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Adomian decomposition method and application on solving nonlinear partial differential equations and nonlinear system partial equation

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Abstract:

In this paper, Adomian decomposition method is discussed and applied to solve the nonlinear partial differential equations and nonlinear system partial equation, a nonlinear partial differential equations can be found in wide variety scientific and engineering applications. Many important mathematical models can be expressed in terms of nonlinear differential equations. Firstly, it will apply this method to solving nonlinear partial differential equations and secondly, it will apply it on nonlinear system partial equation then will see how this method is very suitable to solve this type of equations.

Key words: Adomian decomposition method, partial differential equations, nonlinear partial differential equations, nonlinear system partial equation, Adomian's polynomial, the decomposition method.

INTRODUCTION :

The method is developed in the 1970s by George Adomian, The Adomian decomposition method (ADM) is a semi-analytical method for solving ordinary and partial nonlinear differential equations. The aim of this method is towards a unified theory for the solution of partial differential equations (PDE), an aim which is superseded by the more general theory of the homotopy analysis method. The most general form of nonlinear partial differential equations is given by:

$$F\big(u, u_t, u_x, u_y, x, y, t\big) = 0 \qquad (1)$$

With initial and boundary conditions :

$$u(x,y,0) = \emptyset(x,y) \qquad (2)$$

$$u(x,y,t) = f(x,y,t) \qquad (3)$$

In recent year, much researches and papers have been focused on the numerical solution of nonlinear partial equations by using numerical methods and developing these methods (AL-Safi,2007; Leveque, 2006; Rossler, Husner,1997; Wescot, Rizwan-Uddin,2001). In the numerical methods, which are commonly used for solving this kind of equations large size or difficult of computation appears and usually, the round-off error causes the loss of accuracy. The Adomian decomposition method which needs less computation was employed to solve many problems (Celiket,2006; Javidiand Golbabai,2007). Therefore, we applied the Adomian decomposition method to solve some models of nonlinear partial equations, this study reveals that Adomian decomposition method is very efficient for nonlinear models, and its results give evidence that high accuracy can be achieved.

THE DECOMPOSITION METHOD:

The principle of Adomian decomposition method (ADM), when applied to general nonlinear equation, is in the following form

$$Ly + Ry + Ny = g \qquad (4)$$

The linear term decomposed into Lu+Ru , but the nonlinear terms are represented by Nu, so L is an easily invertible operator, R is the reminder of the linear operator. For convenience, L is taken as the highest order derivative. By inverse operator L, with $L^{-1} = \int_0^t (\,.\,)dt$. equation (4) can be hence as :

$$L^{-1}(Lu) = L^{-1}g - L^{-1}(Ru) - L^{-1}(Nu) \qquad (5)$$

The decomposition method represents the solution of equation (5) as the following infinite series:

$$u = \sum_0^\infty u_n \qquad (6)$$

The nonlinear term N(u)=Ψ(u) will be decomposed by the infinite series of Adomian polynomials

$$N(u) = \sum_0^\infty A_n \qquad (7)$$

Where A_n are Adomian's polynomials, which are defined as

$$A_n = \frac{1}{n!}.\frac{d^n}{d\lambda^n}[\Psi(\sum_{k=0}^\infty \lambda^k y^k)]_{\lambda=0} \qquad (8)$$

Now, substituting equation (6) and (7) into equation (5), we have

$$u = \sum_0^\infty u_n = u_0 - L^{-1}[R(\sum_0^\infty u_n)] - L^{-1}(\sum_0^\infty A_n) \qquad (9)$$

Consequently we can write

$$u_0 = \emptyset + L^{-1}(g)$$
$$u_1 = -L^{-1}R(u_0) - L^{-1}(A_0) \qquad (10)$$
$$u_2 = -L^{-1}R(u_1) - L^{-1}(A_1)$$

.

.

