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Coupled Fixed Point Theorem on Partially Ordered G-metric Space

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Abstract:

Ayedi et al. established coupled coincidence and coupled common fixed point result. Recently Erdal Karapinar, Billur Kayamakcalan and Kenan Tas [19] improved and extend the coupled fixed point of Ayedi et al.[2] Now we prove some recent coupled fixed point theorem in partially ordered G-metric spaces.

Key words: Coupled Fixed Point Theorem, Partially Ordered G-metric Space

INTRODUCTION AND PRELIMINARY:

One of the simplest and the most useful result in the fixed point theory is a Banach Contraction Principal [6].These principal has been generalized in different direction in different spaces by mathematicians.

In [2] Ayedi et al. establised coupled coincidence and coupled common fixed point results for a mixed g-monotone mapping satisfying Non-linear contraction in partially ordered G-metric spaces .These result generalize those of Choudhary and Maity [9]. Consequently Erdal Karapinar ,Billur Kaymakcalan and Kenan Tas improved the result of Ayedi et al.

Definition 1.1 Let X be a non-empty set, and $G : X \times X \times X \rightarrow R+$ be a function satisfying the following properties:
(G1) $G(x, y, z) = 0$, if $x = y = z$,
(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables),
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).
Then the function G is called a generalized metric or, more specially, a G-metric on X, and the pair (X,G) is called a G-metric space.
Every G-metric on X defines a metric dG on X by
$dG(x, y) = G(x, y, y) + G(y, x, x)$, for all $x, y \in X$. (1.1)

Example 1.2 Let (X, d) be a metric space. The function $G: X\times X\times X\rightarrow[0,+\infty)$, defined by
$G(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$
or
$G(x, y, z) = d(x, y) + d(y, z) + d(z, x)$, for all $x, y, z \in X$, is a G-metric on X.

Definition 1.3 Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of
We say that (x_n) is G-convergent to $x \in X$ if lim $n,m\rightarrow+\infty$ $G(x, x_n, x_m) = 0$, that is, for any $\varepsilon > 0$, there exists $N \in N$ such that $G(x, x_n, x_m) < \varepsilon$, for all $n,m \geq N$. We call x the limit of the sequence and write $x_n\rightarrow x$ or lim $n\rightarrow+\infty$ $x_n = x$.

Proposition 1.4 Let (X,G) be a G-metric space. The following are equivalent:

(1) $\{x_n\}$ is G-convergent, to x
(2) $G(x_n, x_n\, x,) \to 0$ as $n \to +\infty$,
(3) $G(x_n, x, x) \to 0$ as $n \to +\infty$,
(4) $G(x_n, x_n, x) \to 0$ as $n, m \to +\infty$.

Definition 1.5 Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called a G-Cauchy sequence if, for any $\varepsilon > 0$, there is $N \in N$ such that $G(x_n, x_m, x_l) < \varepsilon$ for all $m, n, l \geq N$, that is, $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

Proposition 1.6 Let (X, G) be a G-metric space. Then the following are equivalent:
(1) The sequence $\{x_n\}$ is G-Cauchy,
(2) For any $\varepsilon > 0$, there exists $N \in N$ such that $G(x_n, x_m, x_m) < \varepsilon$, for allm, $n \geq N$.

Proposition 1.7 Let (X,G) be a G-metric space. A mapping $f : X \to X$ is G-continuous at $x_0$if and only if it is G-sequentially continuous at x_0, that is, whenever (x_n) is G-convergent tox_0, the sequence $(f(x_n))$ is G-convergent to $f(x_0)$.

Definition 1.8 A G-metric space (X,G) is called G-complete if every G-Cauchy sequence is G-convergent in (X,G).

Definition 1.9 Let (X, G) be a G-metric space. A mapping $F: X \times X \to X$ is said to be continuous if for any two G-convergent sequences $\{x_n\}$ and $\{y_n\}$ converging to x, y respectively, $\{F(x_n, y_n)\}$ is G-convergent to F(x, y).
Let $(X, \leq)$ be a partially ordered set and (X, G) be a G-metric space, $g : X \to X$ be a mapping.
A partially ordered G-metric space, $(X, G, \leq)$, is called g-ordered complete if for each convergent sequence $\{x_n\}_{n=0}^{\infty} \subset X$, the following conditions hold:
(1) if $\{x_n\}$ is a non-increasing sequence in X such that $x_n \to x$ implies $gx \leq g\, x_n$, $\forall n \in N$,

(2) if $\{y_n\}$ is a non-decreasing sequence in X such that $y_n \to y$ implies $gy \geq gy_n$, $\forall n \in N$.
Moreover, a partially ordered G-metric space, $(X,G, \leq)$, is called ordered complete when g is equal to identity mapping in the above conditions (1) and (2).

