



# Enhancing the Controller for Quarter Car Semi-Active Suspension System

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#### Abstract

The aim of this paper is achieve the stability of a quarter car semi-active suspension system by designing PID and LQR controllers of semi-active suspension system approach. The vehicle suspension systems are typically rated by its ability to provide good road handling and improve passenger comfort. Here, the hydraulic damper is replaced by a magneto-rheological damper and a controller is developed for controlling the damping forces of the suspension system. Two control techniques, Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) for quarter car semi active suspension system have been designed. Dynamic system used in this study considered as a two-degree-of-freedom (2-DOF) linear system, which can apprehend basic performance parameters of a suspension system like body and suspension travel and give results in terms of rise time, settling time and over-shoot. A comparison between uncontrolled suspension and semi active suspension with PID and LQR controllers with road profile as input has been made. Simulation study has been made on the quarter car semi active suspension system using MATLAB/SIMULINK. Based on the simulation, it has been found that PID and LQR controllers were better in suppressing the vibrations, providing good road handling and improve passenger comfort as compared to uncontrolled suspension system. The results showed that the semi active suspension with controllers had reduced the rise time, settling time and overshoot of sprung mass displacement, sprung mass acceleration, wheel displacement and suspension deflection.

**Keywords:** Proportional Integral Derivative (PID), Linear Quadratic Regulator (LQR), Controller, MR damper, Quarter car semi active suspension system.

### I. INTRODUCTION

In recent years, researches in the field of automotive vehicles are focused on improving driving safety and passenger comfort. These two parameters are mainly influenced by the design of the suspension system. The suspension system works between vehicle chassis and wheels, and its main goal is to reduce motion of the vehicle body called sprung mass [1]. A vehicle suspension system performs two major tasks, isolate the vehicle body from external road disturbances for the sake of passenger comfort and control the vehicle body attitude and maintain a firm contact between the road and the type to provide guidance along the track [2]. Suspension systems are classified by the control system in passive, semi-active and active, which depend on the operation mode to improve vehicle ride comfort, vehicle safety, road minimization and the overall performance. Passive damage suspension systems consist of conventional springs and dampers, whose properties are fixed, and there is no external energy source in the system. Semi-active suspension systems, generally, consist of controllable dampers and passive springs without requiring large power sources, so the control system is not destabilized. Different types of semi-active dampers have been investigated; the most representative is magneto rheological dampers, whose response varies with the magnetic field applied. To study the behavior of suspension systems, different vehicle models have been used. The most used model is the quarter car model because it takes into account the most important features of suspension system preserving the simplicity of the model. It is a model of two degrees of freedom (2DOF) which considers the vertical dynamics of a single wheel [3]. The suspension control arms or links allow wheel movement independent of the body. This provides a mechanism to isolate body from the road bumps. The springs manipulate the frequency of road disturbances and try to bring them into a more manageable band. They also provide damping through friction (spring ends and the seat) and own hysteresis. The damper dissipates the energy of the dynamic load coming through the road bumps. Together, they try to eliminate the effects of road undulations on the ride as well as stability of the vehicle.

