

## DC Motor Speed Control by Coefficient Diagram Method(CDM)

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### Abstract

*In this study, modeling of a direct current (DC) motor and speed control has been realized. The best control of dc motors is one of the objectives of control methods in general. Because these engines have a wide range of applications, especially industry. Therefore, it is proposed to use a polynomial approach called the Coefficient Diagram Method (CDM) to obtain a better behavior for these motors. Controller design with CDM; it is based on determining the coefficients of the characteristic polynomial of the closed-loop system against the appropriate behavioral characteristics such as equivalent time constant, stability index and stability limit index. Also, this method was compared with the PID control method. Simulation of the CDM and PID controlled system was carried out in noisy environments. A second-order low-pass filter is designed to increase the performance of the controllers in the noise environment. These methods were used to monitor the speed of the DC motor. As a result, the simulation results show that the proposed methods, the controller of the dc motor, has been successfully achieved*

**Key words:** DC motor; Mathematic Model; Speed control; Coefficient Diagram Method (CDM)

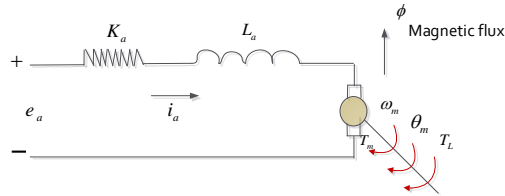
### 1.INTRODUCTION

Direct current (DC) motors are used frequently because of their features such as cost, ease of control, long life and quiet operation. DC

motors for robotics, defense industry and automotive applications etc. are used in many fields [1-9]. The control loop is generally designed to keep the output of a system around a reference value that can take various values within a given range of variation. In this study, our main goal is to design a suitable controller for these motors with many application areas. Controller design with CDM; it is based on determining the coefficients of the characteristic polynomial of the closed-loop system against the appropriate behavioral characteristics such as equivalent time constant, stability index and stability limit index. Although KDY is a new method, some basic principles have been used in control design for years and have found applications in different areas of the industry for a long time. Although the CDM core principles have been known since the 1950s, the first systematic method was proposed by Shunji Manabe [10]. He developed a new method that created a target characteristic polynomial to meet the desired time response. CDM is an algebraic approach that combines classical and modern control theories and uses polynomial representation in a mathematical expression. The advantages of classical and modern control techniques are obtained using previous experience and knowledge of control design, in this method, it is integrated into the basic principles. Therefore, an effective and efficient control method control systems have emerged as a configurable tool without much experience and without any problems. Many control systems have been successfully designed using CDM. It is very easy to design a controller under conditions of stability, time-domain performance and durability. The close relationship between these conditions and the characteristic polynomial coefficients can be easily detected. This means that the CDM control system is effective not only for design but also for setting controller parameters [11-14]. Its most important features are that it combines the advantages of classical and modern control theories. One of the most used and best results among these methods is the Coefficient Diagram Method (CDM), which was put forward by Shunji Manabe[15-16]. CDM is a control system design method that has been obtained as a result of combining the advantages of classical and modern control theories in an appropriate way by making use of past ideas and experiences. CDM is a very effective method, the design procedure is easier than most methods, and a lot of experience is not required for the design [16-19].

## 2. MODELING OF DC MOTOR

Direct current motor is the most commonly used motors in the control systems. They may provide rotation and offset movement. DC motor model is shown in Figure 1.



**Fig. 1 Dc motor model**

Moment received from the electric motor;

$$T_m(t) = K_m \cdot \Phi \cdot i_a(t) = K_i \cdot i_a(t) \quad (1)$$

$$\frac{di_a}{dt} = \frac{1}{L_a} \cdot e_a - \frac{R_a}{L_a} \cdot i_a - \frac{1}{L_a} \cdot e_b \quad (2)$$

$$T_m = K_i \cdot I_a \quad (3)$$

$$e_b = K_b \cdot \frac{d\theta_m}{dt} = K_b \cdot \omega_m(t) \quad (4)$$

$$J_m \frac{d^2\theta_m}{dt^2} = T_m - T_L - B_m \cdot \frac{d\theta_m}{dt} \quad (5)$$

If we take our variables as  $i_a$ ,  $\theta_m$  and  $\omega_m$ , the equations of state from the first order can be written as follows.

$$\dot{x} = Ax + Bu \quad (6)$$

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega_m}{dt} \\ \frac{d\theta_m}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_i}{J_m} & -\frac{B_m}{J_m} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} \cdot e_a - \begin{bmatrix} 0 \\ \frac{1}{J_m} \\ 0 \end{bmatrix} \cdot T_L(t) \quad (7)$$

$$y = Cx + Du \quad (8)$$

The parameters of the dc motor are shown in the Table 1.