Definition 1.10 An element $(x, y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Definition 1.11 An element $(x, y) \in X \times X$ is called a coupled coincidence point of a mapping
$F : X \times X \to X$ and $g : X \to X$ if $F(x, y) = g(x)$, $F(y, x) = g(y)$.
Moreover, $(x, y) \in X \times X$ is called a common coupled coincidence point of F and g if $F(x, y) = g(x) = x$, $F(y, x) = g(y) = y$.

Definition 1.12 Let $F : X \times X \to X$ and $g : X \to X$ be mappings. The mappings F and g are said to commute if $g(F(x, y))= F(g(x), g(y))$, for all $x, y \in X$.

Definition 1.13 Let $(X,\leq)$ be a partially ordered set and $F : X \times X \to X$ be a mapping.
Then F is said to have mixed monotone property if F(x, y) is monotone non-decreasing in x and is monotone non-increasing in y, that is, for any $x, y \in X$,
$x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$, for $x_1, x_2 \in X$,
And $y_1 \leq y_2 \Rightarrow F(x, y_2) \leq F(x, y_1)$, for $y_1, y_2 \in X$.

Definition1.14 Let $(X,\leq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \to X$ be two mappings. Then F is said to have mixed g-monotone property if F(x, y) is monotone g-non-decreasing in x and is monotone g-non-increasing in y, that is, for any $x, y \in X$,
$g(x_1) \leq g(x_2) \Rightarrow F(x_1, y) \leq F(x_2, y)$, for $x_1, x_2 \in X$,) and (1.2)
$g(y_1) \leq g(y_2) \Rightarrow F(x, y_2) \leq F(x, y_1)$, for y_1 , $y_2 \in X$. (1.3)

Let $\emptyset$ denote the set of functions $\emptyset^{-1}$: [0,∞)→[0,∞) satisfying
(a) $\emptyset^{-1}(\{0\}) = \{0\}$,
(b $\emptyset$(t)< t for all t > 0,
(c) lim r →t+ $\emptyset$(r)< t for all t > 0.

Main Result:

Theorem-2.1 Let (X, ≤) be a partially ordered set and G be a G-metric on X such that (X ,G) is a complete G-metric .Suppose that there exist Φϵ**Φ,** f:X x X →X and g : X →X such that

$$[G(f(x,y),f(u,v),f(w,z))\ ]+[G(f(y,x),f(v,u),f(z,w))] \le$$
$$[G(gx,gu,gw)+\ G(gy,gv,gz)]\ -\ \Phi\,[G(gx,gu,gw)+G(gy,gv,gz)] \qquad (2.1)$$

For all x, y, u, v, w, zϵ X with $gw \le gu \le gx$ and $gy \le gv \le gz$.suppose also that f is continuous and has the mixed g-monotone property,$f(X \times X) \subseteq g(x)$ and g is continuous and commutes with f. If there exist $x_0, y_0 \epsilon X$ such that $gx_0 \le f(x_0, y_0)$ And $f(y_0, x_0) \le gy_0$. then f and g have a coupled coincident point ,that is there exist $(x, y) \epsilon\, X\, x\, X$ such that $gx = f(x, y)$ and $gy = f(y, x)$

Proof: Given $x_0, y_0 \epsilon X$ satisfying $gx_0 \le f(x_0, y_0)$ and $f(y_0, x_0) \le gy_0$,we shall construct iterative sequence (x_n) and (y_n) in the following way;$f(X \times X) \subseteq g(X)$,we can choose
$x_1, y_1 \epsilon X$ such that $gx_1 = f(x_0, y_0)$ and $gy_1 = f(y_0, x_0)$. similarly we can choose
$x_2, y_2 \epsilon X$ Such that $gx_2 = f(x_1, y_1)$ and $gy_2 = f(y_1, x_1)$. Since f has the mixed g −manotone property, we conclude that $gx_0 \le gx_1 \le gx_2$ and $gy_2 \le gy_1 \le gy_0$, we get from above
$gx_n = f(x_{n-1}, y_{n-1}) \le gx_{n+1} = f(x_n, y_n)$ and $gy_{n+1} = f(y_n, x_n) \le gy_n = f(y_{n-1}, x_{n-1})$
If for some n_0 we have $(gx_{n0+1}, gy_{n0+1}) = (gx_{n0}, gy_{n0})$, then
$f(x_{n0}, y_{n0}) = gx_{n0}$ and $f(y_{n0}, x_{n0}) = gy_{n0}$, that is f and g have a coincidence point. So we assume that $(gx_{n+1}, gy_{n+1}) \neq (gx_n, gy_n)$ for all $n \in N$,Thus we have either

