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The Expansion of Al-Zughair Transform for Solving Euler's Equation

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Abstract:

In this paper, we introduce Expansion of Al-Zughair transformation. Also, we introduce properties, theorems, proofs and transformations of the constant functions, logarithms functions and other functions. Also, we introduce how we can use this transform and it's inverse to solve the L.O.D.E with variables coefficients (Euler's equation) which has the general form:

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = f(x) \ ,$$

where $a_0, a_2, \dots, a_n$ are constants and f(x) is a function of x. (Al-Zughair transformation is discovered by prof. Ali Hassan Mohammed).see[1]

Key words: Expansion of Al-Zughair transformation

The Expansion of Al-Zughair transform

Definition(1):[1] *Al-Zughair transform* for the function $f(x)$, we denote by $\text{Z}[f(x)]$,where $x \in [1, e]$ is defined by the following integral :

$$\text{Z}[f(x)] = \int_1^e \frac{(lnx)^p}{x} f(x)dx = F(p) \qquad \cdots (1)$$

Such that this integral is convergent , $p > -1$

From (1) ,

Let $lnx = y \qquad \Rightarrow \frac{1}{x} dx = dy$

$if \ x = e \Rightarrow lne = y \Rightarrow y = 1 \ and \ if \ x = 1 \Rightarrow ln(1) = y \Rightarrow y = 0$

Hence ,we can define

$$\wp[f(x)] = \int_0^1 x^p \ f(x) \, dx$$

This formula call *Expansion of Al-Zughair transform.*

Property(1) : (Linear property)

$\wp[Af(x) \pm Bg(x)] = A\wp[f(x)] \pm B\wp[g(x)]$, where A and B are constant , the functions $f(x)$ and $g(x)$ are defined when $x \in [0,1]$

Proof:

$$\wp[Af(x) \pm Bg(x)] = \int_0^1 x^p \ [Af(x) \pm Bg(x)]dx$$

$$= \int_0^1 x^p A\, f(x) \, dx \pm \int_0^1 x^p B\, g(x) \, dx = A \int_0^1 x^p \ f(x) \, dx \pm B \int_0^1 x^p \, g(x) \, dx$$

$$= A \, \wp[f(x)] \pm B \, \wp[g(x)]$$

Transformations for some functions:

We are going to find the *Expansion of al-Zughair transform* for some functions, like the constant functions, logarithm functions, polynomial functions and other functions.

1- If $f(x) = 1$, $p > -1$, then

$$\wp[1] = \frac{1}{p+1}$$

Proof:

$$\wp[1] = \int_0^1 x^p \,.(1)dx = \left.\frac{x^{p+1}}{p+1}\right|_0^1 = \frac{1}{p+1}$$

2- If $f(x) = k$, $p > -1$ and k is constant, then

$$\wp[k] = \frac{k}{p+1}$$

Proof:

$$\wp[k] = \int_0^1 x^p \,.k\, dx = k \int_0^1 x^p\, dx = k \left.\frac{x^{p+1}}{p+1}\right|_0^1 = \frac{k}{p+1}$$

3- If $f(x) = x^n$, $p > -(n+1)$, then

$$\wp[x^n] = \frac{1}{p+(n+1)}$$

Proof:

$$\wp[(x)^n] = \int_0^1 x^p\, x^n dx = \int_0^1 x^{p+n}\, dx = \left.\frac{x^{p+n+1}}{p+n+1}\right|_0^1 = \frac{1}{p+(n+1)}$$

4- If $f(x) = (lnx)^n$, $p > -1$, $n \in \mathbb{N}$, then

$$\wp[(lnx)^n] = \frac{(-1)^n\, n!}{(p+1)^{n+1}}$$

Proof:

if n=1

$$\Longrightarrow \qquad \wp[lnx] = \int_0^1 x^p\, lnx\, dx$$

Integrate by part,

Let $u = lnx \implies du = \frac{1}{x}\, dx$, $dv = x^p dx \implies v = \frac{x^{p+1}}{p+1}$

$$\int_0^1 x^p\, lnx\, dx = \left.\frac{x^{p+1}}{p+1}\, lnx\right|_0^1 - \int_0^1 \frac{x^{p+1}}{p+1}\, \frac{1}{x}\, dx = \frac{-1}{p+1}\int_0^1 x^p\, dx = \frac{-1}{(p+1)^2}$$

