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The Use of Extreme Value Theory in the Financial Area: An Approach Geared to Long-Short Strategies in the Brazilian Stock Market

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Abstract

Many investors have opted for funds classified as hedge funds with the aim of boosting their performance by using arbitrage strategies, such as long-short strategies. In this article the Extreme Value Theory (EVT) was used to model series of prices of assets traded on the São Paulo stock exchange. To model the spread, we use the stock price series of Itaú Unibanco Holding S.A. (ITUB4.SA) and Banco ABC Brasil S.A. (ABCB4.SA) - base assets for the application of the pair trading strategy. The long-short strategy was built on the spread, considering the amounts resulting from its modeling, in order to obtain distinct limits for assembling and disassembling the operations performed. Since the long-short strategy was set up, for the different types of limits, the cumulative return was calculated in order to identify the best approach to generate a stable degree of profitability. The results obtained were satisfactory.

Keywords: Finance, long-short strategies, extreme value theory, stock market

1. INTRODUCTION

Extreme Value Theory (EVT) is considered a probabilistic branch of statistical support that deals with a better way to quantify the probabilities of occurrence of infrequent (rare) or even never observed events. The fundamentals of extreme value theory were first exposed by Fisher and Tippett (1928), who introduced the three possible types of asymptotic distributions of extreme value, known as Gumbel, Fréchet and Weibull. However, the first to study and formalize the statistical application of the above distributions was Gumbel (1954), whose methodology has been widely applied. Other important contributions to the study of extreme value were made by Gnedenko (1943), who showed the conditions necessary for the existence of asymptotic distributions of extreme value.

An approach aimed at applying extreme value theory is directly linked to long-short strategies. Among the funds that use the long-short strategy are the hedge funds, which due to their performance, had great repercussion in the financial market. The long-short strategy comprises the simultaneous maintenance of stocks and their derivatives - bought and sold - in which the investor maintains a position sold in one stock (or its derivatives) and bought in another, always with the purpose of obtaining a financial residual of the operation when liquidating it.

2. HEDGE FUNDS

There is no single meaning for hedge funds, however, Anson (2006) presents a broad definition for this denomination, which conceptualizes hedge funds as private investment vehicles to manage concentrated portfolios of public and private securities, and their respective derivative instruments, which may hold both long and short positions and may use leverage.

Hedge funds were created in 1949 by Alfred W. Jones. Jones central idea, according to Connor and Woo (2004), was directly linked to the strategy of making trades to protect other contracted operations, and thus provide protection for the portfolio against market risks. These funds were based on the purchase of undervalued stocks and short selling of overvalued stocks, in order to protect the portfolio from negative market fluctuations, in other words, to reduce the risk of the stock portfolio through short positions in other stocks, and it was due to this strategy that Jones significantly reduced his exposure to the stock market.

Although hedge funds do not exist in Brazilian law, there are funds that sometimes act as such, these are called Multimarket Funds. According to Ferreira (2017), considering the regulation still in force (CVM Instruction no. 409/2004), Anbima (Brazilian Association of Financial and Capital Market Entities) calculates a hedge fund index in Brazil - IHFA, which encompasses seven of the ten types of multimarket funds. According to this association's estimate, the net worth of these funds was in January 2015 equivalent to 2.7% of the sector's total.

3. LONG SHORT STRATEGIES

In general, the long-short strategy consists of a simultaneous operation, where the investor maintains a long position in one stock and a short position in another with the objective of obtaining positive returns at the end of the operation. According to Dutra (2016), a long-short strategy combines two operations, the simultaneous purchase and sale of share pairs with the objective of obtaining gains from the relative variations in the pair's prices. Investors who make use of this strategy plan to eliminate or reduce the exposure of their investments to so-called "systemic risks", which affect the economy as a whole, since they refer to the risk of collapse of an entire financial system or market, thus having a strong impact on interest rates, exchange rates and asset prices, among other variables. For Caldeira and Portugal (2010), most of the time long-short strategies seek to obtain returns without directional exposure to the market, so they are called market neutral strategies.

