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Wind Energy Potential for Karachi (Pakistan) and Estimation of Weibull Distribution function parameters

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Abstract:

The daily wind speed data measured at 00:00 hours and 12:00 hours, respectively, over a period of 10 years (2002-2011) is obtained from Karachi Meteorological office. A detailed analysis of the measured data is presented and wind energy potential for Karachi is estimated. Measured data is fitted to a Weibull probability distribution function. Weibull shape and scale parameters are computed using six different statistical methods, i.e. Method of Least Squares (MLE), Maximum Likelihood Method (MLM), Modified Maximum Likelihood Method (MMLM), Method of Moments (MoM), Empirical Method (EM) and Energy Pattern Factor Method (EPFM). Various error analysis techniques are employed and it is concluded that the MLM, MMLM, MoM and EM performed better, whereas MLE and EPFM are comparatively lagging in performance. Weibull probability density function (PDF) and cumulative distribution function (CDF) are also determined. In order to consolidate the results Monte Carlo simulations are also performed.

Key words: wind speed data, Karachi Meteorological office, wind energy potential, Weibull distribution function parameters.

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1. Introduction

The major contribution of world's energy requirement today comes from burning huge quantities of fossil fuels and which consequently releases large quantities unwanted waste in the atmosphere. This is one of the factors for abnormal weather conditions observed in various parts around the world. The number of adverse effects of using fossil fuel, such as acid rain, polluted fog (urban smog), localized haze, more frequent tornados in certain parts of the globe etc, has forced scientists all around the world to use clean and reliable renewable energy resources. These include wind, solar radiation, biomass, and geothermal energy. Wind energy is one of the renewable energy sources, i.e. it is sustainable and can be harnessed commercially. Using renewable energy sources, such as wind energy, reduces the consumption of fossil fuel and which in turn leads to an effective reduction of environmental pollution. Therefore by installing a wind engineering project, the cost of electrical power generation can be reduced to a significant level. Nevertheless, for an effective planning and realization of a wind power engineering project requires a detailed understanding of wind characteristics.

Wind energy is the by-product of solar energy. Solar radiations are absorbed by the earth's atmosphere which leads to expansion and convection of air and this subsequently give rise to wind pattern. On a larger scale these thermal effects and Earth's rotation, produce wind distribution around the globe. Annually, the estimated amount of solar energy reaching the Earth's surface is 1.5×10^{18} kWh [1] and 2 % of this value is translated in to energy of motion of air. Even with such low conversion efficiency, power estimated is c.a. 4×10^{12} kWh. Furthermore, the frictional drag of the surface of Earth, of vegetations and of building decreases with altitude and leads to an increase in wind speed with increasing altitude above the Earth's surface. Furthermore, since wind power is proportional to the cube of wind speed therefore at higher altitudes more energy can be extracted.

A suitable statistical model for wind speed distribution is required for constructing a better and reliable wind energy conversion system. Weibull distribution function, named after Waloddi Weibull [2], is widely used function to model wind speed data. In most of the commercially available wind energy estimation computer programs, for instance Wind Atlas Analysis and Application Program [3], Weibull distribution function is used and is characterised by two parameters, known as shape and scale parameters. In a plot of wind speed data distribution the Weibull scale parameter gives the control over the abscissa of the plot, i.e. it gives the range of the distribution. The Weibull shape parameter controls the width of the data distribution, i.e. larger the shape parameter narrower the distribution and higher is its peak value. Thus shape parameter gives flexibility to the Weibull distribution to model variety of data.

Sopian et al. (1995) [4] investigated the wind speed data for wind potential in Malaysia and used the conventional graphical method for computing the Weibull parameters. Garcia et al. (1998) [5] used Weibull distribution graphical method and lognormal models for wind speed data. Seguro and Lambert (2000) [6] recommended the modified maximum likelihood method for the estimation of Weibull parameters using the time series wind data. The method of estmation they used for estimating Weibull parameters is based on a wind speed data set of 3 days and they suggested that a true estimation using modified maximum likelihood method would require wind speed data spanned over a period of many months or years. Wind characteristics of Oman were studied by Sulaiman et al. (2002) [7] using the graphical method to determine the Weibull parameters. Several groups [8-17] have used different statistical methods to estimate Weibull parameters, such as graphical method, method of moments, maximum likelihood method, modified maximum likelihood method, empirical method and energy pattern factor method. Jowder [18] analyzed the wind power density at anemometer heights of 10, 30, and 60 m using the empirical and graphical methods in the Kingdom of Bahrain. They estimated Weibull shape and scale parameters at these heights and compared their results. It was reported that the empirical method perform better as compared to graphical method in predicting average wind speed and power density. Dorvlo [19] investigated the wind data for four stations in Oman. Based on Kolmogorov–Smirnov statistic, he concluded that the Chi-square method provides better estimations for Weibull parameters as compared to the moment and graphical methods. The numerical methods mentioned are applicable provided the wind speed data follow the Weibull probability function.