if n=2

$$\wp[(lnx)^2] = \int_0^1 x^p\,(lnx)^2\,dx$$

Integrate by part,

$$\Longrightarrow \text{ Let } u = (lnx)^2 \quad \Longrightarrow du = 2\,lnx\,.\frac{1}{x}\;dx\,, dv = x^p dx \Longrightarrow v = \frac{x^{p+1}}{p+1}$$

$$\int_0^1 x^p\,(lnx)^2\,dx = \frac{x^{p+1}}{p+1}(lnx)^2\Big|_0^1 - \int_0^1 \frac{x^{p+1}}{p+1} 2\,lnx\,.\frac{1}{x}\,dx = \frac{-2}{p+1}\cdot\frac{-1}{(p+1)^2}$$

$$= \frac{2}{(p+1)^3}$$

if n=3

$$\wp[(lnx)^3] = \int_0^1 x^p\,(lnx)^3\,dx$$

Integrate by part ,

$$\Longrightarrow \text{ Let } u = (lnx)^3 \quad \Longrightarrow du = 3\,(lnx)^2\,\cdot\frac{1}{x}\;dx\,, dv = x^p dx \quad \Longrightarrow v = \frac{x^{p+1}}{p+1}$$

$$\int_0^1 x^p\,(lnx)^3\,dx = \frac{x^{p+1}}{p+1}(lnx)^3\Big|_0^1 - \int_0^1 \frac{x^{p+1}}{p+1} 3\,(lnx)^2\,\cdot\frac{1}{x}\,dx$$

$$= \frac{-3}{p+1}\,.\frac{(-2)(-1)}{(p+1)^3} = \frac{-3.\,(-2)(-1)}{(p+1)^4} = \frac{-3!}{(p+1)^4}$$

if n=4

$$\wp[(lnx)^3] = \int_0^1 x^p\,(lnx)^3\,dx$$

Integrate by part ,

$$\Longrightarrow \text{ Let } u = (lnx)^4 \quad \Longrightarrow du = 4\,(lnx)^3\,\cdot\frac{1}{x}\;dx\,, dv = x^p dx \quad \Longrightarrow v = \frac{x^{p+1}}{p+1}$$

$$\int_0^1 x^p\,(lnx)^4\,dx = \frac{x^{p+1}}{p+1}(lnx)^4\Big|_0^1 - \int_0^1 \frac{x^{p+1}}{p+1} 4\,(lnx)^3\,\cdot\frac{1}{x}\,dx$$

$$= \frac{-4}{p+1}\,.\frac{(-1)(-2)(-3)}{(p+1)^3} = \frac{4!}{(p+1)^5}$$

Thus ,

$$\wp[(lnx)^n] = \frac{(-1)^n\,n!}{(p+1)^{n+1}} \quad , n = 1,2,3,\dots$$

6- If $f(x) = sin(a\,lnx)$, $p > -1$, $a \in \mathbb{R}$, then

$$\wp[sin(a\ lnx)] = \frac{-a}{(p+1)^2 + a^2}$$

Proof:

$$\wp[sin(a\ lnx)] = \int_0^1 x^p\ sin(a\ lnx)\ dx = \int_0^1 x^p \left(\frac{e^{ialnx} - e^{-ialnx}}{2i}\right) dx$$

$$= \int_0^1 x^p \left(\frac{x^{ia} - x^{-ia}}{2i}\right) dx \ = \frac{1}{2i}\left(\int_1^e x^{p+ia}\ dx - \int_1^e x^{p-ia}\ dx\right)$$

$$= \frac{1}{2i}\left[\frac{x^{p+ia+1}}{p+ia+1} - \frac{x^{p-ia+1}}{p-ia+1}\right]_0^1 = \frac{1}{2i}\left[\frac{1}{p+ia+1} - \frac{1}{p-ia+1}\right]$$

$$= \frac{-a}{(p+1)^2 + a^2}$$

7-If $f(x) = cos(a\ lnx)\,,\ p > -1\ ,a \in \mathbb{R}$, then

$$\wp[cos(a\ lnx)] = \frac{p+1}{(p+1)^2 + a^2}$$

Proof:

$$\wp[cos(a\ lnx)] = \int_0^1 x^p\ cos(a\ lnx)\ dx = \int_0^1 x^p \left(\frac{e^{ialnx} + e^{-ialnx}}{2}\right) dx$$