.

$$u_n = -L^{-1}R(u_{n-1}) - L^{-1}(A_{n-1})$$

Where $\emptyset$ is the initial condition, Hence all the terms of u are calculated and the general solution obtained according to ADM as $u = \sum_0^\infty u_n$. the convergent, of this series has been proved in. However, for some problems this series cannot be determined, so we used an approximation of the solution from truncated series

$$U_m = \sum_{n=0}^{M} u_n \qquad (11)$$

With

$$\lim_{M\to\infty} U_M = u \qquad (12)$$

Example 1. *Consider the following hyperbolic nonlinear problem*

$$\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x} \qquad 0 < x \leq 1\,, 0 \leq t \leq 1 \qquad (13)$$

With the initial condition

$$u(x,0) = \frac{x}{10} \qquad 0 < x \leq 1 \qquad (14)$$

Equation (14) has the exact solution

$$u(x,t) = \frac{-x}{(t-10)}$$

Now, we use ADM to solve equation (1). in this problem we have

$$Nu = \Psi(u) = u\frac{\partial u}{\partial x}\ ,\ g(x,t) = 0\ ,\ Ru = 0\ \ ,\ Lu = \frac{\partial u}{\partial t}$$

And

$$\emptyset = u(x,0) = \frac{x}{10}$$

By using equation (10) Adomian's polynomials can be derived as follows :

$$A_0 = u_0\frac{\partial u_0}{\partial x}$$

$$A_1 = u_1\frac{\partial u_0}{\partial x} + u_0\frac{\partial u_1}{\partial x}$$

$$A_2 = u_2\frac{\partial u_0}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_0\frac{\partial u_2}{\partial x}$$

$$A_3 = u_3\frac{\partial u_0}{\partial x} + u_2\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_2}{\partial x} + u_0\frac{\partial u_3}{\partial x}$$

…

…

And so on.

The rest of polynomials can be constructed in similar manner. By using equation (10), we have

$$u_0 = \frac{x}{10}$$

$u_1 = \frac{x}{10}\left(\frac{t}{10}\right)$
$u_2 = \frac{x}{10}(\frac{t}{10})^2$
$u_3 = \frac{x}{10}(\frac{t}{10})^3$
…
…
$u_n = \frac{x}{10}(\frac{t}{10})^n$

Substituting these individual terms in equation (6), we get
$u(x,t) = \frac{x}{10}\left[1 + \left(\frac{t}{10}\right) + (\frac{t}{10})^2 + (\frac{t}{10})^3 + \cdots + (\frac{t}{10})^n + \cdots\right]$
This gives the exact solution (10). This result can be verified through substitution.

Example 2. *Consider the following nonlinear problem*
$\frac{\partial u}{\partial t} = x^2 - \frac{1}{4}(\frac{\partial u}{\partial x})^2 \quad 0 < x \leq 1 \ , \ 0 \leq t \leq 1$ (15)
With the initial condition
$u(x,0) = 0 \quad 0 < x \leq 1$ (16)
Equation (15) has the exact solution
$u(x,t) = x^2 \tanh(t)$ (17)

Now, we use ADM to solve equation (15). In this problem we have
$Nu = \Psi(u) = (\frac{\partial u}{\partial x})^2 \ , g(x,t) = x^2 \ , Ru = 0 \ , Lu = \frac{\partial u}{\partial t}$
And
$\emptyset = u(x,0) = 0$
By using equation (10) Adomian's polynomials can be derived as follows :
$A_0 = (\frac{\partial u_0}{\partial x})^2$
$A_1 = 2\frac{\partial u_0}{\partial x} + \frac{\partial u_1}{\partial x}$
$A_2 = (\frac{\partial u_1}{\partial x})^2 + 2\frac{\partial u_0}{\partial x} + \frac{\partial u_2}{\partial x}$
$A_3 = 2\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} + 2\frac{\partial u_0}{\partial x}\frac{\partial u_3}{\partial x}$

$A_4 = (\frac{\partial u_2}{\partial x})^2 + 2\frac{\partial u_1}{\partial x}\frac{\partial u_3}{\partial x} + 2\frac{\partial u_0}{\partial x}\frac{\partial u_4}{\partial x}$

…

…

And so on.

The rest of polynomials can be constructed in similar manner.

By using equation (10), we have

$u_0 = x^2 t$

$u_1 = -\frac{1}{3}x^2 t^3$

$u_2 = \frac{2}{15}x^2 t^5$

$u_3 = -\frac{17}{315}x^2 t^7$

$u_4 = \frac{62}{2835}x^2 t^9$

$u_5 = -\frac{1382}{155925}x^2 t^{11}$

…

….

