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Probability Analysis of Maximum Monthly Precipitation of Upper Indus Basin

ASIF IQBAL
Department of Mathematics, University of Karachi
SYED AHMAD HASSAN[1]
Department of Mathematics, University of Karachi

Abstract

Statistics of the magnitude of extreme precipitation for a specific probability of occurrences is essential for the design of hydraulic structures. The capacity of such structures is generally designed sequentially to prevent flooding; thereby reducing the loss of life and infrastructure is achieved. In this study the probability analysis of maximum monthly precipitation data of seven weather stations (Astore, Chitral, Chilas, Gilgit, Gupis, Skardu and Swat) of upper Indus basin for a period of 30 years (1986-2015) is performed by the frequency factor approach. The expected maximum monthly precipitations were evaluated for the probability distributions via: Normal, Log-Normal, Log-Pearson type-III and Gumbel distributions for return periods of 2, 5, 10, 25, 50 and 100 years. The return period of the observed maximum monthly precipitation magnitude at each station is determined by the Weibull's plotting position method. The best-fit probability distribution was selected by using Chi-square and Kolmogorov Simonov test. Moreover, the exponential equations of the form $y = ae^{bx}$ *are fitted with coefficient of determination respectively to the observed magnitude of maximum monthly precipitation with return period T for each station. It was observed that the log-normal and log-Pearson type-III are the best-fit probability distribution to predict the maximum monthly precipitation of the upper Indus region for different return periods. To be consistent, the forecasted maximum precipitations for the selected return periods are assessed in*

[1] Corresponding author: ahmedhassan@uok.edu.pk

comparison with the results of the plotting position. This study suggested that plotting position methods can provide reliable estimates of precipitation for higher return periods.

Key Words: Precipitation analysis, Return period, Probability distribution, Plotting Position

INTRODUCTION:

Several studies explore that the mean temperature of the earth's surface has been rising due to human activities. However, this may support the preservation of atmospheric water capacity through expanding vaporization from the sea and land surfaces prompts reinforcing normal global precipitation [1]. Precipitation is a primary source of the hydrological cycle that deal with the water resources and ecological integrity [2]. In the last couple of decades, intensity of precipitation varies in several region of the world[3]. Moreover, past studies on seasonal and annual precipitation of global and local extent uncovers that the annual precipitation is extremely variable over many regions of the world [3]. This variability may rely upon the environmental change and the length of the observed period. They may bring sometime hazardous flood and dry spell occasions that have tremendous impact of socio-economic effects on human life. The frequency analysis is the most widely recognized approach in reducing flood damages and economic losses [4, 5]. Various past studies utilizes different techniques of frequency analysis such as interval method, ranking method and applying theoretical frequency distributions [6]. Although, the length of the time series depend on the temporal variability in precipitation while a period of 30 years or more is usually considered satisfactory [7].

Throughout the previous 30 years, flood events are higher in Bangladesh, China, India, Pakistan, Philippines, and

Thailand. Pakistan consistently experiences disastrous that cause immense misfortune as far as human's lives, economy and foundations. The glaciers of Indus Basin generated more than 70% of water which is the only sole source of freshwater supply for the ecosystem, agriculture and domestic use in Pakistan[8]. The Indus River originates from the Tibetan Plateau and the Himalayas and covers six major tributaries: the Sutlej, Ravi, Beas, Jhelum, Chenab and Kabul rivers in Pakistan. It is one of the principal water reserves and its preservation is extremely fundamental for the millions of people living in downstream territories. The upper Indus basin (UIB) is located at the boundary between tropical and continental climate influences and it is the main area for climatological studies in Pakistan[9]. The UIB extends from the Tibetan Plateau to northeast Afghanistan whereas the Jhelum, Chenab, and Sutlej rivers originate in the Karakoram, Hindu Kush, and Himalayan mountain ranges respectively [10]. Generally, Pakistan received significant amount of precipitation during summer/monsoon (July-September) season[11]. However, most of the annual precipitation of UIB falls in winter and spring and initiates from the west[12]. Monsoonal season bring unpredictable rain to trans-Himalayan areas but not entirely precipitation generated from monsoon sources even during summer season[13]. The decreasing precipitation pattern is found over UIB and expanding drift over Chitral and the northwest Karakoram, while the greater part of the precipitation changes found are statistically insignificant[14, 15].The assessment of the possible weather influences on water resources over the UIB would be helpful in understanding the water issues in Pakistan particularly for the zones of lower Indus basin[16-18].