$$= \int_0^1 x^p \left(\frac{x^{ia} + x^{-ia}}{2}\right) dx = \frac{1}{2}\left(\int_0^1 x^{p+ia}\ dx + \int_0^1 x^{p-ia}\ dx\right)$$

$$= \frac{1}{2}\left[\frac{x^{p+ia+1}}{p+ia+1} + \frac{x^{p-ia+1}}{p-ia+1}\right]_0^1 = \frac{1}{2}\left[\frac{1}{p+ia+1} + \frac{1}{p-ia+1}\right]$$

$$= \frac{p+1}{(p+1)^2 + a^2}$$

8- If $f(x) = sinh(a\ lnx)\,,\ |p+1| > a\,,\ a \in \mathbb{R}$, then

$$\wp[sinh(a\ lnx)] = \frac{-a}{(p+1)^2 - a^2}$$

Proof:

$$\wp[sinh(a\ lnx)] = \int_0^1 x^p\ sin\,h(a\ lnx)\ dx \ = \int_0^1 x^p \left(\frac{e^{alnx} - e^{-alnx}}{2}\right) dx$$

$$= \int_0^1 x^p \left(\frac{x^a - x^{-a}}{2}\right) dx = \frac{1}{2}\left(\int_0^1 x^{p+a}\, dx - \int_0^1 x^{p-a}\, dx\right)$$

$$= \frac{1}{2}\left[\frac{x^{p+a+1}}{p+a+1} - \frac{x^{p-a+1}}{p-a+1}\right]_0^1 = \frac{1}{2}\left[\frac{1}{p+a+1} - \frac{1}{p-a+1}\right]$$

$$= \frac{-a}{(p+1)^2 - a^2}$$

9- If $f(x) = cosh(a\, lnx)$, $|p+1| > a$, $a \in \mathbb{R}$, then

$$\wp[cosh(a\, lnx)] = \frac{p+1}{(p+1)^2 - a^2}$$

Proof:

$$\wp[cosh(alnx)] = \int_0^1 x^p\, cosh(a\, lnx)\, dx = \int_0^1 x^p \left(\frac{e^{alnx} + e^{-alnx}}{2}\right) dx$$

$$= \int_0^1 x^p \left(\frac{x^a + x^{-a}}{2}\right) dx = \frac{1}{2}\left(\int_0^1 x^{p+a}\, dx - \int_0^1 x^{p-a}\, dx\right)$$

$$= \frac{1}{2}\left[\frac{x^{p+a+1}}{p+a+1} + \frac{x^{p-a+1}}{p-a+1}\right]_0^1 = \frac{1}{2}\left[\frac{1}{p+a+1} - \frac{1}{p-a+1}\right]$$

$$= \frac{p+1}{(p+1)^2 - a^2}$$

Theorem(1):

If $\wp[f(x)] = F(p)$ and a is constant , then $\wp[x^{\pm a} f(x)] = F(p \pm a)$

Proof:

$$\wp\,[x^{\pm a} f(x)] = \int_0^1 x^p\, x^{\pm a} f(x)\; dx = \int_0^1 x^{p \pm a}\, f(x)\; dx = F(p \pm a)$$

For example:

$$1 - \wp\,[x^2 \cos(5\, lnx)] = \frac{p+3}{(p+3)^2 - 25}$$

$$2 - \wp[x^{-8}\, (lnx)^6] = \frac{6!}{(p-7)^7}$$

Definition(2):

Let $f(x)$ be a function where , $x \in [0,1]$ and $\wp[f(x)] = F(p)$, $f(x)$ is said to be an inverse for the *Expansion of al-Zughair transform* and written as $\wp^{-1}[F(p)] = f(x)$ where $\wp^{-1}$ returns the transformation to the original function.

