

## Implementations of Sensitivity Analysis Technique to Atmospheric Boundary Layer Heights

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### Abstract

*The prediction of local air quality became greatly important by knowing the heights of atmospheric boundary layer. These heights give valuable information's about lower atmospheric transport and dispersion of pollutants and basically for adapting dispersion models, which will be used to study pollutants released from different sources. The sensitivity analysis techniques applied for parameter values of atmospheric boundary layer heights model equations will be performed for three successive stability criteria such as neutral, stable, and unstable conditions. The necessary model parameters are sampling utilizing Latin Hypercube Sampling technique and their sensitivity indices will be identified as partial correlation coefficient and partial rank correlation coefficient. The results show that in a neutral condition, the following parameters of proportionality constant, wind speed and roughness height are strongly dependent. In stable condition, Monin-Obukhov length, proportionality constant, wind speed and roughness height are strongly dependent variables. While for unstable conditions, wind speed, roughness height, day time and temperature are strongly dependent. All of these parameters will be ranked from higher to lower values.*

**Keywords:** Atmospheric boundary layer height, Dispersion of pollutants, Dispersion models. Latin Hypercube Sampling Technique, partial correlation coefficient, partial rank correlation coefficient

## 1. INTRODUCTION

Stability of the atmosphere within ABL largely determines intensity of turbulence and, then the diffusion processes, which affect effluents released into this layer [1]. The unstable boundary layer characterized by large eddies, convective plumes, a capping inversion and well-mixed appearance of wind speed and potential temperature. The boundary layer in stable condition is shallower and has smaller eddies and steeper vertical gradients in wind speed and potential temperature and [2].

## 2. ATMOSPHERIC BOUNDARY LEVELS

Many empirical formulae for ABL heights are available in literatures and all of them rely on the type of atmospheric stability conditions. They are depending mainly on the type of atmospheric stability conditions, which varies hourly during the day time [3].

### 2.1 Neutral Conditions

The empirical equation used for ABL heights calculated as [4]:

$$h_n = C_n \frac{u_*}{|f|}, \text{ when } \left| \frac{u_*}{f L} \right| < 4 \quad (1)$$

and;

$$u_* = k u \left( \ln \left( \frac{z}{z_0} \right) \right)^{-1} \quad (2)$$

Where:

$C_n$	Proportionality constant
$u_*$	Friction velocity
$f$	Coriolis parameter = $2 \Omega \sin \phi$
$\Omega$	Earth's rotation = $7.292 \times 10^{-5}$ rad/s
$\phi$	Site latitude
$L$	Monin-Obukhov length

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$k$	Von Karman's constant = 0.41
$u$	Wind velocity at height $z$
$z$	Reference height =10 m
$z_0$	Roughness height

For small  $\phi$  (i.e. the tropics), this equation, yields unrealistic values, a minimum value of  $\phi \cong 20^\circ \cong 0.35$  rad will be used and  $C_n = 0.2$  [4]. Other authors choose values between 0.07 and 0.5, which leads to systematic differences in the estimate of heights [5].

## 2.2 Stable Conditions

The following empirical equation for ABL height was given as [6]:

$$h_s = C_s \left[ u_* \left( \frac{L}{|f|} \right) \right]^{1/2}, \text{ when } \left| \frac{u_*}{f L} \right| > 4 \quad (3)$$

Where,  $C_s$  is a proportionality constant equal to 0.4. While the friction velocity is given from the equation:

$$u_* = k u \left[ \ln \left( \frac{z}{z_0} \right) + \beta \left( \frac{z}{L} \right) \right]^{-1} \quad (4)$$

Where  $L$  is Monin-Obukhov length [7], while  $\beta$  is a constant equal to 5 [8].

## 2.3 Unstable Conditions

The convective ABL heights were determined [9-11]. For buoyancy-generated turbulence, the differential equation for convective  $h_b$ , was given by the following equation:

$$\frac{dh_b}{dt} = (2\beta + 1) \frac{(\overline{\omega' \theta'_o})}{\gamma h_b} \quad (5)$$

Assuming that the heat flux varies linearly with height and the entrainment heat flux at  $z = h_b$  is proportional to the heat flux of the surface  $\overline{\omega' \theta'_h} = -\beta \overline{\omega' \theta'_o}$ . When mechanically generated shear turbulence is dominant, the heat flux of the surface is zero. In this case, assuming once again a linear variation of heat flux with height and

specifying the entrainment heat flux at  $z = h_m$  as  $\overline{\omega' \theta'_h} = -\frac{\alpha \theta u_*^3}{g h_m}$ .