Monte Carlo simulations can be used to check the accuracy of the numerical methods. Ghosh [20] wrote a program in FORTRAN to generate random samples following Weibull function using specific shape and scale parameters. Genc et al. [21] and Kantar et al. [22] calculated Weibull parameters using some numerical techniques and compared their results with the data generated through Monte Carlo simulations. However, through out their calculations the scale parameters they used were all below 1.5 m/s which is lower than the cut-in speed for most of the commercially available wind generators. Furthermore, for the estimation of the parameters they used sample sizes of c.a. 100 and are much less than the number of hours in a month, i.e. 720 hours. Such a small sample size is not applicable for the yearly or long-term assessment of wind energy.

Wind speed data for Karachi, Pakistan, recorded at the anemometer height of 10 m and at 0000 hours and 1200 hours, respectively, for 10 years during the period 2002 to 2011 in Karachi Metrological Office, are studied. In this study Weibull probability distribution function is used for calculating wind energy potential in Karachi. Various statistical methods are employed for estimation of Weibull parameters. These methods include Method of Least Squares (MLE), Maximum Likelihood Method (MLM), Modified Maximum Likelihood Method (MMLM), Method of Moments (MoM), Empirical Method (EM) and Energy Pattern Factor Method (EPFM). The accuracy of the six methods for estimating Weibull parameters has been discussed. The actual wind speed data is analysed and Weibull parameters are calculated using the aforementioned statistical methods. Furthermore the performance of these methods is investigated using Monte Carlo simulation by generating wind speed data for given shape and scale parameters.

2. Theoretical Background

To calculate wind energy potential, the measured wind speed distribution is modelled to a theoretical distribution function. Weibull distribution is one such theoretical model for modelling wind speed data. It is characterized by a velocity function of two parameters (k, c) [23]. Weibull distribution is described by the probability density function *f(v)* and cumulative distribution function *F(v)* given by:

$$f(v)=\frac{k}{c}\left(\frac{v}{c}\right)^{k-1} exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

$$F(v)=1-exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (2)$$

where v is the wind speed, k the dimensionless shape parameter, and c the scale parameter having the same dimension as for v. The distribution is named Rayleigh distribution if its shape parameter k is 2. Weibull mean wind speed v_m and wind energy density based on Weibull probability distribution are given by equations 3 and 4 [24, 25]. Wind power density P_{obs} based on wind speed data can be estimated using equation 5, i.e.,

$$v_m = c\Gamma\left(1+\frac{1}{k}\right) \tag{3}$$

$$\frac{E_d}{A} = \frac{\rho_a c}{2}\Gamma\left(\frac{k+3}{k}\right) \tag{4}$$

$$P_{obs} = \frac{1}{2}\rho_a v^3 A \tag{5}$$

In addition to the above wind power density and wind energy density can also be calculated using the formulas developed by Weibull distribution analysis [26], i.e.,

$$\frac{P_m}{A} = \frac{\rho_a c^3}{2}\Gamma\left(\frac{k+3}{k}\right) \tag{6}$$

$$\frac{E_{dm}}{A} = \frac{\rho_a c^3}{2}\Gamma\left(\frac{k+3}{k}\right)T \tag{7}$$

where ρ_a is the standard air density at sea level with a mean temperature of 20 °C and 1 atmosphere, that is, 1.225 kg/m^3 and A is blade sweep area. The air density depends on altitude, pressure and temperature. T is the specific time period for which the wind energy density is calculated. Γ() is the Gamma function given by

$$\Gamma(x) = \int_0^{\infty} t^{x-1}\exp(-t)dt \tag{8}$$

Weibull shape and scale parameters are estimated using the six statistical methods, which are briefly described in the following sections.