3.1. Pair trading strategy

In accordance with Gatev, Goetzmann and Rouwenhorst (2006), the strategy known as pair trading was formulated around the 1980s by a wall street investor named Nunzio Tartaglia who, according to De Freitas (2007) had as main objective to create a technique based on statistical methods that would be able to identify and select asset pairs that presented a temporal relation, that is, the adopted assets would have to perform in agreement with each other in a certain period of time. Its strategy was summarized in the search for stock pairs that presented a long term trend, then a daily monitoring of the relative behavior of the prices of the respective assets was performed, with the purpose of analyzing the existence of anomalies related to the price differential in the short term. Consequently, if such discrepancies in relative pricing behavior were discovered, Tartaglia would arbitrate with the assets in question until the relative difference previously presented was corrected. This tactic soon became known as pair trading. Caldeira (2010) applied the pair strategy to daily data in the Brazilian stock market using the cointegration approach of Engle and Granger (1987) and Johansen (1988), to select the asset pairs.

Spread analysis is extremely important to make decisions about pair trading strategy. This involves selling a stock whose price is higher than normal and buying another stock that is devalued, conjuring that the price distortion will correct itself in the future. It is exactly this mutual price distortion between the two stocks that is captured by the spread.

Being X_t and Y_t two time series referring to stock prices, the spread can be defined as:

$$spread_t = \log(Y_t) - (\alpha + \beta \log(X_t)). \quad (1)$$

The equilibrium relationship between the prices of the two assets X and Y is assumed to be , the interpretation of which is directly linked to selling (buying) a share of Y and buying (selling) ß shares of asset X in order to obtain a marketable pair. However, this interpretation is unfortunately not true if the time series of the price is transformed using the logarithm.

According to Harlacher (2016), the use of logarithm involves a relationship of balance of form:

$$Y_t = c\,(X_t)^{\hat{\beta}}\,, \quad (2)$$

where $c = e^{\hat{\alpha}}$.

4. EXTREME VALUE THEORY

The theory of extreme value has been much studied over the years. Fundamental results for the univariate case were obtained in 1928 by Fisher and Tippett (1928), who presented the three possible types of asymptotic distributions of extreme value, currently called Gumbel, Fréchet and Weibull. Yet, the first advances for the bivariate case date from the late 1950s.

There are numerous applications of EVT in the financial area, for example, McNeil (1998) studied the estimation of the tail index and calculated amounts using maximums collected in blocks and Embrechts (2000) explored the limitations of using VaR (Value-at-Risk) as a tool in financial risk management.

The definitions and main results in this section can be found in Mendes (2004), and the theorem demonstrations are available in Embrechts, Kluppelberg and Mikosch (1997).

4.1. Exact maximum distribution

Being $X_1, X_2, \dots, X_n$ independent and identified random variables distributed (i.i.d.) with the F distribution function. When ordering the terms of this set in ascending order, we have:

$$X_{(n)} = max(X_1, \dots, X_n) \quad (3)$$

$$X_{(1)} = min(X_1, \dots, X_n) \quad (4)$$

Definition 4.1: Random variables $X_1, \dots, X_n$ are the aleatory variables $X_{(1)}, \dots, X_{(n)}$ obtained from a reordering of $X_1, \dots, X_n$.

Theorem 4.2: Is entitled the distribution function of the maximum a

$$F_{X_{(n)}}(x) = [F_X(x)]^n \,, \; x \in \mathbb{R} \;\; e \;\; n \in \mathbb{N}. \quad (5)$$

Observation 4.3: To obtain the minimum distribution, use the relation:

$$min(X_1, \dots, X_n) = -max(-X_1, \dots, -X_n). \quad (6)$$

It is intuitive that the maximum value are located near the upper limit of the support (x_{F_X}) of the x distribution. Therefore, it is concluded that the asymptotic behavior of the maximum is directly related to the upper tail of F_X, in other words, the tail of F_X near the x_{F_X} . Then the following results are obtained:

- If $x < x_{F_X}$, then

$$P\left(X_{(n)} \leq x\right) = [F_X(x)]^n \xrightarrow{n \to \infty} 0 \; ; \tag{7}$$

- If $x \geq x_{F_X}$, considering $x_{F_X} < \infty$, you get

$$P\left(X_{(n)} \leq x\right) = [F_X(x)]^n = 1. \tag{8}$$

It is concluded that the maximum converges in probability to the upper limit of the support, or rather $X_{(n)} \xrightarrow{P} x_{F_X}$ when n is large enough and considering $x_{F_X} < \infty$.

As the sequence ($X_{(n)}$) is non-decreasing at n, another important result is that

$$X_{(n)} \xrightarrow{p.q.c} x_{F_X} \tag{9}$$

where $n \to \infty$.