Precipitation is one of the most important natural resources for human survival and a major component that ensures the ecosystem. Several applications in water resources engineering require appropriate estimate of precipitation depth and its return period from available historical data[19]. Different studies were used to apply probability distribution for precipitation data all over the world. Fowler and Kilsby [20] suggested two approaches to deal with measure precipitation extremes, the primary approach utilizes a percentile or quantile technique and the second approach uses statistical distributions to assess extreme precipitation and their return period on annual basis[21-23]. In addition, the log-Pearson type III, generalized extreme value and Gumbel distribution are the most commonly used for extreme events analysis [24, 25]. Several studies have examined different plotting position formulas in selecting a suitable frequency distribution [26]. However, the plotting positions have been used in estimating the magnitude of hydrological events and their corresponding return periods. Various plotting position methods are available in literature for the evaluation of exceedance probabilities [27-33]. All plotting position relationships give same values close to the centre of the distribution however; it might be differ significantly in the tails and satisfying Gumbel five criteria of plotting position relationship. The objective of this study is to evaluate the precipitation magnitude corresponding to different return periods. This study is attempted to predict the expected maximum monthly precipitation and its variability over seven weather stations (Astore, Chitral, Chilas, Gilgit, Gupis, Skardu and Swat) of upper Indus basin by using different probability distributions via Normal, Log-Normal, Gumbel and Log-Pearson type III distributions for different return periods corresponding to various probability levels.

Materials and Methods:

In order to analyse the amount of maximum monthly precipitation data, it is important to classify the best fit distribution to the data. The magnitudes of the maximum monthly precipitations data were taken for the purpose of frequency analysis. This study incorporates seven weather stations (Astore, Chitral, Chilas, Gilgit, Gupis, Skardu and Swat) of upper Indus basin (Fig.1) in order to predict the probable maximum monthly precipitation for different return periods based on theoretical probability distributions. The statistical analysis was performed by using maximum monthly precipitation data for the period of 30 years (1986-2015) by using four probability distributions (Normal, Lognormal, Gumbel and Log-Pearson type III). In determining the best distributions for maximum monthly precipitation estimation, the Weibull's plotting position method and four probability distributions were plotted against the monthly precipitation dataset. The goodness of fit can be observed by a Chi-square and Kolmogorov Smirnov tests. Furthermore, the coefficients of determination R^2 employed to determine how well the predicted maximum precipitation had the capacity to predict the observed maximum precipitation. In addition, the statistical parameters such as mean, standard deviation, coefficient of variation and coefficient of skewness were computed in order to judge the variability of maximum monthly precipitation. For the present study, the monthly precipitation data of seven weather station of upper Indus basin for the period of 30 years (1986-2015) is obtained from the Pakistan Meteorological Department (PMD).

Return Period:

The return period or recurrence interval (R_t) is the average interval of time within which any extreme event of a given magnitude will be equalled or surpassed in any event once [34]. In this study the return period was determined by Weibull's plotting position method[28] by arranging precipitation data in the descending order and giving their respective rank as :

$$R_t = \frac{N+1}{\hat{r}} \qquad (1)$$

Where N is the total number of precipitation years recorded and $\hat{r}$ is the rank of observed precipitation magnitudes arranged in the descending order.

Weibull's plotting position method was utilized for the estimation of the observed maximum monthly precipitation in mm for the return periods of 2, 5, 10, 25, 50, 100 and 200 years.

Probability analysis using frequency factor method:

The extent of extreme natural disasters such as flood, drought and heavy storm with their number of occurrences with time can be predicted by using frequency factor method. The frequency factor equation (α_{R_t}) for calculating the expected values of the precipitation corresponding to the return period R_t is defined as:

$$\alpha_{R_t} = \overline{M} + f_{R_t}\sigma \qquad (2)$$

where, $\overline{M}$ is the arithmetic mean of annual precipitation series, (f_{R_t})is the frequency factor and σ is the standard deviation. The evaluation of the expected values or frequency of occurrence of precipitation can be expressed in term of the frequency factor f_{R_t}.The frequency factor f_{R_t} assumes frequency distribution and depends on the return period R_t. In this study the values of f_{R_t} were calculated by Normal, Log-Normal, Gumbel and Log-Pearson type III distributions.