Therefore, the rate equation for  $h_m$  becomes:

$$\frac{dh_m}{dt} = 2 \alpha \left( \frac{\theta}{g} \right) \left( \frac{u_*^3}{\gamma h_m^2} \right) \tag{6}$$

Where:

$\beta, \alpha$  are constants and equal 0.2 and 2.5, respectively

$\overline{\omega' \theta'_e}$  is the entrainment heat flux =  $-u_*^3 / k L (g / \theta)$

$\gamma$  is the potential temperature gradient above inversion layer  
 $(\partial \theta / \partial z) = 0.005 \text{ (km}^{-1}\text{)}$

$g / \theta$  is the buoyancy parameter

$g$  is the acceleration due to gravity =  $9.78 \text{ ms}^{-2}$

$\theta$  is the potential temperature at height  $z$

The differential equations (5) and (6) are solved analytically, we obtains:

$$h_b^2 = 2 (2 \beta + 1) \frac{(\overline{\omega' \theta'_e})}{\gamma} t + \lambda_1 \tag{7}$$

$$h_m^3 = 6 \alpha \left( \frac{\theta}{g} \right) \left( \frac{u_*^3}{\gamma} \right) t + \lambda_2 \tag{8}$$

Where  $\lambda_1$  and  $\lambda_2$  are constants was given by the initial values of  $h_b$  and  $h_m$ . While the friction velocity was given by the following equation:

$$u_* = k u \left\{ \ln \left( \frac{z}{z_0} \right) - 2 \ln \left( 0.5 \left( 1 + \frac{I}{\varphi_m} \right) \right) - \ln \left( 0.5 \left( 1 + \frac{I}{\varphi_m^2} \right) \right) + 2 \tan^{-1} \left( \frac{I}{\varphi_m} \right) - \left( \frac{\pi}{2} \right) \right\}^{-1} \tag{9}$$

and;  $\varphi_m = (1 - 16 (z / L))^{-1/4}$  (10)

Monin-Obukhov length was determined [12], and the empirical curves were fitted using the power law functions, so that we have the following equation [13]:

$$L^{-1} = a z_a^b \tag{11}$$

Given the initial values for  $h_b$  and  $h_m$ , equations (7) and (8) was solved numerically [14] and the proposed interpolation formula were given as follows:

$$h_{us} = (h_b^3 + h_m^3)^{1/3} \tag{12}$$

Where  $h_b$  and  $h_m$  are the boundary layer depths determined for the ideal cases of complete mechanically or complete buoyancy-generated turbulence, respectively.

## **SENSITIVITY ANALYSES TECHNIQUES**

A LHS is commonly used and applied to many computer models [15]. The techniques give the relation between dependent output values and the correlation coefficient between independent values. A LHS with applications was given [16-17] and a test example was found [18]. The analysis determined by rank-transformed values are more effective for representing a variety of relation between independent values and dependent model outputs, therefore (PRCC) will be used to minimizing the effects of extreme values than those calculated from real values.

The independent parameters are sampling by utilizing LHS technique [19-20], and their result indices are given as PRCC. The selective sensitive independent parameters for different stability conditions are shown in Table 1, assuming that the other parameters are constant, LHS code generate 100 value of independent variables and another 100 values of dependent variables. The flow diagrams of the complete processes are given in Figure 1, a developed code RRR giving almost similar results [21]. A computer program is developed for model equations; each model treated each case of study separately. The PCC code are used to correlate output with input variables. A PCC and PRCC are determined, where the result values  $< 0.6$  in absolute value are not recommended.

For neutral model, a correlation analysis of influential input parameters where (PRCC  $\geq 0.60$ ) shows strong correlations with the proportionality constant (0.96), wind speed (0.89) and roughness height (0.59), while neglecting the effects of site latitude, as shown in Figure 2.

In case of stable condition, the technique is repeated for other sets of input parameters. The results of PRCC are declaring that the proportionality constant (0.93), Monin-Obukhov length (0.92), wind speed (0.58), all having strong correlations while roughness length and site latitude having neglecting effects.