**Table 1. Physical parameters of Dc motor**

Sembol	Description	Units	Value
m	Body Mass	<i>kg</i>	10
J	Body Inertia	<i>kgm<sup>2</sup></i>	0.171
$K_m$	Motor Constant	<i>Nm/A</i>	3520
$R_a$	Motor Resistance	<i>Ohm</i>	55
$l_o$	Leg Length	<i>m</i>	0.323
$L_m$	Motor electric inductance	<i>H</i>	0.3
$B_m$	Damping ratio of the system friction constant	<i>Nm.</i>	0.097

### 3. CONTROLLER DESIGN

The main objective of designing the control system of the Dc motor is stability and low tracking error. Coefficient Diagram Method (CDM) and PID control method were used for control of Dc motor. The aim of the control methods used is to ensure that the output value of the system tracks the targeted (reference) value. The error was tried to be minimized with the controller applied to the system.

#### 3.1. Coefficient Diagram Method (CDM)

Controller design with CDM; it is based on determining the coefficients of the characteristic polynomial of the closed-loop system against the appropriate behavioral characteristics such as equivalent time constant, stability index and stability limit index. Unlike other methods, the Coefficient Diagram Method (CDM) is a method that directly takes into account closed loop control behavior in the controller design. Polynomial representation is easy to understand, such as the transfer function, in CDM. The relationship between the characteristic polynomial and the controller is determined by the linear and easy-to-solve diophantine equation and the coefficient diagram [20-22]. The relationship is effectively explained mathematically and graphically, which allows the simultaneous design of the controller and the characteristic equation. In the first phase, the closed-loop transfer function and controller are determined in the first phase, and the remaining parameters are determined during the design. With this feature of CDM, simple and durable controllers that can provide the desired closed-loop response can be easily designed. The graphical tool of the method, the CDM shows the three main elements of the control system design, the stability, response shape, and durability, on a single diagram, which allows the

designer to fully control the control system. In the CDM, the controllers are designed based on the stability index known as  $\gamma_i$  and equivalent time constant known as  $\tau$  which are synthesized from the characteristic polynomial of the closed-loop transfer function in Equation 9..

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \tag{9}$$

The choice of stability index  $\gamma_i$  due to the control design specifications must satisfy the following inequality

$$\gamma_i > 1.5 \gamma_i^* \tag{10}$$

where  $\gamma_i$  is the stability limit and is described as

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}}; \tag{11}$$

From the characteristic polynomial  $P(s)$  given in equation 9, the stability index  $\gamma_i$  and equivalent time constant  $\tau$  are respectively defined in general term as in equation 12 and 13

$$\gamma_i = \frac{a_i^2}{a_{i-1} a_{i+1}}, \quad (i=1.2\dots,n-1) \tag{12}$$

Finally, the characteristic polynomial known as the wanted characteristic polynomial can be explained as

$$P_m(s) = \frac{\prod_{j=1}^{n-1} \gamma_j^{n-j}}{\tau^n} \left[ \left\{ \sum_{i=2}^n \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] \\ = s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \tag{13}$$

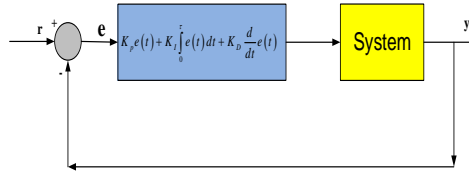
where,  $a_n, a_{n-1}, \dots, a_0$  are the coefficients of the wished characteristic polynomial.

### 3.2. PID Control Method

Even though the PID (proportional integral derivative) control method used in many practices is an old method, it displays a good performance [23]. As it is easier to adjust compared to the other controllers and it is a simple control mechanism, it is widely used. It was observed that it is not a vital necessity to know the mathematical model of the process that will be controlled by PID control completely

and that the control parameters can be adjusted favorably. Basic structure of the PID control method can be seen in the Equation 9[24].