Normal distribution

The frequency factor f_{R_t} was determined by using the normal distribution defined by the following relation as proposed by [35].

$$f_{R_t} = \frac{\left(\alpha_{R_t} - \overline{M}\right)}{\sigma} \tag{3}$$

The values of f_{R_t} for the normal distribution is equal to standard normal variable $\hat{v}$ defined as:

$$\hat{v} = \omega - \frac{2.515517 + 0.802853\omega + 0.010328\omega^2}{1 + 1.432788\omega + 0.189269\omega^2 + 0.001308\omega^3} \tag{4}$$

$$\omega = \left[\ln\left(\frac{1}{\rho^2}\right)\right]^{\frac{1}{2}} \qquad \left(0 < \rho < 0.5\right) \tag{5}$$

whereas, if $\rho > 0.5$ then $1 - \rho$ is to be substituted instead of ρ in Eq.(5).

The value of $\hat{v}$ estimated by using the above equations is given a negative sign [36] however, the value of f_{R_t} is equal to $\hat{v}$ as mentioned above.

Lognormal distribution

The lognormal distribution is commonly used probability distribution for hydrological data whose logarithm has a normal distribution. The frequency factor f_{R_t} for the lognormal distribution is determined in a similar way as in the normal distribution with the special case that the magnitudes of extreme events dependent on the logarithm of the data. However, the value of f_{R_t} is evaluated by considering the coefficient of skewness as zero [19]. The maximum generated precipitation α_{R_t} along with return period R_t can be calculated by using Eq.(6) and Eq.(7).

$$\alpha_{R_t} = \exp\left(X_{R_t}\right) \tag{6}$$

$$X_{R_t} = \overline{X}\left(1 + C_v\,\alpha_{R_t}\right) \tag{7}$$

where, $\overline{X}$ is the mean and C_v represent the coefficient of variation of data.

Log Pearson Type III distribution

In recent years, the Log Pearson type III (LP3) distribution is considered as the most highly recommended distribution especially for regional frequency analysis. In this distribution first the measured values are transformed into logarithmic form and then mean and standard deviation are determined by using this data [37]. The simplified expression for this distribution is defined as:

$$\log\alpha = \log\overline{\alpha} + f_{R_t}\,\sigma_{\log\alpha} \tag{8}$$

where, $\log\overline{\alpha}$ is the average logarithmic values of observed precipitation, f_{R_t} represents the Pearson frequency factor and $\sigma_{\log\alpha}$ is the standard deviation of the precipitation data.

Furthermore, the value of the coefficient of skewness S_k is required to extract the values of f_{R_t} which can be derived from Eq.(9) and Eq.(10).

$$f_{R_t} = \frac{2}{S_k}\left[\left\{\left(\hat{v} - \frac{S_k}{6}\right)\frac{S_k}{6} + 1\right\}^3 - 1\right] \quad (9)$$

$$S_k = \left(\frac{1}{\sigma^3}\right)\left\{\frac{N}{(N-1)(N-2)}\right\}\sum_{i}^{n_i}\left(X_i - X_{avg}\right)^3 \quad (10)$$

Gumbel distribution

The Gumbel distribution is widely applied for extreme value analysis of maximum precipitation as well as for other extreme events such as flood and drought. Eq.(11) is used in order to calculate the maximum generated precipitation α_{R_t} corresponding to return period for this distribution.

$$\alpha_{R_t} = \overline{M}(1 + C_v f_{R_t}) \quad (11)$$

where, $\overline{M}$ is the mean and f_{R_t} represent the Gumbel frequency factor which can be calculated by the equation proposed by [31] :

$$f_{R_t} = -0.7797\left\{0.5772 + \ln\left[\ln\left(\frac{R_t}{R_t - 1}\right)\right]\right\} \quad (12)$$

Goodness of fit test:

The performance of the distribution evaluated by using goodness of tests namely Chi-square and Kolmogorov Smirnov test. The Chi-square test is a commonly used test for testing the goodness of fit of empirical data to specific theoretical distribution whereas Kolmogorov-Smirnov is used to decide if a sample derives from a population with specific distribution [32, 38]. The goodness of fit test has been conducted at significance level $(\alpha = 0.05)$ for selecting the best distribution[39].