For example:

$$\wp^{-1}\left[\frac{k}{p+1}\right] = k; \qquad p > -1\,, \text{ since } \quad \wp[k] = \frac{k}{p+1}$$

$$\wp^{-1}\left[\frac{1}{p+(n+1)}\right] = x^n \;;\; p > -(n+1), \qquad \text{since } \;\wp[x^n] = \frac{1}{p+(n+1)}$$

$$\wp^{-1}\left[\frac{(-1)^n\, n!}{(p+1)^{n+1}}\right] = (lnx)^n \;;\; p > -1\,, \text{ since } \;\wp[(lnx)^n] = \frac{(-1)^n\, n!}{(p+1)^{n+1}}$$

$$\wp^{-1}\left[\frac{-a}{(p+1)^2+a^2}\right] = sin(a\, lnx) \qquad ;\; p > -1$$

$$\text{since}\,, \quad \wp[sin(a\, lnx)] = \frac{-a}{(p+1)^2+a^2}$$

$$\wp^{-1}\left[\frac{p+1}{(p+1)^2-a^2}\right] = cosh(a\, lnx) \qquad ;\; |p+1| > a$$

$$\text{since}\,, \quad \wp[cosh(a\, lnx)] = \frac{p+1}{(p+1)^2-a^2}$$

$\wp^{-1}$ has the linear property as it is for *Expansion of Al-Zughair transform* .i.e

$$\begin{aligned}\wp^{-1}&[a_1F_1(p) \pm a_2F_2(p) \pm \cdots \pm a_nF_n(p)\,] \\ &= a_1\wp^{-1}[F_1(p)] \pm a_2\wp^{-1}[F_2(p)] \pm \cdots \pm a_n\wp^{-1}[F_n(p)] \\ &= a_1f_1(x) \pm a_2f_2(x) \pm \cdots \pm a_nf_n(x)\end{aligned}$$

Where $a_1\,, a_2\,, \dots\,, a_n$ are constants, the functions $f_1(x), f_2(x), \cdots, f_n(x)$ are defined when $x \in [0,1]$

Theorem(2): If $\wp^{-1}[F(p)] = f(x)$,then $\wp^{-1}[\,F(p \pm a)] = x^{\pm a}f(x)$

Where a is constant.

Proof:

$$\wp^{-1}[F(p \pm a)] = x^{\pm a}f(x) = x^{\pm a}\,\wp^{-1}[F(p)]$$

For example:

$$1 - \;\wp^{-1}\left[\frac{1}{(p-3)^2+49}\right] = \frac{-1}{7}x^{-4}sin(7lnx)$$

$$2 - \;\wp^{-1}\left[\frac{1}{(p+4)^{13}}\right] = \frac{1}{12!}x^3\,(lnx)^{12}$$

Definition(3):[2]

The equation

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = f(x)$$

where $a_0, a_2, \dots, a_n$ are constants and $f(x)$ is a function of x , is called

Euler's equation

Theorem(3):

If the function $y(x)$ is defined for $x \in [0,1]$ and its derivatives $y^{(1)}(x)\,, y^{(2)}(x)\,, \dots, y^{(n)}(x)$ are exist then:

$$\wp\left[x^n y^{(n)}\right] = y^{(n-1)}(1) + (-1)^n (p+n) y^{(n-2)}(1)$$
$$+(-1)^{n-1}(p+n)\big(p+(n-1)\big) y^{(n-3)}(1) + \cdots$$
$$+(p+n)\,\big(p+\ (n-1)\big) \dots (\,p+2) y(1) + (-1)^n (p+n)!\,\wp[y]$$

Proof:

If n=1 ,

$$\wp[x\, y'] = \int_0^1 x^p \cdot x\ \cdot y'\, dx = \int_0^1 x^{p+1} \cdot\ y'\, dx$$

Let $u =\ x^{p+1}\ \Rightarrow\ du = (p+1)\, x^p dx\ , dv = y'\, dx \Longrightarrow\quad v = y$

$$\int_0^1 x^{p+1} \cdot\ y'\, dx = x^{p+1} \cdot y|_0^1 - (p+1) \int_0^1 x^p \cdot y\ dx$$
$$= y(1) - (p+1)\, \wp[y]$$

If n=2 ,

$$\wp[x^2\, y''] = \int_0^1 x^p \cdot x^2 \cdot\ y''\, dx = \int_0^1 x^{p+2} \cdot\ y''\, dx$$

Let $u =\ x^{p+2}\ \Rightarrow\ du = (p+2)\, x^{p+1} dx\ , dv = y''\, dx \Longrightarrow\quad v = y'$

$$\int_0^1 x^p \cdot x^2 \cdot\ y''\, dx = x^{p+2} \cdot y'|_0^1 - (p+1) \int_0^1 x^{p+1} \cdot y'\ dx$$

$$\int_0^1 x^{p+2} \cdot\ y''\, dx = x^{p+2}. y'|_0^1 - (p+2) \int_0^1 x^{p+1} \cdot\ y'\, dx$$

$$= y'(1) - (p+2)\,\wp[x^2\,.\,y'']$$
$$= y'(1) - (p+2)y(1) + (p+2)(p+1)\,\wp[y]$$