4.2. Maximum distribution limits

Fisher and Tippett (1928) determined that the distribution of maxima, after they were standardized, converges to some limit distributions, called extreme value distributions. The fundamental theorem of Fisher-Tippett (1928) provides this result.

Theorem 4.4: Being (X_n) a sequence of random variables i.i.d. If there are sequences of normalizing constants $c_n > 0$ e $d_n \in \mathbb{R}$ and a non-degenerated distribution *H* such that

$$\frac{X(n) - d_n}{c_n} \xrightarrow{d} H \tag{10}$$

where $\xrightarrow{d}$ means convergence in distribution, then the only possible forms of *H* are Gumbel, Fréchet and Weibull distributions.

Extreme value distributions can be classified as type I (Gumbel), type II (Fréchet) and type III (Weibull). Rental parameters (μ) and scale (σ) can also be added to the distributions, obtaining the following results (11), (12) and (13):

- Gumbel

$$H_{I,\mu,\sigma}(x) = e^{-e^{-\frac{(x-\mu)}{\sigma}}} \, , \quad x \in \mathbb{R} \, ;$$

- Fréchet

$$H_{II,\mu,\sigma,\alpha}(x) = \begin{cases} 0 \ , & se\ (x-\mu) \leq 0 \\ e^{-\left(\frac{(x-\mu)}{\sigma}\right)^{-\alpha}} \ , & se\ (x-\mu) > 0\ , \alpha > 0\ ; \end{cases}$$

- Weibull

$$H_{III,\mu,\sigma,\alpha}(x) = \begin{cases} e^{-\left(-\frac{(x-\mu)}{\sigma}\right)^{-\alpha}} \ , & se\ (x-\mu) \leq 0 \\ 1 \ , & se\ (x-\mu) > 0\ , \alpha < 0. \end{cases}$$

4.3. Max-stability

Definition 4.5: Two random variables X_1 and X_2 are considered equal in distribution
($X_1 \stackrel{d}{=} X_2$) if the following sentence is true

$$P(X_1 \leq x) = P(X_2 \leq x)\ ,\ \ x \in \mathbb{R}. \qquad (14)$$

Definition 4.6: Whether $X_1, X_2, \dots$ are random variables i.i.d. with an *F*-distribution function, and both $d_n \in \mathbb{R}$ and c_n are positive constants. It is stated that *F* is max-stable if the following equality in distribution is met

$$max(X_1, \dots, X_n) \stackrel{d}{=} c_n X + d_n. \qquad (15)$$

Theorem 4.7: The class of distributions with max-stability coincides with the class of all possible (non-degenerated) limit distributions for the standardized maximum of random variables i.i.d.

4.4. Attraction domain

Fisher-Tippett's Theorem has the following direct implication: If $[F_X(c_n X + d_n)]^n$ is
non-degenerate when 0 is large enough, for certain positive constants c_n and $d_n \in \mathbb{R}$, then

$$\left|[F_X(x)]^n - H\left(\frac{x-d_n}{c_n}\right)\right| \to 0\ , n \to \infty \qquad (16)$$

for some *H* belonging to the set of extreme value distributions, that is $H \in \{H_I, H_{II}, H_{III}\}$.

Effectively, this fact allows to define a collection of $F_X{}'s$ that have the same limit distribution.

Definition 4.8: Since (16) is confirmed, then it is stated that the F_X in question belongs to the area of attraction of the maximum distribution of extreme value *H* and the $F_X \in MDA(H)$ notation is used for any $H \in \{H_I, H_{II}, H_{III}\}$.

Definition 4.9: Two distributions *F* and *G* are called tail equivalents if both have the same upper limit, namely $x_F = x_G$, and $\lim_{x \uparrow x_F} \frac{1-F(x)}{1-G(x)} = c$, for any constant in the $(0, \infty)$ range.

4.5. Generalised extreme Value distribution (GEV)

The three types of distributions, Gumbel, Fréchet and Weibull, according to Escobar (2018), are part of a single family of distributions: the standard generalized extreme value distribution, which refers to the distributions of extreme Value within a single family, parameterized only by the parameter ξ. The GEV is denoted by H_ξ and is presented in the following equation.