$gx_{n+1} = f(x_n, y_n) \neq gx_n$ or $gy_{n+1} = f(y_n, x_n) \neq gy_n$,

We define $s_n = G(gx_{n+1}, gx_n, gx_n) + G(gy_{n+1}, gy_n, gy_n)$ (2.2)

for all $n \in \mathbb{N}$.Due to the property (G2).we have $s_n > 0$ for all $n \in \mathbb{N}$.By using inequality (2.1),

$$\begin{aligned} G(gx_{n+1}, gx_n, gx_n) + G(gy_{n+1}, gy_n, gy_n) &= G\big(f(x_n, y_n), f(x_{n-1}, y_{n-1}), f(gx_{n-1}, gy_{n-1})\big) \\ &\quad + G\big(f(y_n, x_n), f(y_{n-1}, x_{n-1}), f(gy_{n-1}, gx_{n-1})\big) \\ &\leq \big[G\big(gx_n, gx_{n-1,} gx_{n-1}\big) + G\big(gy_n, gy_{n-1,} gy_{n-1}\big)\big] \\ &\quad - \emptyset\big[G\big(gx_n, gx_{n-1,} gx_{n-1}\big) + G\big(gy_n, gy_{n-1,} gy_{n-1}\big)\big] \quad (2.3) \end{aligned}$$

$$s_n \leq s_{n-1} - \emptyset(s_{n-1}) \quad (2.4)$$

Since $\emptyset(t) < t$ for all $t > 0$, it follows that s_n is monotone decreasing. Therefore, there is some $s \geq 0$ such that $\lim_{n\to\infty} s_n = s$.

Now, we assert that $s = 0$. Suppose, on contrary, that $s > 0$.Letting $n \to +\infty$

$$s = \lim_{n\to+\infty} s_n \leq \lim_{n\to+\infty} s - \emptyset(s) < s$$

This is a contradiction. Thus $s = 0$. Hence

$$\lim_{n\to+\infty} s_n = \lim_{n\to+\infty} G(gx_{n+1}, gx_n, gx_n) + G(gy_{n+1}, gy_n, gy_n) = 0 \quad (2.5)$$

Next we prove that $(gx_n), (gy_n)$ are Cauchy sequence in G metric space (X, G). suppose on contrary, that at least one of $(gx_n), (gy_n)$ is not a Cauchy sequence in (X,G). then there exist $\epsilon > 0$ and sequence of natural number (m_k) and (n_k) such that for every natural number k, $(m_k) > (n_k) \geq k$ and

$$r_k = G(gx_{m_k} gx_{n_k}, gx_{n_k}) + G\left(gy_{m_k,} gy_{n_k} gy_{n_k}\right) \geq \epsilon \quad (2.6)$$

Now corresponding to (n_k), we choose (m_k) to be smallest for which (2.6) holds. Hence

$$G(gx_{m_{k-1}}, gx_{n_k}, gx_{n_k}) + G\left(gy_{m_{k-1},} gy_{n_k} gy_{n_k}\right) < \epsilon$$

Using rectangular inequality and G_5 ,we get

$$\begin{aligned} \epsilon &\leq r_k \\ &\leq G(gx_{m_k} gx_{m_{k-1}}, gx_{m_{k-1}}) + G(gx_{m_{k-1}}, gx_{n_k}, gx_{n_k}) \\ &\quad + G\left(gy_{m_k,} gy_{m_{k-1}} gy_{m_{k-1}}\right) + G(gy_{m_{k-1}}, gy_{n_k}, gy_{n_k}) \\ &= G(gx_{m_{k-1}} gx_{n_k}, gx_{n_k}) + G(gy_{m_{k-1}} gy_{n_k}, gy_{n_k}) + s_{m_{k-1}} \\ &< \epsilon + s_{m_{k-1}} \quad (2.7) \end{aligned}$$