2.1 Method of Least Squares

Least square method is extensively used for estimating Weibull distribution parameters in engineering and mathematical problems. In the present study this method is used for estimating the Weibull shape and scale parameters using wind speed time-series data. In order to use this method the time-series wind speed data must be sorted into bin. Weibull shape parameter k is estimated using the following equation [3, 24]

$$k = \left\{ N\sum_{i=1}^{N} Y_i X_i - \left(\sum_{i=1}^{N} X_i\right)\left(\sum_{i=1}^{N} Y_i\right)\right\}\left\{ N\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2 \right\}^{-1} \tag{9}$$

where

$$Y_i = \ln\{-\ln[1 - P_i]\} \tag{10}$$

$$X_i = ln\left(v_{max,i}\right) \tag{11}$$

The n measured wind speeds are classified into N intervals whose cumulative relative frequencies are contained in a vector P with components P_i. $v_{max,i}$ are the maximum measured wind speed values within each N intervals. The scale parameter c is estimated using the following equation,

$$c = \exp\left(-\frac{a}{k}\right) \tag{12}$$

where a is estimated through the following equation,

$$a = \left\{ \sum_{i=1}^{N} Y_i \sum X_i^2 - \left(\sum_{i=1}^{N} X_i\right)\left(\sum_{i=1}^{N} X_i Y_i\right)\right\}\left\{ N\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2 \right\}^{-1} \tag{13}$$

2.2 Maximum Likelihood Method (MLM)

Maximum likelihood method was formulated by Stevens et al. [27] and is used for estimating Weibull shape (k) and scale (c) parameters. The likelihood function indicates how well a distribution describes the observed data $V = v_1, v_2, ..., v_n$. The best fit is obtained provided the model parameters maximize the log-likelihood function. In this condition the derivative of the respective model parameters become zero. The Shape and scale parameters are determined using the following equations

$$k = \left(\frac{\sum_{i=1}^{n} v_i^k ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} ln(v_i)}{n} \right)^{-1} \quad (14)$$

$$c = \left(\frac{1}{n} \sum_{i=1}^{n} v_i^k \right)^{1/k} \quad (15)$$

where n is the number of data points with wind speed greater than zero and v_i is the wind speed in the i^{th} interval.

2.3 Modified Maximum Likelihood Method (MMLM)

In modified maximum likelihood method, the wind speed data is first converted into frequency distribution format and the equations for shape and scale parameters modified. Using the modified form of equations [28], the shape and scale parameters are determined using following equations

$$k = \left(\frac{\sum_{i=1}^{n} v_i^k ln(v_i) P(v_i)}{\sum_{i=1}^{n} v_i^k P(v_i)} - \frac{\sum_{i=1}^{n} ln(v_i) P(v_i)}{f(v \geq 0)} \right)^{-1} \quad (16)$$

$$c = \left(\frac{1}{f(v \geq 0)} \sum_{i=1}^{n} v_i^k P(v_i) \right)^{1/k} \tag{17}$$

where v_i is the central value of wind speed in the i^{th} class, $P(v_i)$ is the frequency of wind speeds in the i^{th} class, and n is the number of classes.

2.4 Method of Moments (MoM)

In the method of moments [29], the measured mean wind speed v_{mean} and standard deviation σ of the wind speed data are first calculated. These two statistical measures are then equated to the corresponding Weibull parameters. After solving the formulated system of two equations, the estimate of shape and scale parameters are obtained

$$v_{mean} = c\Gamma(1+1/k) \tag{18}$$

$$\sigma = c\left[\Gamma(1+2/k) - \Gamma^2(1+1/k)\right]^{1/2} \tag{19}$$

where

$$v_{mean} = \frac{1}{n} \sum_{i=1}^{n} v_i \tag{20}$$

$$\sigma = \left[\frac{1}{n-1} \sum_{i=1}^{n} (v_i - v_{mean})^2 \right]^{1/2} \tag{21}$$

where k and c are the respective Weibull shape and scale parameters.

2.5 Empirical Method (EM)

Empirical method [30] is based on the collection of data points and can be considered as derived from the method of moments, described in section 2.4. Weibull shape parameter can be estimated by

$$k = \left[\frac{\sigma}{v_{mean}} \right]^{-1.086} \tag{22}$$

and the scale parameter is estimated using the following equation

$$c = \frac{v_{mean}}{\Gamma(1+1/k)} \tag{23}$$

2.6 Energy Pattern Factor Method (EPFM)

Energy pattern factor method [31, 32] is a simple method of estimating Weibull parameters. Energy pattern factor is defined as

$$E_{pf} = \frac{(v^3)_{mean}}{(v_{mean})^3} \tag{24}$$

where $(v^3)_{mean}$ is the mean of wind speed cubes and $(v_{mean})^3$ is the cube of wind speed data. The shape parameter is obtained by the following equation

$$k = 1 + \frac{3.69}{E_{pf}^2} \tag{25}$$

The scale parameter is obtained from equation (23).