# II. RELATED WORKS.

Colina, G. Lerma, et al (2014) developed the modeling of semi active suspension system of (2DOF), the results showed that the semi-active suspension system controlled by a logical strategy minimizes vertical acceleration experienced by passengers compared to passive suspension system [1]. Bhise, A., Desai, R., Yerrawar, (2016) designed quarter car model with passive and semi-active suspension system, semiactive designed with MR damper, control performance was compared between passive and semi-active, so the semi-active had gave lower value of maximum sprung mass acceleration for above (Half Sine & Step) road excitation and the semiactive suspension MR damper performance was found to be better than passive damper system [4]. Ahmed A. Abougarair1, Muawia M. A. Mahmoud (2017) (PD), (LQR) and Fuzzy logic tune PD controllers' techniques implemented to the active suspensions system for a quarter car active suspension system model, the comparison between these controllers by means of the reduction of body displacement trajectory under influence road profiles and variation of body mass, so that the using of the three proposed control techniques were effective in controlling a vehicle [5]. Abd El-Nasser S. Ahmed1, Ahmed S. Ali2, et al (2015) obtained a mathematical model for passive and active suspensions systems of quarter car model and constructed an active suspension control using PID controller. Active suspension posed the ability to reduce the traditional design as a compromise between road handling and comfort performed using road profile, so the designing of an active suspension PID controller improved performance compared to passive suspension system [6]. D. Hanafi (2010) developed semi-active suspension by installing a variable shock absorber parallel with the passive suspension system and controlled using PID controller. A high fidelity mathematical model for capturing the realistic dynamic of car passive suspension system for the basic element of semi-active suspension system was determined using the intelligent system identification. The result showed that the PID controller was successfully control the variable shock absorber in order to eliminate the road surface disturbances effect to the car body, then the PID controller is one type of suitable controller for semi-active suspension system in practical implementation [7]. Truong Nguyen Luan Vu, Do Van Dung, et al (2017) designed (PID) controller for active vehicle suspension system (AVSS) of a quarter car model that proposed on the basis of Internal Model Control (IMC) theory in order to compromise two conflicting criteria, ride comfort and road handling, the road input signals selected as unit step, sine wave, and white noise signals. The results indicated that the performance of AVSS had significantly improved in terms of reducing the peak overshoot of sprung mass displacement and sprung mass acceleration in compared with traditional suspension systems [8]. M. S. Yahaya, and M. R. A. Ghani, (2000) worked to improve the ride quality and handling performance within a given suspension stroke limitation. The ride quality measured by the vertical acceleration, sprung mass, and the handling performance determined by the type deflection. This study analyzed the aspects of passive and active suspensions and focus on the ride quality improvement, and the LQR control was selected to control an actuator in active suspension. Accordingly, the ride quality can improved using an active suspension with LQR controller that can be considered one of the solutions for excellent comfort ride and good handling of cars [9]. S. M. S. M. Putra, F. Yakub, Z. A. Rasid et al (2018). The suspension system was required in an automobile to absorb shock that comes from various types of disturbances such as irregular road profile, engine vibrations and wheel. LQR controller design was required to minimize the vibrations, an active suspension model that considers only vertical movement was utilized in the suspension system, and the control design approach was compared with the classical control (PID) that set as a benchmark control. The results obtained from the simulations showed that the responses of the quadratic based approach give the significant improvement in minimizing the vibration and fast settling time [10]. El Majdoub K, Ouadi H, Touati A. (2014), The semi-active suspension system involves a (MR) damper with a hysteretic behavior captured through the Bouc-Wen model, this device can be integrated in the electric or hybrid electric, the control design was performed using the LQR theory and Lyapunov control design tools; it included an observer providing online estimated of the hysteresis internal state. The theoretical result was confirmed by LQR method and compared with passive suspension [11]. Preeti Singh and M. P. R. Prasad. (2020) Shaia Kaid Mosleh Mohammed Almadrahi, Heisum M. Ewad- Enhancing the Controller for Quarter Car Semi-Active Suspension System

studied improving the ride quality and to minimize the semi-active suspension vibrations within tolerable limits by means of applying control technique. Robust PID controller is used to control vibration for high speed train. Simulation studies have been carried out using Matlab/Simulink. The paper focused on dynamic modeling and control of high speed train. High speed train model complex due to hunting stability, derailment, ride quality etc. Motion control of high speed train was a big challenge in transportation system. PID Controller was applied on motion control of high speed train. Performance analysis of high speed train (lateral acceleration of car body, front and rear body) has been carried out. These simulation results using PID were comparatively better than the system results without controller [12].

# III. MODELING OF QUARTER CAR SEMI ACTIVE SUSPENSION SYSTEM

The idea of this paper is to design PID and LQR controllers for quarter car semi active suspension system, and study the stability of the system. When these controllers designed, four parameters are needed to be acknowledged, sprung mass acceleration, sprung mass displacement, unsprung displacement and suspension deflection, and the hydraulic damper is replaced by a magneto-rheological damper and a controller is developed for controlling the damping force of the suspension system. Modeling of suspension system has been taking into account the following observations: . The suspension system modeled here is considered 2DOF and assumed to be a linear or approximately linear system for a quarter cars. • For the ease of the design, certain minor factors, like backlash and movement in various gear systems, linkages and joints and the vehicle chassis flex have been disregarded to reduce the complexity, these have been neglected in the model. • Tyre material is considered as having damping property as well as stiffness. The quarter-car model contains two vertical displacements: 1) The unsprung mass Mu, and 2) The sprung mass Ms, the equations of vertical direction motion for the quarter car suspension system can be represented by applying Newton's second law of motion with the help of following mathematical model.