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-\frac{1}{\xi}}} & , \ se\ \xi \neq 0,\ 1+\ \xi > 0 \\ e^{-e^{-x}} & , \ se\ \xi = 0. \end{cases} \qquad (17)$$

When $\xi = 0$, that occurs in the $\xi \to 0$ condition, the H_ξ fits the Gumbel distribution. When $\xi < 0$ or $\xi > 0$, occurs at H_ξ coincides with the Weibull or Fréchet distributions, in this order. The rental and stopover family, equivalent to $H_{\xi,\mu,\sigma}$, is achieved by replacing *x* by $\frac{(x-\mu)}{\sigma}$, with $\mu \in \mathbb{R}$ and $\sigma > 0$, as follows:

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} e^{-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{-1}{\xi}}} & se\ \xi \neq 0,\ 1+\xi\left(\frac{x-\mu}{\sigma}\right) > 0, \\ e^{-e^{-\left(\frac{x-\mu}{\sigma}\right)}} & se\ \xi = 0. \end{cases} \quad (18)$$

The density of the generalized distribution (GEV) is denoted by $h_{\xi,\mu,\sigma}$.

5. Estimation

It is essential to use methods for the estimation of the parameters ξ, μ and σ of the $H_{\xi,\mu,\sigma}$. For this purpose, $M_1, M_2, \dots, M_m$ random variables are considered i.i.d. representing the maximums (or minimums processed) collected in blocks of size *n* and $m_1, m_2, \dots, m_m$ a sample of *m* maximums.

5.1. Estimation by maximum likelihood method

The maximum likelihood estimators of the parameters, ξ, μ and σ from GEV, can be
obtained numerically by maximizing the likelihood function below.

$$L\left(\xi, \mu, \sigma; (m_1, \dots, m_m)\right) = \prod_{i=1}^{m} h_{\xi,\mu,\sigma}(m_i)\ \ I_{\left\{1+\frac{\xi}{\sigma}(m_i-\mu)>0\right\}} \quad (19)$$

and $(m_1, \dots, m_m)$ refers to a maximum sample.

6. STATISTICAL TESTS

6.1. Gumbel distribution adequacy test

If the estimates of the GEV parameters were taken for maximum likelihood, the following test statistic can be used to test the assumption that the data actually follow the Gumbel distribution: the Kolmogorov-Smirnov, D statistic (see Chandra (1981)). This is defined as,

$$D^+ = max_i \left\{\frac{i}{m} - H_{I,\hat{\mu},\hat{\sigma}}\left(m_{(i)}\right)\right\},$$

$$D^- = max_i \left\{H_{I,\hat{\mu},\hat{\sigma}}\left(m_{(i)}\right) - \frac{i-1}{m}\right\},$$

$$D = max\ (D^+, D^-). \quad (20)$$

Where $m_{(i)}$ refers to the ordered maximums and $H_{I,\hat{\mu},\hat{\sigma}}$ represents the Gumbel distribution with the parameters estimated by the maximum likelihood method.

The following hypotheses need to be tested:

a) Null hypothesis: the distribution of extremes is Gumbel;

b) Alternative hypothesis: the distribution of the extremes is not Gumbel.

Table 1 displays the critical value for samples of m=10, m=20, m=50 and m=∞. The significance levels addressed were 10%, 5%, 2.5% and 1%.

The test is carried out by comparing the obtained test statistic, multiplied by $\sqrt{m}$, with the value presented in Table 1. Given that the value of the test statistic exceeds the proposed value in the table, the null hypothesis is rejected at the level of significance α adopted.

Table 1 – Critical Value for $\sqrt{m}D$

Significance Level	$\sqrt{m}D$
m = 10	
10%	0,760
5%	0,819
2,5%	0,880
1%	0,944
m = 20	
10%	0,779
5%	0,843
2,5%	0,907
1%	0,973
m = 50	
10%	0,790
5%	0,856
2,5%	0,922
1%	0,988
m = ∞	
10%	0,803
5%	0,874
2,5%	0,939
1%	1,007

Source: Chandra (1981)

6.2. Augmented Dickey-Fuller Test

The augmented Dickey-Fuller Test, or more commonly called the ADF Test, is a unit root test for time series.

Hypotheses to be tested:

a) Null hypothesis: the series is not stationary;

b) Alternative hypothesis: the series is stationary.