If n=3 ,

$$\wp[x^3\,y'''] = \int_0^1 x^{p+3}\cdot y'''\ dx$$

Let $u = x^{p+3} \Rightarrow du = (p+3)\,x^{p+2}dx\ ,dv = y'''\,dx \Longrightarrow\quad v = y''$

$$\int_0^1 x^{p+3}\cdot y'''\ dx = x^{p+3}\cdot y''|_0^1 - (p+3)\int_0^1 x^{p+2}\ y''\,dx$$
$$= y''(1) - (p+3)\,\wp[x^2\ \cdot y'']$$
$$= y''(1) - (p+3)y'(1) + (p+3)(p+2)\,y(1) - (p+3)\,(p+2)\,(p+1)\,\wp[y]$$

Then ,

$$\wp\left[x^n y^{(n)}\right] = y^{(n-1)}(1) + (-1)^n(p+n)y^{(n-2)}(1) + (-1)^{n-1}(p+n)\big(p+(n-1)\big)y^{(n-3)} + \cdots + (p+n)\,\big(p+\ (n-1)\big)\ldots(\,p+2)y(1) + (-1)^n(p+n)!\,\wp[y]$$

Solving the Linear Ordinary Differential Equations with Variable Coefficients

One of the most important applications of the *Expansion of Al-Zughair transform* is solving the linear differential equations with variable coefficients. Suppose we have a linear ordinary differential equation of order (n) with variable coefficients and subject to some initial conditions, which general structure can be written as:

$$a_0x^ny^{(n)} + a_1x^{n-1}y^{(n-1)} + \cdots + a_{n-1}x\,y' + a_ny = f(x) \qquad \cdots(2)$$

Where $a_0, a_1, \ldots, a_n$ are constants, $y^{(\mathrm{n})}$ is the nth derivative of the function $y(x), f(x)$ is a continuous function whose the *Expansion of Al-Zughair transform* can be determined , and $y(1)$,…,$y^{(n-1)}$ (1) are defined. To find a solution of DE**.**(2) we take the *Expansion of Al-Zughair transform* ($\wp$) to both sides of (2) , after simplification we can put $\wp$(y) as follows :

$$\wp(y) = \frac{h(p)}{k(p)} \; ; \; k(p) \neq 0 \qquad \dots (3)$$

Where h , are polynomials of p , such that the degree of h is less than the degree of k and the polynomial k with known prime cofactors. By taking $\wp^{-1}$ to both sides of equation (3) we will get:

$$y = \wp^{-1}\left[\frac{h(p)}{k(p)}\right] \qquad \dots (4)$$

Equation (4) represents the general solution of the differential equation (2) which is form is given by :

$$y = A_0 k_0(x) + A_1 k_1(x) + \cdots + A_m k_m(x) \qquad \dots (5)$$

Such that k_0 , k_1 ,..., k_m are functions of x and that A_0, A_1,..., A_m are constants, whose number equals to the degree of $k(p)$. To find the values of constants of A_1 , A_2 ,..., A_m we will use partial fractions decomposition.

Example(1): To find the solution of the differential equation

$x\,y' - 2y = sin(lnx) \quad ; \quad y(1) = -1$

We take$\wp$-transform to both sides of above equation we get:

$$\wp[x\,y'] - 2\,\wp[y] = \wp[sin(lnx)]$$

$$y(1) - (p+1)\,\wp[y] - 2\,\wp[y] = \frac{-1}{(p+1)^2+1}$$

$$-1 - (p+3)\,\wp[y] = \frac{-1}{(p+1)^2+1}$$

$$\wp[y] = \frac{-p^2-2p-1}{(p+3)\,[(p+1)^2+1]}$$

By taking $\wp^{-1}$-transform to both sides of above equation we get:

$$y = Z^{-1}\left[\frac{A}{(p+3)} + \frac{Bp+C}{(p+1)^2+1}\right]$$

$A + B = -1$

$2A + 3B + C = -2$

$2A + 3C = -1$

Hence ,

$$A = \frac{-4}{5} \;,\; B = \frac{-1}{5} \;,\; C = \frac{1}{5}$$

$$y = \wp^{-1}\left[\frac{\frac{-4}{5}}{(p+3)} + \frac{\frac{-1}{5}p + \frac{1}{5}}{(p+1)^2 + 1}\right]$$