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{d}{dt} e(t) \tag{9}$$



**Figure 2** Block diagram of PID feedback system

$u$ ,  $K_p$ ,  $K_i$ ,  $K_d$ , and  $e$  are named respectively as controller output, modulating gain, integral gain, differential gain and error signal. Block diagram of PID feedback system is shown in the Figure 2. In this study, Ziegler-Nichols method suggested by John G. Ziegler and Nathaniel B. Nichols is used to find the PID coefficients and closed loop control type is used within this method [24]. the Table 2 is shown Ziegler-Nichols method.

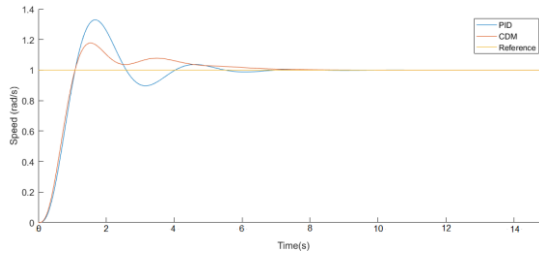
**Table 2.** Control Parameters of Dc motor by using Ziegler-Nichols

Control type	$K_P$	$K_i$	$K_D$
P	$0.5 * K_{cr}$	$\infty$	0
PI	$0.4 * K_{cr}$	$0.8 * P_{cr}$	0
PID	$0.6 * K_{cr}$	$0.5 * P_{cr}$	$0.125 * P_{cr}$

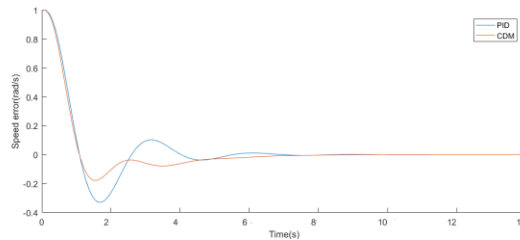
#### 4. SIMULATION RESULTS

In this section, simulation studies were conducted by using the obtained model equations of DC motor. The performance values of the control method were given graphically. The performance of the CDM and PID control method were tested of motor. The control variable of the speed is rotary angle values. The simulation run time was accepted as 14 seconds. In this paper, the simulation results of dc motor control are obtained by using the MATLAB packet program. Dc motor control was simulated using the parameters shown in Table 1. Speed monitoring was conducted in the simulation studies and the performance was evaluated. The simulation in Figure 3 shows reference, CDM controller is the satisfying specification of the performance of the control system and clarifies about adjustment of

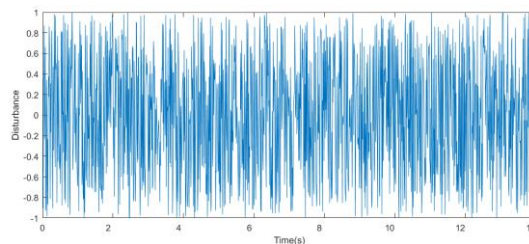
response speed and its overshoot by the CDM technique. The PID control method gave more amplitude and response. Error responses of control methods are given in Figure 4. Figure 5 shows the disruptive input signal affecting the dc motor.



**Figure 3. The speed results of numerical simulation**



**Figure 4. The speed error results of numerical simulation**



**Figure 5. Disturbance input signal affecting the motor**

## 5. DISCUSSION AND CONCLUSION

In this study, CDM and PID control method design was made for dc motor speed control. Simulation of the CDM and PID controlled system was carried out in noisy environments. A second-order low-pass filter is designed to increase the performance of the controllers in the noise environment. Then CDM and PID controllers were compared. According to the results of the study, it was observed that the peak amplitude and overflow rate criteria of the CDM controlled method were more successful than the PID controlled method. In

future studies, the proposed methods are intended to be improved and performed on a real system.

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## APPENDIX

Nomenclature	
$i_a$	Motor current
$R_a$	Motor resistance
$e_b$	emf
$T_L$	Load torque
$\phi$	Magnetic flux
$J_m$	Moment of inertia
$B_m$	Viscous damping coefficient
$L_a$	Motor inductance
$e_a$	Motor voltage
$K_b$	emf constant
$\theta_m$	Angular rotation of rotor
$K_i$	Torque constant
$w_m$	Angular velocity of rotor