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Problem of calculating fluid velocity through porous areas and free area located between two porous areas

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Abstract:

In this paper we discussed the problem of finding velocity Component through Three zones. The first is porous zone and the second is free zone Located between two porous areas and a third porous zone. The velocity Component was found in porous areas

Key words: fluid velocity, Darcy's law, continuity equation, pressure, porous

1. INTRODUCTION

A porous medium can be defined as a solid, or collection of solid bodies, with sufficient open space in or around the solids to enable a fluid to pass through or around them.

There are various conceptual ways of describing a porous medium. One concept is a continuous solid with holes in it. Such a medium is referred to as consolidated, and the holes may be unconnected (impermeable) or connected (permeable). Another concept is the collection of solid particles in a packed layer, where the fluid can pass through the voids between the particles, which is referred to as unconsolidated. Both of these concepts have been used as the basis for developing the equations which describe fluid flow behavior. Example for porous medium the passage of petroleum materials into the tube another example Pores on human skin and Flow of an ideal gas through a homogeneous porous medium so on . Henry Darcy (1803–1858) studied the movement of water through sand and from empirical observations defined the basic equation, universally known as Darcy's law, that governs groundwater flow in most alluvial and sedimentary formations.

Interface conditions at a boundary between two porous media are derived directly from Biot's equations of poroelasticity by replacing the discontinuity surface with a thin transition layer, in which the properties of the medium change rapidly yet continuously, and then taking the limit as the thickness of the transition layer approaches zero.

In our problem consider that we have Three free zones contain fluids. Two free zones and a central area porous between the two free zones. These regions are governed by Darcy laws and continuity equations this is based on the type of area where it is a porous or free zone:

1- the continuity of normal velocities

2- The flow of each fluid can be described by the extended Darcy formula including the relative permeability coefficient.

3- Both fluid phases are bar tropic, i.e. each phase density depends only on the pressure in the respective phase

2- PROBLEM DESCRIPTION

We have the continuity equation

$$\nabla . q = 0 \qquad \rightarrow 1$$

The porous regions apply Darcy's law

Darcy's law states that the fluid velocity (q) will be defined by

$$q = -\frac{k}{\gamma}\nabla p$$

From spherical polar coordinates

$$\nabla p = \frac{\partial \mathrm{p}}{\partial \mathrm{r}}\mathrm{i}_{-\mathrm{r}} + \frac{1}{\mathrm{r}}\frac{\partial \mathrm{p}}{\partial \theta}\mathrm{i}_{-\theta}$$

And

$$q = i_{-r} q_r + i_{-\theta} q_\theta$$

$$q = -\frac{k}{\gamma}\left(\frac{\partial p}{\partial r} i_{-r} + \frac{1}{r}\frac{\partial p}{\partial \theta} i_{-\theta}\right)$$

Where i_{-r} *and* $i_{-\theta}$ are local unit vectors

→2

Where

K = constant permeability

p = pressure

μ = viscosity of the fluid

q = fluid velocity

Also we have the equation

$$r^2 \frac{\partial^2 p}{\partial r^2} + 2r\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial \theta^2} + \cot\theta \frac{\partial p}{\partial \theta} = 0$$

→3

3- Mathematical Solution

Let p= R(r) $\Theta\ (\theta)$ in equation 3

$$\frac{r^2}{R}\frac{d^2 R}{dr^2} + \frac{2r}{R}\frac{dR}{dr} = \frac{-1}{\Theta}\frac{d^2\Theta}{d\theta^2} - \frac{1}{\Theta}\cot\theta\frac{d\Theta}{d\theta}$$

By using separation method

$$\frac{r^2}{R}\frac{d^2 R}{dr^2} + \frac{2r}{R}\frac{dR}{dr} = c$$

Or can be written as

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - cR = 0$$

→4

$$\frac{-1}{\Theta}\frac{d^2\Theta}{d\theta^2} - \frac{1}{\Theta}\cot\theta\frac{d\Theta}{d\theta} = c$$

Or can be written as

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{d\Theta}{d\theta}\right]+c\Theta=0$$

→5

Where $c > 0$

To solve equation (4) Let R= r^{λ}

Then The solution is

$$\lambda_1 = \frac{1}{2}\sqrt{1+4c} - 1$$

$$\lambda_2 = -\frac{1}{2}\sqrt{1+4c} - 1$$

→6

To solve equation 5 let

$$\Theta\,(\theta) = A^*\sin\theta + B^*\cos\theta$$

Then the solution is

$$\Theta = B^*\cos\theta$$

→7

From equations 6 and 7 the solution is

$$p\,(r,\theta) = \left(c_1 r^{\lambda_1} + c_2 r^{\lambda_2}\right)\cos\theta$$

In regions I,III

But c = 2 → $\lambda_1 = 1\,, \lambda_1 = -2$

Then

$$p_I = p_\infty + \gamma\, UErcos\theta$$

→8

And

$$p_{III} = p_\infty + \frac{\gamma\, UF}{r^2}cos\theta$$

Where p_∞ is constant pressure at infinity

→9

In region I by comparing 2 equation 8 and

$$p_I = p_\infty + \gamma\, UErcos\theta \qquad \text{and} \qquad q = -\frac{k}{\gamma}\nabla p$$

$$q = -\frac{k}{\gamma}\nabla[p_\infty + \gamma\, UErcos\theta]$$

$$\to -\frac{k}{\gamma}\left[\gamma\, UEcos\theta\ r^{\wedge} - \frac{1}{r}\gamma\, UErsin\,\theta\,\theta^{\wedge}\right]$$

$$\to [-k\ UEcos\theta\ r^{\wedge} + k\ UErsin\,\theta\,\theta^{\wedge}]$$

Then the velocity field for region I

$$q_r = -k\, E\, Ucos\theta$$

And

$$q_\theta = k\, E\, U\, sin\theta$$

Similarly we can get q_r , q_θ In region III

In region III by comparing 2 equation 9 and

$$P = p_\infty + \frac{\gamma\, UF}{r^2}cos\theta \quad And \qquad q = -\frac{k}{\gamma}\nabla p$$

$$q = -\frac{k}{\gamma}\nabla\left[p_\infty + \frac{\gamma\, UF}{r^2}cos\theta\right]$$

$$\to -\frac{k}{\gamma}\left[-2\,\gamma\, UF\, r^{-3}cos\theta\ r^{\wedge} - \frac{1}{r}\frac{\gamma\, UF}{r^2}sin\,\theta\,\theta^{\wedge}\right]$$

$$\left[\frac{-2\,k\,F}{r^3}\ U\,cos\theta\ r^{\wedge} + \frac{k\,F}{r^3}U\,sin\,\theta\,\theta^{\wedge}\right]$$

Then the velocity field for region III

$$q_r = \frac{-2k\,F}{r^3}\ Ucos\theta$$

And

$$q_\theta = \frac{k\,F}{r^3}\ \sin\theta$$

3. CONCLUSION:

In this problem we found importance of Darcy's law to find velocity Component in porous areas. We can find the Component velocity in the free area by using continuity equation and comparing between areas .

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