2.7. Random Number Generator

The set of data points for the Weibull distribution are randomly generated [33, 34] using the given or estimated shape and scale parameters, under the condition that the cumulative distribution function for the continuous variable varies uniformly in the range [0, 1]. Therefore a random variable, characterizing Weibull distribution with the given shape (k) and scale (c) parameters, are obtained by solving equation 2 for wind speed v as below

$$v = c\left[\ln\left(\frac{1}{1-R_n}\right)\right]^{1/k} \tag{26}$$

where R_n is a random variable in the range [0,1].

2.8. Statistical Error Analysis and Goodness of Fit

In order to examine the viability and accuracy of six methods of estimation and to test the goodness of fit of the measured data to the Weibull function, following tests are performed: RMSE (Root Mean Square Error), χ^2, and R^2 (analysis of variance or efficiency of the method). These test are defined as

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{1/2} \tag{27}$$

$$\chi^2 = \frac{\sum_{i=1}^{N}(y_i - x_i)^2}{N-n} \tag{28}$$

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - z_i)^2 - \sum_{i=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(y_i - z_i)^2} \tag{29}$$

where N is the number of observations, y_i is the observed frequency for the bin i, x_i is the expected frequency for bin i and is calculated using Weibull distribution, z_i is the mean of y_i.

Root Mean Square Error (RMSE) test is the square root of the variance of residuals. RMSE gives the absolute measure of the fit of the model to the measured data. Lower value of RMSE indicates a better fit. In comparison to RMSE, the R-Squared test gives the relative measure of the fit of the model to the measured data. A value of R-Square closer to 1 indicates that a greater proportion of variation in data is being explained by the model.

χ^2 is a widely used statistical test for comparing the actual measured data with the data generated by the model. In this test the goodness of fit to the statistical model is investigated with the help of a statistical hypothesis known as a null hypothesis. The null hypothesis implies that there is no significant difference between the expected and observed result. Based on the computed probability or p-value, the null hypothesis is either accepted or rejected.

The general distribution of measured wind speed data can be visualised by a histogram. Histogram is particularly useful in comparing the distribution of the wind speed variations with the modelled Weibull distribution. Since the shape of the histogram depends on the bin size, therefore the choice of bin size is critical. The following empirical expression [35] is used to determine the best bin size (B):

$$B = \frac{v_{max}}{\left[3.3\ln(n) + 1\right]} \tag{30}$$

where v_{max} is the maximum wind speed in the data set and n is the number of data points in the set.

3. Results and Discussion

In estimating wind energy potentials for Karachi, a bi-daily wind speed data at 0000 hours and 1200 hours measured at Karachi Meteorological Office for a period of 10 years (2002-2011) is used. The measured data is fitted to a Weibull distribution function, Weibull parameters were estimated using six statistical methods, i.e., MLE, MLM, MMLM, MoM, EM, and EPFM. Tables 1 and 2 give mean monthly wind speeds for Karachi at 0000 hours and 1200 hours. Table 3 lists calculated mean wind speed, the standard deviation, skewness and Kurtosis at 0000 hours and 1200 hours, respectively. Table 4 lists the statistical results of the six numerical methods of estimation of Weibull shape and scale parameters. Columns 2-5 list the calculated Weibull parameters, i.e., shape parameter k, scale parameter c, Weibull mean wind speed, Weibull energy density. Columns 6-8 list values of three statistical tests for each of the six estimation methods. Tables 5-8 give a comparative account between actual and measured wind parameters where k and c are measured using the four methods of estimation, i.e. MLM, MMLM, MoM and EM. Columns 2 and 3 list the estimated mean monthly k and c values, columns 4 and 5 list the actual or observed mean monthly velocity and standard deviation, columns 6 and 7 list the measured Weibull mean wind speed and columns 8 and 9 list wind power densities using actual wind speed and measured k and c values. Columns 2 to 9 list values of wind speed measured daily at 0000 hours and whereas columns 10 to 17 list similar measurements but wind speed measured at 1200 hours daily. Tables 9-11list the relative errors between the actual weibull parameters and Weibull parameters estimated using Monte Carlo simulation of wind speed data after repeating 100 times, each containing 100, 1000 and 10000 data points.