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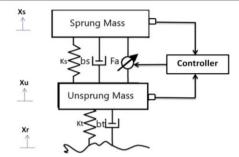


Figure 1 Quarter car semi active suspension model

Free body Diagram Performs force analysis shows the Forces apply to Ms and Mu:

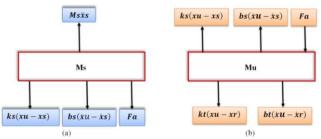


Figure 2 Free body diagrams of a) sprung mass Ms, b) unsprung mass Mu.

From the figure 2 (a), the mathematical equation of the spring mass part is:

$$\begin{split} Ms\ddot{x}s &= Fa - ks(xs - xu) - bs(\dot{x}s - \dot{x}u)\\ \ddot{x}s &= \frac{1}{Ms}[Fa - ks(xs - xu) - bs(\dot{x}s - \dot{x}u)] \end{split} \tag{1}$$

From the figure 2 (b), the mathematical equation of the unspring mass part is:

-Mužu = Fa + bs( $\dot{x}u - \dot{x}s$ ) + ks(xu - xs) + bt( $\dot{x}u - \dot{x}r$ ) + kt(xu - xr) Mužu = bs( $\dot{x}s - \dot{x}u$ ) + ks(xs - xu) – bt( $\dot{x}u - \dot{x}r$ ) – kt(xu - xr) – Fa Then, for unsprung-mass motion, the equation is:  $\ddot{x}u = \frac{1}{Mu} [bs(\dot{x}s - \dot{x}u) + ks(xs - xu) - bt(\dot{x}u - \dot{x}r) - kt(xu - xr) - Fa]$  (2) **Where:** Ms: Mass of vehicle's body (kg). Mu: Mass of wheel (kg). Ks: stiffness of spring (N/m) Kt: stiffness of tyre (N/m). bs, bt: damping coefficient of damper (N/m). xs: displacement of vehicle's body(m). xu: displacement of wheel (m).

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xr: displacement of road profile (m).

Fa: controllable damping force of the control semi-active suspension. After choosing State variables. Let:

 $x_1(t) = xs(t) - xu(t)$   $x_2(t) = xu(t) - xr(t)$   $x_3(t) = \dot{x}s(t)$  $x_4(t) = \dot{x}u(t)$ 

#### Where,

From equation (1), we have:

$$\begin{split} \dot{x}_3(t) &= \frac{1}{Ms} \Big[ Fa - ksx_1(t) - bs \big( x_3(t) - x_4(t) \big) \Big] \\ \text{Disturbance caused by road roughness,} \\ &w(t) = \dot{x}r(t) \quad (3) \\ \text{From equations (2) and (3), we have:} \\ &\dot{x}_4(t) = \frac{1}{Mu} \Big[ bs \big( x_3(t) - x_4(t) \big) + ksx_1(t) - bt \big( x_4(t) - w(t) \big) - ktx_2(t) - Fa \Big] \end{split}$$

#### 3.3.2 State space of the model

$$\begin{split} \dot{x}_1(t) &= x_3(t) - x_4(t) \\ \dot{x}_2(t) &= x_4(t) - W(t) \\ \dot{x}_3(t) &= -\frac{ks}{Ms} * x_1(t) - \frac{bs}{Ms} * x_3(t) + \frac{bs}{Ms} * x_4(t) + \frac{Fa}{Ms} \\ \dot{x}_4(t) &= \frac{ks}{Mu} * x_1(t) - \frac{kt}{Mu} * x_2(t) + \frac{bs}{Mu} * x_3(t) - \left(\frac{bs+bt}{Mu}\right) * x_4(t) + \frac{bt}{Mu} * w(t) - \frac{Fa}{Mu} \end{split}$$

Based on the state space equation method of a single input multiple outputs, Equations (1) and (2) can be rewritten as:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{BF}(t)$$
$$\mathbf{y}(t) = \mathbf{C} \mathbf{x}(t) + \mathbf{DF}(t)$$

Where the state vector  $\mathbf{x}(t)$  = [x1(t), x2(t), x3(t), x4(t)]T ,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{ks}{Ms} & 0 & -\frac{bs}{Ms} & \frac{bs}{Ms} \\ \frac{ks}{Mu} & -\frac{kt}{Mu} & \frac{bs}{Mu} & -(\frac{bs+bt}{Mu}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{Ms} \\ -\frac{1}{Mu} \end{bmatrix} Fa + \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{bt}{Mu} \end{bmatrix} W$$

Where:

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{ks}{Ms} & 0 & -\frac{bs}{Ms} & \frac{bs}{Ms} \\ \frac{ks}{Mu} & -\frac{kt}{Mu} & \frac{bs}{Mu} & -(\frac{bs+bt}{Mu}) \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{Ms} \\ -\frac{1}{Mu} \end{bmatrix} Bw = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{bt}{Mu} \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where x &(t), x(t), y(t), A, B, C and D are the matrices of various orders. Matrix A is called the state matrix, B is the input matrix, C is the output matrix and D is the direct transmission matrix.