Chi-square Test:

The Chi-square test statistics is defined as a relation between the observed number of occurrence o_i and expected number of occurrence ξ_i with $(n-k-1)$ degree of freedom which is expressed as:

$$\chi^2 = \sum_{i=1}^{v} \frac{(o_i - \xi_i)^2}{\xi_i} \qquad (13)$$

The Chi-square value can be determined for each distribution for a particular return period and its value corresponding to different degrees of freedom against distribution percentages can be extract from χ^2 table. In general, the probability distribution that provides the least Chi-square value is considered as a best-fitted probability distribution.

Kolmogorov-Smirnov Test:

The Kolmogorov-Smirnov test statistic is based on the largest vertical difference between the theoretical and the empirical cumulative distribution function (E_N) defined as:

$$d = \max\left[F(S_i) - \frac{i-1}{N}, \frac{i}{N} - F(S_i) \right];$$

$$1 \le i \le N \qquad (14)$$

$$E_N = \frac{n(i)}{N} \qquad (15)$$

where, S_i is a random sample, $n(i)$ is the number of points less than S_i and N is ordered data points $S_1, S_2, ..., S_N$.

This test is used to decide if a sample derived from a hypothesized continuous distribution [39, 40].

Regression model:

The regression models were developed for estimating the maximum monthly precipitation of each station corresponding to different return periods and derive the coefficient of determination R^2.

Result and Discussion:

Selecting the best probability position method is an important step for determining the best estimates for different return period as well as for estimating parameters of distribution. The maximum monthly precipitation data extracted from 30 years (1986-2015) precipitation dataset of seven weather stations of upper Indus basin was analysed using probability distribution as discussed above. The distribution of maximum precipitation received during different months in a year is depicted in Fig.2. From Fig. 2, the maximum amount of precipitation of selected stations was mostly observed in winter (Dec-March) and spring (April-June) season. Furthermore, the statistical parameters used to estimate the maximum monthly precipitation with the help of different probability distributions, summarized in Table 1. The observed maximum monthly precipitation for different return periods of 2, 5, 10, 25, 50 and 100 were obtained by Weibull's plotting position method. However, the expected maximum monthly precipitation corresponding to different return periods with their probability levels were calculated by Normal, Lognormal, Gumbel and Log-Pearson type III distribution depicted in Table 2.

The analysis revealed that log-Pearson type III distribution found to be the best fitted distribution with minimum values of Chi-square and Kolmogorov Smirnov test for Astore, Chitral, Gilgit and Chilas whereas the lognormal distribution found best fitted for Gupis, Skardu and Swat as summarized in Table 3.According to log-Pearson distribution, the least maximum monthly precipitation of

Astore(124.370mm), Chitral (124.8097mm), Gilgit (38.0713mm) and Chilas (50.3385mm) can be expected to occur with 50% probability with two year return period respectively. However, the expected maximummonthlyprecipitation of Astore (247.1046 mm), Chitral (409.4921 mm), Gilgit (231.5360 mm) and Chilas (331.8958 mm) may be received with 1% probability with 100 years return period respectively. Moreover, the lognormal distribution provide the least maximum monthly precipitation of Gupis (65.2561 mm), Skardu (69.5239 mm) and Swat (69.5239 mm) that can be expected to arise with 50% probability with two year return period while the expected maximum monthly precipitation of Gupis (350.2102 mm), Skardu (177.9734 mm) and Swat (558.3962 mm) can be received with 1% probability with 100 years return period respectively. Furthermore,the regression model was developed from the observed maximum monthly precipitation corresponding to different return period by using Weibull's plotting position methods. The trend analysis for the prediction of monthly maximum precipitation against different return period provide exponential trend line with significant coefficient of determination $\left(R^2\right)$ for Astore(0.9649),Chitral(0.9154), Chilas(0.9342), Gilgit(0.9947), Gupis(0.9404), Skardu(0.96) and Swat(0.969) respectively depicted in Fig.3.

CONCLUSION:

This study employs Weibull's plotting position method to estimating return period of maximum monthly precipitation of seven weather stations (Astore, Chitral, Chilas, Gilgit, Gupis, Skardu and Swat) of upper Indus basin for the period of 30 years (1986-2015). The maximum monthly precipitation data indicates large variations of precipitation every month and years. This study found log-Pearson Type III distribution as the significant distribution for maximum monthly precipitation of Astore, Chitral, Gilgit and Chilas whereas log-normal distribution found to be the best fit distribution for Gupis, Skardu and Swat with minimum Chi-square and Kolmogorov Smirnov test values respectively. Our calculation suggest that log-Pearson distribution confer the least maximum monthly precipitation of Astore, Chitral, Gilgit and Chilas that can be expected to occur with 50% probability with two year return while the expected maximum monthly precipitation can be received with 1% probability with 100 years return period respectively. Moreover, the log-normal distribution provide the least maximum monthly precipitation of Gupis, Skardu and Swat that can be expected to arise with 50% probability with two year return period whereas the expected maximum monthly precipitation can be received with 1% probability with 100 years return period respectively.