The analysis of actual wind speed data is show in Figure 1 which is a plot showing histogram of the actual wind speed data overlapped by calculated Weibull probability density function (pdf) each with particular shape and scale parameters estimated by using four different statistical methods. This is the reason that all six methods are not applicable for estimating the shape and scale parameters for Weibull distribution. Figures 2a - 2d show the plots histogram of the generated data having 7000 random variables. The histograms are overlapped by Weibull probability distribution function (pdf) and cumulative probability distribution function (cdf) calculated with specific shape and scale parameters for the actual wind speed data as well as for the generated data. These figures show a good agreement between the histograms, pdf and cdf of the actual time series data and of generated data where Weibull shape and scale parameters calculated using 4 statistical methods, i.e., MLM, MMLM, MoM and EM.

Table 1. Mean monthly wind speed (m/s) for Karachi at 0000 hours

Month	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
January	3.1	1.8	1.9	2.1	1.4	0.8	1.5	3.2	1.9	2.1
February	1.6	3.1	2.1	4	1.6	1.4	2.2	1.2	2.7	2.1
March	2.7	3.4	2.1	3.9	1.6	2.3	2.3	2.5	2.6	2.8
April	5.3	3.6	3.9	3.1	4.5	2.3	3.4	2.6	4	3.1
May	8	6.1	6	4.3	7.2	3.9	7.7	3.4	6.3	7.1
June	8.2	7.6	8.1	5.5	6.7	3.9	4.7	5	7.3	8.2
July	10.1	4.7	8.6	8.3	8.6	3.5	6.8	6.4	5.7	7.5
August	6.1	6.6	9.5	6.1	5.9	4.5	6.4	6.5	6	5.6
September	6.5	4.9	5.4	5.1	4	4.1	5.5	5.5	4.6	4.3
October	1.1	0.8	2.3	1.6	3	1	2.3	1.8	1.7	2.3
November	1.3	1.7	0.4	0.3	0.8	0.7	1.1	2.5	2	0.9
December	1.6	2.2	2.4	0.7	1.6	1.7	2.1	2.7	2.3	1.3

Table 2. Mean monthly wind speed (m/s) for Karachi at 1200 hours

Month	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
January	6	7.1	6.3	4.6	4.5	4	4.5	7.4	5	5.4
February	6.9	8.3	7.4	6.9	6	7.5	7.6	7.3	6.8	6.6
March	8.6	9.4	7.8	7.9	7.8	6.8	8.2	7.8	7.7	6.5
April	10.9	9.1	10.2	9.2	8.9	7.5	10.3	8.9	8.3	8
May	12.6	11.2	11.7	10.3	10.4	10.2	12.6	9.9	9.8	11.7
June	13.1	10.5	12.2	9.9	10.8	10.3	8.8	9.7	9.3	11.7
July	13.9	7.9	12.5	10.9	11.4	8.6	11	9.4	8.8	10.8
August	9.7	9.6	11.9	10.3	8.5	9.5	9.2	9.3	7.2	8.5
September	11	7.3	9.4	8.1	6.4	8	8.7	9	6.5	7.4
October	5.6	6.2	6.7	5.5	7.9	5.8	6.5	6.1	6.2	6.3
November	5.8	4.7	3.8	4.4	4	6.2	4.9	5	4.9	5.4
December	5.1	4	4.9	4.3	4.8	3.3	3.9	3.9	5	4.4

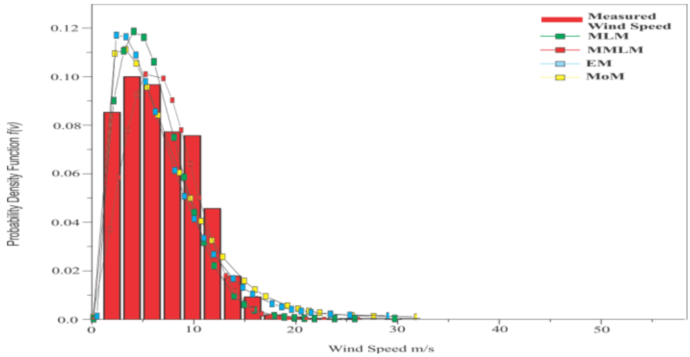

Figure-1: Comparision of actual wind speed data with Weibull pdfs each with particular shape and scale parameters estimated by using four different numerical methods.