B substituting the values of the parameters:

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$\mathbf{A} \!=\! \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -55.80 & 0 \\ 269.70 & -3166.66 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -3.45 & 3.45 \\ 16.67 & -16.67 \end{bmatrix},$	$B = \begin{bmatrix} 0 \\ 0 \\ 0.0034 \\ -0.0167 \end{bmatrix},$	$Bw = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 16.67 \end{bmatrix}$
$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	D=[0 0 0 0]		

The performances of ride comfort, road holding ability, and suspension deflection are three main performance criteria in any vehicle suspension design, which are exactly the multi- objective of robust control the ride comfort and the vehicle body's vertical acceleration, have a direct relationship. Consequently, the sprung mass acceleration (SMA), should be as small as possible [13].

Table 1 Parameters used in system simulation, the parameter values are taken from [14] with some adjustments

	Parameter	Symbol	Quantities	Unit
1	Mass of vehicle body	Ms	500	kg
2	Mass of the tyre and suspension	Mu	60	kg
3	Coefficient of suspension spring	Ks	13100	N/m
4	Coefficient of tyre material	Kt	252000	N/m
5	Damping coefficient of the dampers	bs	400	N-s/m

#### **IV. PID CONTROLLER DESIGN**

PID control is very simple control hence it is widely used in many research and industrial applications. PID basically has a three control i.e. Proportional, Integral and Derivative. A PID is unity feedback controller which calculates error between desired values called set point (SP) and measured value (MV). The PID aims to minimize the error by manipulating the control variables. For best performance of PID controller, their parameters must be tuned depending upon the nature of the system. The three terms of PID controller performs the different control action. P control decreases the rise time of a response, while there is no improvement in offset. I control basically used to eliminate the offset and steady state error but increases the settling time, thus the transient behavior of the system get worse and finally D control action used to get better transient response but stand-alone derivative control introduce a large steady state error. The transfer function of PID controller is:

(4)

 $u(t) = Kp e(t) + Ki \int_{0}^{t} e(t)dt + Kd \frac{de(t)}{dt}$ 

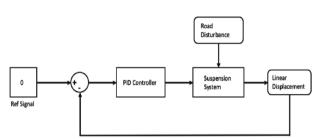


Figure 3 Block diagram of semi active suspension system with PID controller.

The above selected values of gains are taken into account by adjusting it manually in Simulink. Now the performance of the designed semiactive quarter car suspension system under the road excitation i.e. Jerk (step input) is evaluated through computer simulation. The input signal type is a step signal as an imitation of bump type of real road surface the semi-active suspension system has a feedback mechanism to control the damping coefficient of the damper. This feedback is fed to a PID controller which generates its response and as a result adjusts the damping stiffness of the damper used.

# V. LQR CONTROLLER DESIGN

Consider a state variable feedback regulator for the system given as: u(t) = K x(t) K is the state feedback gain matrix. The optimal controller of given system is defined as controller design which minimizes the following performance index.

$$J = \frac{1}{2} \int_0^\infty (X^T(t)Qx(t) + U^T Ru(t)dt)$$

The matrix gain K is represented by:

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}$$

The matrix P must satisfy the reduced-matrix equation given as:

$$A^{\mathrm{T}}P + PA - PBR^{-1}B^{\mathrm{T}}P + Q = 0$$

Then the feedback regulator U:

$$\mathbf{u}(\mathbf{t}) = -(\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P})\mathbf{x}(\mathbf{t})$$

u(t) = -K x(t)

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The selection of Q and R determines the optimality in the optimal control law. The choice of these matrices depends only on the designer. Generally, preferred method for determining the values for these matrices is the method of trial and error in simulation. As a rule of thumb, Q and R matrices are chosen to be diagonal.