Letting $k \to +\infty$ in the above inequality and using (2.6), we get

$$\lim_{k\to\infty} r_k = \epsilon \quad (2.8)$$

Again, by rectangle inequality, we have

$$r_k = G(gx_{m_k}gx_{n_k}, gx_{n_k}) + G\left(gy_{m_k}, gy_{n_k}gy_{n_k}\right)$$
$$\leq G(gx_{m_k}gx_{m_{k+1}}, gx_{m_{k+1}}) + G(gx_{m_{k+1}}gx_{n_{k+1}}, gx_{n_{k+1}}) + G(gx_{n_{k+1}}gx_{n_k}, gx_{n_k}) +$$
$$G(gy_{m_k}gy_{m_{k+1}}, gy_{m_{k+1}}) + G(gy_{m_{k+1}}gy_{n_{k+1}}, gy_{n_{k+1}}) + G(gy_{n_{k+1}}gy_{n_k}, gy_{n_k}) +$$
$$= s_{n_k} + G(gx_{m_k}gx_{m_{k+1}}, gx_{m_{k+1}}) + G(gy_{m_k}gy_{m_{k+1}}, gy_{m_{k+1}})$$
$$+G(gx_{m_{k+1}}gx_{n_{k+1}}, gx_{n_{k+1}}) + G(gy_{m_{k+1}}gy_{n_{k+1}}, gy_{xn_{k+1}})$$

Using the fact that$G(x,y,y) \leq 2G(y,x,x)$ for any $x, y \in X$,we obtain

$$r_k \leq s_{n_k} + 2\, G(gx_{m_k}gx_{m_k}, gx_{m_{k+1}}) + 2G(gy_{m_k}gy_{m_k}, gy_{m_{k+1}})$$
$$+G(gx_{m_{k+1}}gx_{n_{k+1}}, gx_{n_{k+1}}) + G(gy_{m_{k+1}}gy_{n_{k+1}}, gy_{n_{k+1}})$$
$$= s_{n_k} + 2s_{m_k} + G(gx_{m_{k+1}}gx_{n_{k+1}}, gx_{n_{k+1}}) + G(gy_{m_{k+1}}gy_{n_{k+1}}, gy_{n_{k+1}})$$

Next, Using inequality (2.1), we have

$$G(gx_{m_{k+1}}gx_{n_{k+1}}, gx_{n_{k+1}}) + G(gy_{m_{k+1}}gy_{n_{k+1}}, gy_{n_{k+1}})$$
$$= G\left(f(x_{m_k,}y_{m_k,}), f(x_{n_k,}y_{n_k,}), f(x_{n_k,}y_{n_k,})\right) + G\left(f(y_{n_k,}x_{n_k,}), f(y_{m_k,}x_{m_k,}), f(y_{m_k,}x_{m_k,})\right)$$
$$\leq \left(G(gx_{m_k}gx_{n_k}, gx_{n_k}) + G\left(gy_{m_k}, gy_{n_k}gy_{n_k}\right)\right)$$
$$- \emptyset\left(G(gx_{m_k}gx_{n_k}, gx_{n_k}) + G\left(gy_{m_k}, gy_{n_k}gy_{n_k}\right)\right)$$
$$\leq r_k - \emptyset(r_k)$$

(2.9)

By using (2.5), (2.8) and letting $k \to \infty$, we get,

$$\varepsilon \leq \lim_{k\to\infty} r_k - \emptyset(r_k) < \varepsilon$$

This is contradiction. So $(gx_n), (gy_n)$ are Cauchy sequence in G metric space (X,G).Since (X,G) is complete then there exist $x, y \epsilon X$ such that (gx_n) and (gy_n) are G-convergent to x and y.

from proposition 1.4, we have

$$\lim_{n\to\infty} G(gx_n, x, x) = 0 \quad \text{and} \quad \lim_{n\to\infty} G(gy_n, y, y) = 0$$

Using continuity of g, we get from proposition 1.7,

$$\lim_{n\to\infty} G(g(gx_n), gx, gx) = 0 \quad \text{and} \quad \lim_{n\to\infty} G(g(gy_n), gy, gy) = 0$$

(2.10)