Moreover, the results indicates that 57% chances log-Pearson III probability distribution and 43% of log-normal for selected seven stations of upper Indus. The log-Pearson distribution shows the increase in length of maximum precipitation as compare to the log-normal. Among the selected stations Swat represents the highest maximum monthly precipitation (July, 2010) during the last 30 years history in which disastrous flood were also reported in Pakistan. The results obtained in this paper show the expected heavy precipitation in future for those stations where log-normal and log-Pearson distributions are adequate.

Furthermore, the results of this study recommended that 2 to 100 years is sufficient return period for water conservation measures, irrigation and drainage works for upper Indus basin region. However, finding of this study also would provide basic information for decision making in minimizing adverse future impacts of flood.

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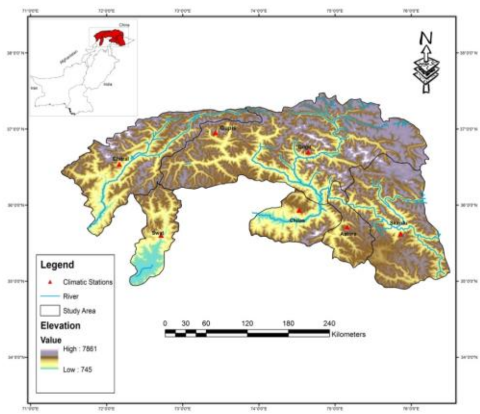

Fig. 1 Location of study area and climatic stations

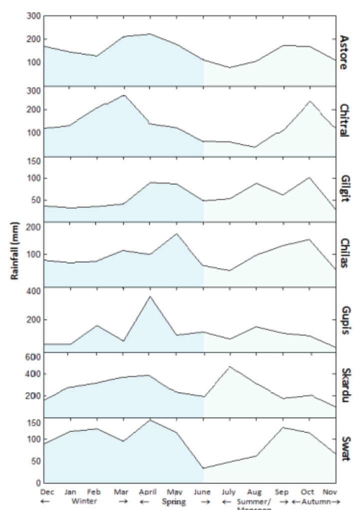

Fig. 2 Seasonal distribution of maximum monthly precipitation of study area

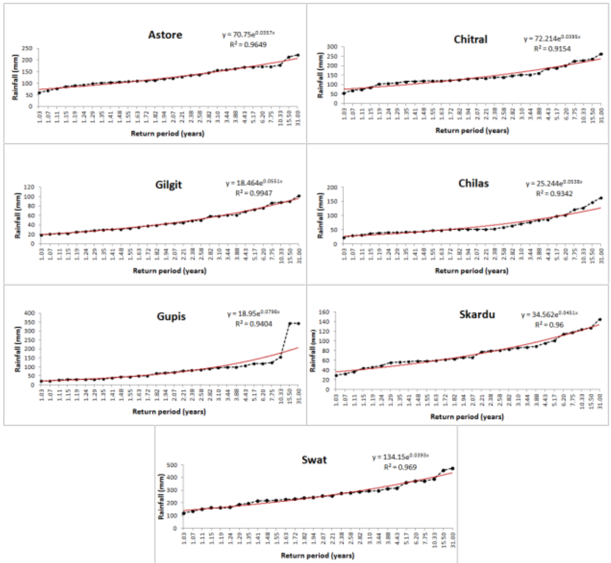

Fig 3 Observed maximum monthly precipitation (mm) with different return periods (years) (Weibull method) of (a) Astore (b) Chitral (c) Chilas (d) Gilgit (e) Gupis (f) Skardu (g) Swat

Table1. Description of statistical parameters for maximum monthly precipitation (mm) of selected stations

	Astore	Chitral	Gilgit	Chilas	Gupis	Swat	Skardu
Mean	129.163	141.770	48.557	65.703	86.167	261.727	75.060
Standard deviation	40.490	50.577	23.520	35.669	78.590	90.707	29.549
Coefficient of variations	0.313	0.357	0.484	0.543	0.912	0.347	0.394
Skewness coefficient	0.474	0.703	0.718	1.339	2.592	1.859	0.625