By using matlab command [k, p, e]=lqr(a,b,q,r); the matrix P equals:

 $P = \begin{bmatrix} 0.13 & -3.60 & 0.036 & 0.0074 \\ -3.60 & 414.37 & -2.20 & -0.37 \\ 0.036 & -2.20 & 0.018 & 0.0033 \\ 0.0074 & -0.39 & 0.0033 & 0.015 \end{bmatrix}$ 

By using the trial and error method the Q and R matrices are obtained as:

Q=	0.000865	0	0	0					
	0	1.81140	0	0					
	0	0	0.01131	0					
	L O	0	0	65.030					
R = [0.00000009]									

The constant feedback gain vector K was computed using MATLAB 'lqr' command given A, B, Q and R and the closed-loop system step responses found by simulation and the values of gain K found equal:

 $K = \begin{bmatrix} 29.67 & -1611010.92 & 8659.13 & -255642.85 \end{bmatrix}$ 

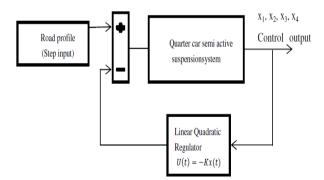


Figure 4 Block diagram for LQR controller Design

The LQR controller has a function to adjust the damping coefficient of the variable shock absorber in order to keep the car body always stable. Adjustable process is based on the characteristic of the road profile. Shaia Kaid Mosleh Mohammed Almadrahi, Heisum M. Ewad- Enhancing the Controller for Quarter Car Semi-Active Suspension System

#### VI. SIMULATION AND RESULTS

The simulation analysis based on the Simulink models for quarter car semi active suspension system was carried out, the suspension deflection, body acceleration, body displacement (body travel) and wheel displacement of the car in terms of linear was taken as the performance parameter. The road profile was taken as input for both the systems (step signal). The performance criteria were taken in terms of rise time (Tr), settling time (Ts), overshoot (OS).

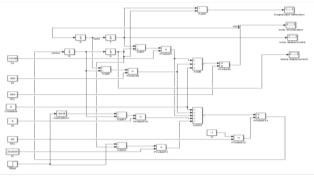


Figure 5 Simulation of suspension system

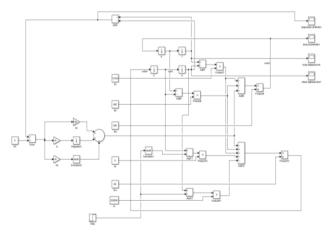


Figure 6 Simulation of suspension system with PID

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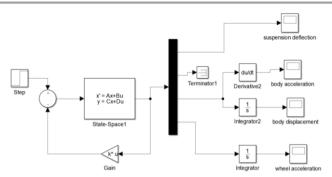


Figure 7 Simulation of suspension system with LQR controller.

The simulation results of the quarter car semi active suspension system with PID controller are shown in figures (8, 9, 10, and 11).

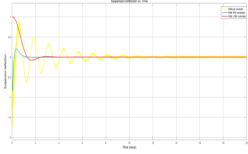


Figure 8 Simulation results of suspension deflection without controller and with PID& LQR controllers.

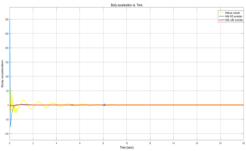
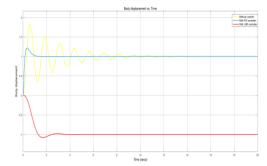


Figure 9 Simulation results of body acceleration without controller and with PID& LQR controllers



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Figure 10 Simulation results of body displacement without controller and with PID& LQR controllers.

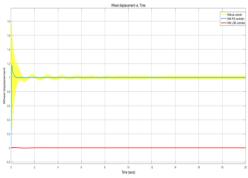


Figure 11 Simulation results of wheel displacement without controller and with PID& LQR controllers.

#### VII. CONCLUSION

Implementation of Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR) controllers in linear system of semi active suspension for a quarter car model is studied successfully and designed using MATLAB/SIMULINK. Both controllers are capable of enhancing very effectively the performance of the suspension system as ride comfort and vehicle stability as compared to uncontrolled suspension system. Based on the results discussed in previous section, it can be observed that semi active suspension system with LQR and PID controller performances was improved in reference with the performance criteria like rise time, settling time and overshoot for suspension deflection, body acceleration, body displacement and wheel displacement. This performance improvement in turn will increase the passenger comfort level and ensure the stability of vehicle. By comparison between two controllers (PID and LQR) the results of simulation showed that the enhancing performance of suspension system was done and the stability of the system was achieved.

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