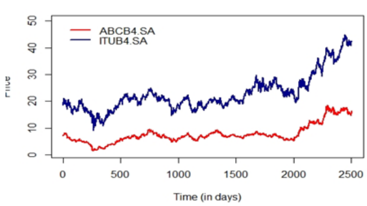
7. GRAPHIC ANALYSIS

Coles (2001) uses graphic devices to test the validity of a GEV model. The probability, quantile and return level plots are based on a comparison of two distribution functions: the empirical cumulative distribution function and the GEV cumulative distribution function with the parameters estimated by the maximum likelihood method. In addition, a diagnosis based on the density function is performed, where the probability density function of a model adjusted with a histogram of the data is compared.

8. RESULTS

To apply the above theory (pair trading and long-short strategies) we use the daily share prices of two companies in the financial sector, Banco Itaú Unibanco Holding S.A. (ITUB4.SA) and Banco ABC Brasil S.A. (ABCB4.SA). The two banks were selected because they are the two largest companies by total assets of the sub-sector (banks). For both series, the period from September 26, 2007 to December 29, 2017 was considered, both having a total of 2509 observations. The variable fixed for the analyses was the "closing", that is, the last value assigned to the share on a given day. Figure 1 represents the daily quotations of ITUB4.SA and ABCB4.SA from September 26, 2007 to December 29, 2017.

Figure 1: Prices of ITUB4.SA and ABCB4.SA.



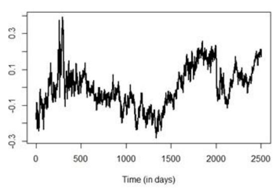
Source: Own preparation.

The augmented Dickey-Fuller Test (ADF) was used to test the non-stationarity hypothesis of the ITUB4.SA and ABCB4.SA series. The p-value were 0,5397 and 0,5595 respectively. As the p-value obtained were higher than the set significance level of 5%, there is no evidence to reject the null hypothesis, that is, both series are non-stationary.

The ADF Test was also used to test the hypothesis of spread stationarity. The p-value obtained was equal to 0,03404, lower than the set significance level of 5%, that is, there is evidence to reject the null hypothesis and it is concluded that the spread is stationary.

Figure 2 corresponds to the spread obtained with the share prices ITUB4.SA and ABCB4.SA.

Figure 2: Spread between prices of ITUB4.SA and ABCB4.SA.



Source: Own preparation

Since the variables referring to asset prices ITUB4.SA and ABCB4.SA are not stationary, although there is at least a linear combination between them that produces a stationary variable, the spread, the hypothesis of cointegration can be confirmed. Thus, there is the following relationship:

$$spread_t = \log(Y_t) - (1{,}96 + 0{,}576 \log(X_t)). \quad (21)$$

Where Y_t is the price of shares ITUB4.SA in period t and X_t is the price of shares ABCB4.SA in period t. The parameters were obtained by simple linear regression, in which the logarithm of Y_t in relation to the logarithm of X_t was considered.

As the ITUB4.SA and ABCB4.SA series are cointegrated, pair trading and long-short strategies can be applied. For this reason, the spread was modeled by extreme value GEV models were adjusted for the minimum and maximum spread, the block size that best suited both models were 20. At first, the models were estimated considering $\xi = 0$, however only the distribution of the maxima adjusted the

Gumbel distribution, since the adequacy test the Gumbel distribution was performed and the value of $\sqrt{m}D$ was approximately 0,609, not exceeding the critical value for a significance level of 5% (0,874). The parameters obtained were $\hat{\mu} = -0{,}0053$ and $\hat{\sigma} = 0{,}1004$. For the distribution of the minimums, the parameters were: $\hat{\mu} = 0{,}0149$, $\hat{\sigma} = 0{,}1112$ and $\hat{\xi} = -0{,}3531$. As $\hat{\xi} < 0$, the distribution of the minimums coincides with the Weibull distribution. The diagnostic graphs recommended by Coles (2001) were used to validate the quality of the fit of both models and can be seen below, in Figures 3 and 4.

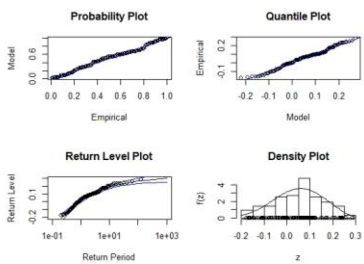


Figure 1: Quality of the adjustment for the distribution of the minimum spread.

Source: Own preparation.