Table 2 Observed and generated maximum monthly precipitation at different return periods and probability level

Station	**Return Period (Probability %)**	**Observed Precipitation**	**Generated Precipitation (mm)**			
			Normal	**LogNormal**	**Gumbel**	**Log-Pearson Type-III**
Astore	**2 (50)**	133.3	129.16	123.06	122.51	124.37
Chitral		138.7	141.77	133.25	133.46	124.81
Chilas		53.1	65.70	58.14	59.84	50.34
Gilgit		49.0	48.56	43.37	44.69	38.07

Gupis		79.7	86.17	65.26	73.26	52.74
Skardu		79.5	75.06	69.52	70.21	70.46
Swat		276.4	261.73	246.85	246.83	215.19
Astore	**5 (20)**	170.0	163.18	161.03	158.29	161.54
Chitral		187.6	184.25	180.86	178.16	174.72
Chilas		97.8	95.67	87.77	91.37	79.07
Gilgit		72.8	68.31	65.26	65.48	59.79
Gupis		118.2	152.18	119.90	142.71	102.76
Skardu		101.0	99.88	97.71	96.32	98.11
Swat		359.1	337.92	331.73	326.99	288.18
Astore	**10 (10)**	177.6	180.99	185.38	181.99	184.08
Chitral		228.8	206.51	212.25	207.75	215.68
Chilas		127.5	111.36	108.90	112.24	110.52
Gilgit		88.1	78.66	80.83	79.24	82.57
Gupis		155.0	186.76	164.90	188.69	168.52
Skardu		124.3	112.88	116.78	113.61	115.74
Swat		387.2	377.83	387.28	380.06	375.87
Astore	**25 (4)**	221.2	200.02	215.47	211.92	210.70
Chitral		262.1	230.28	251.81	245.14	282.48
Chilas		163.3	128.12	137.12	138.61	171.56
Gilgit		102.4	89.72	101.58	96.63	125.15
Gupis		344.5	223.70	231.76	246.79	322.69
Skardu		145.2	126.77	141.27	135.45	137.32
Swat		472.0	420.46	456.92	447.12	550.01
Astore	**50 (2)**	---	212.17	237.19	234.13	229.35
Chitral		---	245.45	280.84	272.88	341.16
Chilas		---	138.82	158.85	158.17	238.74
Gilgit		---	96.77	117.54	109.53	170.52
Gupis		---	247.28	288.01	289.89	525.75
Skardu		---	135.64	159.53	151.66	152.89
Swat		---	447.68	507.79	496.87	743.58
Astore	**100 (1)**	---	223.10	258.59	256.17	247.10
Chitral		---	259.11	309.82	300.41	409.49
Chilas		---	148.46	181.33	177.59	331.90
Gilgit		---	103.12	134.03	122.33	231.54
Gupis		---	268.49	350.21	332.68	855.35
Skardu		---	143.61	177.97	167.75	168.02
Swat		---	472.17	558.40	546.25	1013.16

Table 3: Summary of goodness-of-fit test for maximum monthly precipitation of selected stations

Station	Normal		LogNormal		Gumbel		Log-Pearson Type-III	
	Chi-Square	Kolmogorov Smirnov	Chi-Square	Kolmogorov Smirnov	Chi-Square	Kolmogorov Smirnov	Chi-Square	Kolmogorov Smirnov
Astore	3.7864	0.13802	3.45020	0.10901	5.1815	0.12118	**2.2738**	**0.1016**
Chitral	5.2689	0.15473	0.42807	0.11486	0.4066	0.10859	**0.2293**	**0.1084**
Chilas	2.1533	0.12441	0.41206	0.10559	1.4763	0.10682	**0.1595**	**0.1039**
Gilgit	4.8191	0.23808	8.83620	0.17465	13.3450	0.18660	**0.8730**	**0.1463**
Gupis	2.7192	0.21241	**0.75169**	**0.08673**	2.2324	0.20191	3.1450	0.0953
Skardu	4.6375	0.14731	**0.48490**	**0.08739**	1.4640	0.09029	1.4803	0.0919
Swat	0.6794	0.09447	**0.45893**	**0.08264**	0.6356	0.08865	0.4621	0.0871

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