Since $gx_{n+1} = f(x_n, y_n)$ and $gy_{n+1} = f(y_n, x_n)$,employing the commutativity of f and g ,

$$g(gx_{n+1}) = g\big(f(x_n, y_n)\big) = f((gx_n, gy_n))$$
$$g(gy_{n+1}) = g\big(f(y_n, x_n)\big) = g(f(gy_n, gx_n)). \quad (2.11)$$

Now we shall show that$f(x,y) = gx$ and $f(y,x) = gy$

The mapping f is continuous, and since the sequence (gx_n) and (gy_n) are respectively G-convergent to x and y, Using definition 1.9, the sequence $(f(gx_n, gy_n))$is G-convergent to $f(x,y)$. Therefore from (2.11), $(g(gx_{n+1}))$ is G-convergent to $f(x,y)$ By uniqueness of the limit and using (2.10), we have $f(x,y) = gx$. Similarly, we can show that $f(y,x) = gy$. Hence (x,y) is a coupled coincidence point of f and g.This completes the proof.

Theorem-2.2: Let (X, ≤) be a partially ordered set and G be a G-metric on X such that (X, G,≤) is a complete G-metric. Suppose that there exist$\Phi \epsilon \boldsymbol{\Phi}$, f:X x X →X and g : X →X such that

$$[G(f(x,y), f(u,v), f(w,z))] + [G(f(y,x), f(v,u), f(z,w))] \leq [G(gx, gu, gw) + G(gy, gv, gz)] - \Phi\,[G(gx, gu, gw) + G(gy, gv, gz)] \qquad (2.1)$$

For all x, y, u, v, w, zϵ X with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$.suppose also $(g(x), G)$ is complete, f has the mixed g-monotone property,$f(X \times X) \subseteq g(x)$. If there exist $x_0, y_0 \epsilon X$such that $gx_0 \leq f(x_0, y_0)$ And $f(y_0, x_0) \leq gy_0$.then f and g have a coupled coincident point.

Proof: proceeding exactly as in Theorem 2.1.We have $(gx_n)$and(gy_n) are Cauchy sequence in the complete G-metric spaces$(g(x), G)$. Then there exist $x, y \in X$ such that

$g_{x_n} \to gx$ and $g_{y_n} \to gy$.

$\text{Since}(gx_n)$ is non − decreasing and(gy_n) is non − increasing

Then we have $g_{x_n} \leq gx$ and $gy \leq g_{y_n}$ for all $n \geq 0$. If$g_{x_n} = gx$ and $gy = g_{y_n}$for some n≥0,

Then $gx = gx_n \leq gx_{n+1} \leq gx = gx_n$ and $gy = gy_{n+1} \leq gy_n \leq gy$, which implies that $gx_n = gx_{n+1} = f(x_n, y_n)$ and $gy_n = gy_{n+1} = f(y_n, x_n)$,that is a couple coincidence point of f and g. then we assume that $g(x_n, y_n) \neq (gx, gy)$forall $n \geq 0$.

Then by rectangle inequality ,we have

$$\begin{aligned} G(f(x,y),gx,gx) + G(f(y,x),gy,gy) &\le G(f(x,y),gx_{n+1},gx_{n+1}) + G(gx_{n+1},gx,gx) \\ &\quad + G(f(y,x),gy_{n+1},gy_{n+1}) + G(gy_{n+1},gy,gy) \\ &= G\big(f(x,y),f(x_n,y_n),f(x_n,y_n)\big) + G(gx_{n+1},gx,gx) \\ &\quad + G\big(f(y,x),f(y_n,x_n),f(y_n,x_n)\big) + G(gy_{n+1},gy,gy) \\ &\le \{G(gx,gx_n,gx_n) + G(gy,gy_n,gy_n)\} + \\ &\quad \{G(gx_{n+1},gx,gx) + G(gy_{n+1},gy,gy)\} \\ &\quad - \Phi\{G(gx,gx_n,gx_n) + G(gy,gy_n,gy_n)\} + \\ &\quad \{G(gx_{n+1},gx,gx) + G(gy_{n+1},gy,gy)\} \end{aligned}$$

As $n \to \infty$ in above inequality, we have

$G(f(x,y),gx,gx) + G(f(y,x),gy,gy) = 0$,

Which implies that $gx = f(x,y)$ and $gy = f(y,x)$. Hence (x,y) is a coupled coincident point of f and g.

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