$$y = \wp^{-1}\left[\frac{\frac{-4}{5}}{(p+3)} + \frac{\frac{-1}{5}(p+1)}{(p+1)^2 + 1} + \frac{\frac{2}{5}}{(p+1)^2 + 1}\right]$$

$$y = \frac{-4}{5}x^2 - \frac{1}{5}\cos(lnx) - \frac{2}{5}\sin(lnx)$$

Example(2): To find the solution of the differential equation

$x^3\, y''' + 3x^2\, y'' = x^{-4}\, lnx \quad ; \quad y(1) = y'(1) = y''(1) = 0$

We take $\wp$-transform to both sides of above equation we get :

$$\wp[x^3\, y'''] + 3\,\wp[x^2\, y''] = \wp[x^{-4}\, lnx]$$

$y''(1) - (p+3)y'(1) + (p+3)(p+2)y(1) +$

$$3y'(1) - 3(p+2)y(1) + 3(p+2)(p+1)\wp[y] = \frac{-1}{(p-3)^2}$$

$$\wp[y] = \frac{-1}{p(p+1)(p+2)(p-3)^2}$$

By taking $\wp^{-1}$-transform to both sides of above equation we get:

$$y = \wp^{-1}\left[\frac{A}{p} + \frac{B}{p+1} + \frac{C}{p+2} + \frac{D}{p-3} + \frac{E}{(p-3)^2}\right]$$

$A + B + C + D = 0$

$-3A - 8B - 5C + E = 0$

$-7A - 21B + 3C - 7D + 3E = 0$

$15A - 18B + 9C - 6D + 2E = 0$

$18A = 1$

Hence ,

$$A = \frac{1}{18} \;,\; B = \frac{1}{48} \;,\; C = \frac{-19}{300} \;,\; D = \frac{-47}{3600} \;,\; E = \frac{1}{60}$$

$$y = \wp^{-1}\left[\frac{\frac{1}{18}}{p} + \frac{\frac{1}{48}}{p+1} + \frac{\frac{-19}{300}}{p+2} + \frac{\frac{-47}{3600}}{p-3} + \frac{\frac{1}{60}}{(p-3)^2}\right]$$

$$\Rightarrow \quad y = \frac{1}{18}x^{-1} - \frac{19}{300}x - \frac{47}{3600}x^{-4} - \frac{1}{60}x^{-4}\,lnx + \frac{1}{48}$$

Example (3): To find the solution of the differential equation
$x^2\,y'' + x\,y' - y = \cosh(2lnx); \quad y(1) = 0 \;\; ; \;\; y'(1) = -1$
We take $\wp$-transform to both sides of above equation we get :

$$\wp[x^2\,y''] + \wp[x\,y'] - \wp[y] = \wp[\cosh(2lnx)]$$

$$y'(1) - (p+2)y(1) + (p+2)(p+1)\wp[y] + y(1) - (p+1)\wp[y]$$
$$-\wp[y] = \frac{p+1}{(p+1)^2 - 4}$$

$$-1 + p(p+2)\wp[y] = \frac{p+1}{(p+1)^2 - 4}$$

$$\wp[y] = \frac{p^2 + 3p - 2}{p\,(p+2)[(p+1)^2 - 4]}$$

By taking $\wp^{-1}$-transform to both sides of above equation we get:

$$y = \wp^{-1}\left[\frac{A}{p} + \frac{B}{p+2} + \frac{Cp + D}{(p+1)^2 - 4}\right]$$

$A + B + C = 0$
$4A + 2B + 2C + D = 1$
$A - 3B + 2C = 3$
$-6A = -2$
$18A = 1$
Hence,

$$A = \frac{1}{3} \;\; , \;\; B = \frac{-2}{3} \;\; , \;\; C = \frac{1}{3}, \;\; D = \frac{1}{3},$$

$$y = \wp^{-1}\left[\frac{\frac{1}{3}}{p} + \frac{\frac{-2}{3}}{p+2} + \frac{\frac{1}{3}p + \frac{1}{3}}{(p+1)^2 - 4}\right]$$

$$\Rightarrow \quad y = \frac{1}{3}x^{-1} - \frac{2}{3}x + \frac{1}{3}coh(2lnx)$$

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