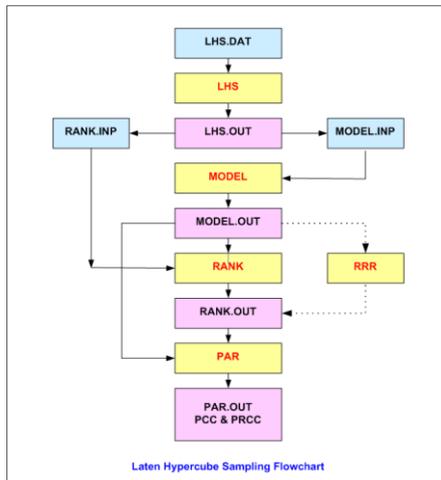
For unstable model equations, the results give the most important parameters by PRCC values as follows: wind speeds (0.89), roughness length (0.88), day time (0.84) and temperature (0.67), neglecting the effect of constants  $\lambda_1$  and  $\lambda_2$  of equation (7) and (8). The performs of the result shows that the day time and temperature

parameters having an effects, so the applied techniques used gives well guaranteed of the model equations.

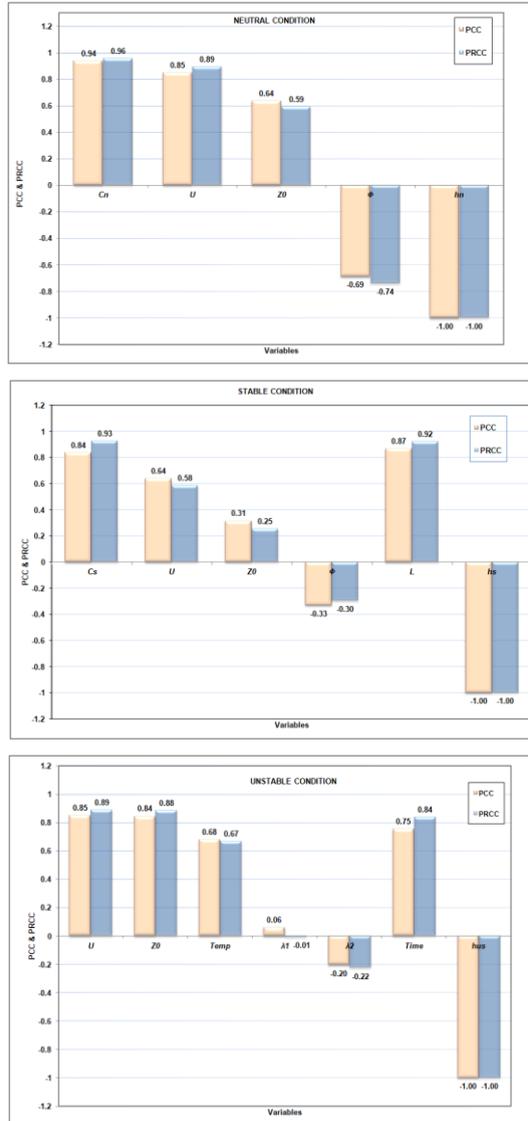
**Table 1 Characteristics of input parameters used in sensitivity analysis**

Parameter	Description	Unit	Range of Value			Type of Distribution	
			Neutral	Stable	Unstable		
$C_n, C_s$	Proportionality constants	-	0.07-0.5	0.07-0.5		UF	
$U$	Wind speed	ms <sup>-1</sup>	0.5-12	0.5-12		NR	
$z_0$	Roughness height	m	0.003-1.0	0.003-1		NR	
$\phi$	Site latitude	rad	0.35-0.7-1-2	0.35-0.7-1-2		TA	
$L$	Monin-Obukhov length	m		2		UD	
				20-0.64			
				100-0.36			
$\theta$	Potential temperature	°K			-7.0-50.0	NR	
$\lambda_1$	Constant	-			100-1000	UF	
$\lambda_2$	Constant	-					
$t$	Time	sec			8		UD
					2	0.125	
					5	0.125	
					8	0.125	
					11	0.125	
					14	0.125	
					17	0.125	
					20	0.125	
			23	0.125			

Note: (TA): triangular distribution, represent minimum, most probable and maximum values, (LN): lognormal distribution, (NR): normal distribution, (UF): uniform distribution, and (UD): user distribution.



**Figure 1 Flow diagram of LHS techniques**



**Figure 2 PCC & PRCC for different stability conditions**

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