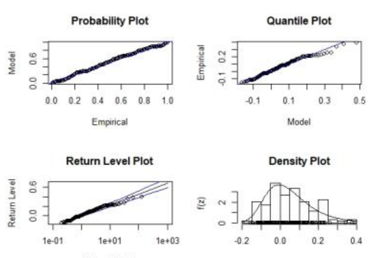


Figure 2: Quality of the adjustment for the distribution of the maximum spread.

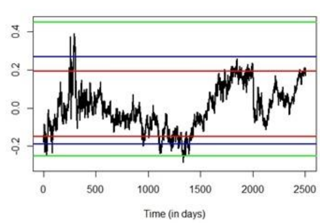
Source: Own preparation

The probability graph and the quantile graph of the two fitted models show that each set of plotted points is almost linear, and because of this, there is no reason to doubt the validity of the model. The return level curve provides a satisfactory representation of the empirical estimates and the density estimation seems consistent with the histogram of the data in both estimated models. Consequently, all four diagnostic graphs in Figures 3 and 4 support the acceptance of the GEV model stipulated for the maximum and minimum spread.

To build the long-short strategy, bands obtained by extreme value from the models found for the maximum and minimum spread were considered, the probability amounts approached were 0,99, 0,995 and 0,999 for the upper limits and 0,01, 0,005 and 0,001 for the lower limits.

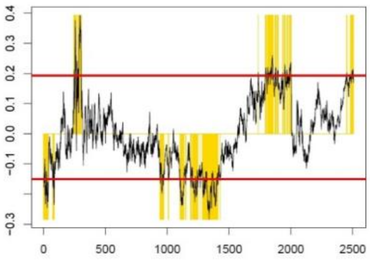
The following graph illustrates this fact.

Figure 5: Outlier bands.

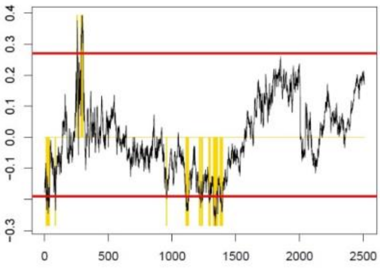


Source: Own preparation.

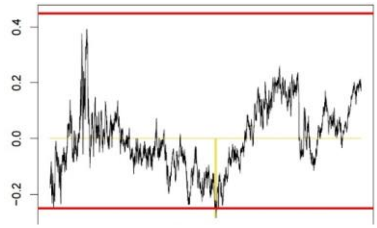
The long-short strategy was created considering the different limits set. In Figure 6 below, the yellow bars indicate where the strategy would be performed.



a) Quantile: 0,99 and 0,01.



b) Quantile: 0,995 and 0,005.



c) Quantile: 0,999 and 0,001.

Figure 3: Long-Short Strategy.

Source: Own preparation.

Whenever the spread reaches the upper band the long-short strategy is set up as follows: sold position in the share ITUB4.SA and bought position in the share ABCB4.SA. The same happens when the spread reaches the lower band, in this case the short position is established in the share ABCB4.SA and the long position is established in the share ITUB4.SA. The "buy and sell" operation is disassembled when the spread, which has exceeded the bands, reaches them again.

Regarding the profitability of operations, the accumulated return was calculated based on the strategies performed considering the different bands obtained by extreme value.

Table 3 shows the cumulative return after 2508 days for the different limits applied.

Table 3 – Cumulative Return

Upper limit	Lower limit	Cumulative Return
0,1933	-0,1511	-33,37%
0,2707	-0,19	28,20%
0,4499	-0,2507	2,67%

Source: Own preparation

9. CONCLUSION

This study aimed at using extreme value theory (EVT) to model asset price series in order to apply pair trading and long-short strategies together to identify (over time) situations where buy and sell positions of the co-integrated stocks would be assembled, with the final objective of detecting the best approach to generate a stable degree of profitability.

As seen in Table 3 , when considering the amounts 0,995 and 0,005, the accumulated return was 28,20%, that is, when using the long-short strategy taking into consideration the limits of 0,2707 and -0,19, both to execute and undo the buy and sell positions, the gain reached was superior to those obtained by other limits. For limits of 0,1933 and -0,1511, the long-short strategy showed an accumulated negative return of -33,37%, showing that although this strategy is considered risk neutral, the limits where buy and sell transactions will be set up and undone significantly influence the financial residual of the operation.

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