0	0	450	0	0	0	0	0	450	0	0	450	0	0	0	0	0	0
$\Phi_{(14,1)}$	150	150	0	150	150	0	0	0	0	150	150	0	0	150	0	0	0	0	0
$\Phi_{(15,1)}$	50	50	50	50	50	50	0	0	0	50	50	50	50	50	50	0	0	0	0
$\Phi_{(16,1)}$	90	0	0	90	0	0	90	0	0	90	0	0	90	0	0	90	0	0	0
$\Phi_{(17,1)}$	30	30	0	30	30	0	30	30	0	30	30	0	30	30	0	30	30	0	0
$\Phi_{(18,1)}$	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	0
$\Phi_{(19,1)}$	1125	0	0	0	0	0	0	0	0	1125	0	0	0	0	0	0	0	0	5

[1,z]	$[x^{150},z]$	$[x^{50},z]$	$[x^{90},z]$	$[x^{30},z]$	$[x^{10},z]$	$[x^{18},z]$	$[x^{6},z]$	$[x^{2},z]$	$[x^{225},z]$	$[x^{75},z]$	$[x^{25},z]$	$[x^{45},z]$	$[x^{15},z]$	$[x^{5},z]$	$[x^{9},z]$	$[x^{3},z]$	$[x,z]$	$[y,z]$
1	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	450
900	450	450	450	450	450	450	450	450	900	450	450	450	450	450	450	450	450	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\Phi_{(1,2)}$	900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	300	300	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	180	0	0	180	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,2)}$	60	60	0	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(6,2)}$	20	20	20	20	20	20	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(7,2)}$	36	0	0	36	0	0	36	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(8,2)}$	12	12	0	12	12	0	12	12	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(9,2)}$	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0
$\Phi_{(10,2)}$	450	0	0	0	0	0	0	0	0	450	0	0	0	0	0	0	0	0	0
$\Phi_{(11,2)}$	150	150	0	0	0	0	0	0	0	150	150	0	0	0	0	0	0	0	0
$\Phi_{(12,2)}$	50	50	50	0	0	0	0	0	0	50	50	50	0	0	0	0	0	0	0
$\Phi_{(13,2)}$	90	0	0	90	0	0	0	0	0	90	0	0	90	0	0	0	0	0	0
$\Phi_{(14,2)}$	30	30	0	30	30	0	0	0	0	30	30	0	0	30	0	0	0	0	0
$\Phi_{(15,2)}$	10	10	10	10	10	10	0	0	0	10	10	10	10	10	10	0	0	0	0
$\Phi_{(16,2)}$	18	0	0	18	0	0	18	0	0	18	0	0	18	0	0	18	0	0	0
$\Phi_{(17,2)}$	6	6	0	6	6	0	6	6	0	6	6	0	6	6	0	6	6	0	0
$\Phi_{(18,2)}$	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
$\Phi_{(19,2)}$	225	0	0	0	0	0	0	0	0	225	0	0	0	0	0	0	0	0	1

900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
300	300	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
180	0	0	180	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	60	0	60	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	20	20	20	20	20	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	36	0	0	36	0	0	0	0	0	0	0	0	0	0	0	0
12	12	0	12	12	0	12	12	0	0	0	0	0	0	0	0	0	0	0
4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0
450	0	0	0	0	0	0	0	0	450	0	0	0	0	0	0	0	0	0
150	150	0	0	0	0	0	0	0	150	150	0	0	0	0	0	0	0	0
50	50	50	0	0	0	0	0	0	50	50	50	0	0	0	0	0	0	0
90	0	0	90	0	0	0	0	0	90	0	0	90	0	0	0	0	0	0
30	30	0	30	30	0	0	0	0	30	30	0	0	30	0	0	0	0	0
10	10	10	10	10	10	0	0	0	10	10	10	10	10	10	0	0	0	0
18	0	0	18	0	0	18	0	0	18	0	0	18	0	0	18	0	0	0
6	6	0	6	6	0	6	6	0	6	6	0	6	6	0	6	6	0	0
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
225	0	0	0	0	0	0	0	0	225	0	0	0	0	0	0	0	0	1

Table(4,4)

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