Substituting these individual terms we obtain

$u(x,t) = x^2\left[t - \frac{1}{3}t^2 + \frac{2}{15}t^5 - \frac{17}{315}t^7 + \frac{62}{2835}t^9 - \frac{1382}{155925}t^{11} + \cdots\right]$

This gives the exact solution.

ADOMIAN DECOMPOSITION METHOD FOR SOLVING NONLINEAR SYSTEM PARTIAL EQUATION

Will apply and explain Adomian Decomposition Method for Solving Nonlinear System Partial Equation by the following example:

Example 3. Consider the nonlinear system of equations

$$\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad (18)$$

With initial condition

$u(x,y,0) = v(x,y,0) = x + y$

Equation (18) has the exact solution

$$u(x,y,t) = v(x,y,t) = \frac{x+y}{1-2t} \qquad (19)$$

In this problem equation (9) can be written as :
$u = L^{-1}(Nu)$, $v = L^{-1}(Nv)$
Where $L(.)\frac{\partial}{\partial t}$, $Nu = \Psi_1(u, v) = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$, $Nv = \Psi_2(u, v) = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$ by using equation (6) the solutions can be written as
$u(x, y, t) = \sum_0^\infty u_n(x, y, t)$
$v(x, y, t) = \sum_0^\infty v_n(x, y, t)$
$u_0 = u(x, y, 0)$ $u_{n+1} = L^{-1}\big(\Psi_1(u_n, v_n)\big)$
$v_0 = v(x, y, 0)$ $v_{n+1} = L^{-1}\big(\Psi_2(u_n, v_n)\big)$
We decompose Ψ_1 and Ψ_2 according to the series $\sum_{n=0}^\infty A_n$ and $\sum_{n=0}^\infty B_n$ respectively, where $A_n$and B_n are calculated by Adomian's polynomials which are defied in equation (10) then we obtain
$A_0 = u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}$
$A_1 = u_0\frac{\partial u_1}{\partial x} + v_0\frac{\partial u_1}{\partial y} + u_1\frac{\partial u_0}{\partial x} + v_1\frac{\partial u_0}{\partial y}$
$A_2 = u_0\frac{\partial u_2}{\partial x} + v_0\frac{\partial u_2}{\partial y} + u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y} + u_2\frac{\partial u_0}{\partial x} + v_2\frac{\partial u_0}{\partial y}$
…
…
Similarly :
$B_0 = u_0\frac{\partial v_0}{\partial x} + v_0\frac{\partial v_0}{\partial y}$
$B_1 = u_0\frac{\partial v_1}{\partial x} + v_0\frac{\partial v_1}{\partial y} + u_1\frac{\partial v_0}{\partial x} + v_1\frac{\partial v_0}{\partial y}$
$B_2 = u_0\frac{\partial v_2}{\partial x} + v_0\frac{\partial v_2}{\partial y} + u_1\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_1}{\partial y} + u_2\frac{\partial v_0}{\partial x} + v_2\frac{\partial v_0}{\partial y}$
…
…
By using equation (6)
$u_0 = x + y$
$v_0 = x + y$
$u_1 = (x + y)(2t)$
$v_1 = (x + y)(2t)$
$u_2 = (x + y)(2t)^2$

$v_2 = (x + y)(2t)^2$
$u_3 = (x + y)(2t)^3$
$v_3 = (x + y)(2t)^3$
$\cdots$
$\cdots$
$u_n = (x + y)(2t)^n$
$v_n = (x + y)(2t)^n$
From equation (9) we get :
$u(x, y, t) = v(x, y, t) = (x + y)[1 + 2t + (2t)^2 + (2t)^3 + \cdots + (2t)^n + \cdots]$
Which gives the exact solution to equation (19).

CONCLUSION:

In this paper we showed how Adomian decomposition method verified to get the solution of a nonlinear partial differential equation and how to apply on nonlinear system partial Equation and also it's one suitable type to solve this kind of equations because it needs less computation to solve many problems.

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