Table 3: Mean wind speed, wind speed standard deviation, skewness and Kurtosis

Month	2002-2011 (0000 hours)				2002-2011 (1200 hours)			
	Mean	Std. Dev.	Skewness	Kurtosis	Mean	Std. Dev.	Skewness	Kurtosis
January	1.98	0.728469	0.455799	0.229847	5.48	1.172651	0.5258	-1.07313
February	2.2	0.861523	1.029544	0.765593	7.13	0.632543	0.070595	0.670146
March	2.62	0.651153	0.658484	0.798894	7.85	0.819553	0.143061	0.808678
April	3.58	0.892935	0.528395	0.200625	9.13	1.073985	0.224508	-0.70243
May	6	1.622755	-0.5119	-1.14797	11.04	1.067916	0.40184	-1.44565
June	6.52	1.615068	-0.44239	-1.50339	10.63	1.349115	0.591432	-0.38659
July	7.02	2.008205	-0.33159	-0.436	10.52	1.865952	0.335415	-0.42167
August	6.32	1.266491	1.754066	5.293663	9.37	1.233829	0.40731	1.787359
September	4.99	0.773807	0.513987	0.043908	8.18	1.406177	0.668841	0.403144
October	1.79	0.696738	0.151969	-0.70443	6.28	0.682805	1.496415	3.250809
November	1.17	0.710321	0.692111	-0.32694	4.91	0.750481	0.232283	-0.39844
December	1.86	0.594792	-0.58564	0.117639	4.36	0.589161	-0.35484	-0.79069

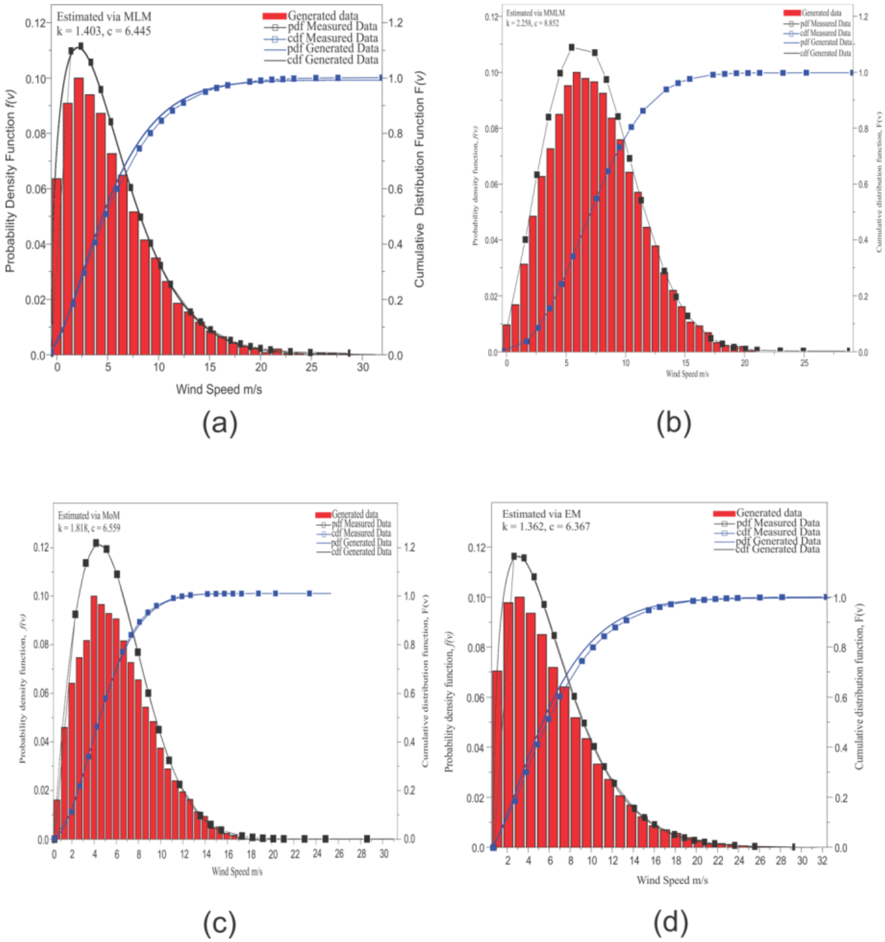

Figure 2: Plots of Weibull probability density and cumulative distribution function (a) Maximum likelihood method (b) Modified maximum likelihood method (c) Method of Moment (d) Empirical method

Table 4: Statistical Analysis

Numerical Method	Weibull Parameter				Statistical Tests		
	k	c (m/s)	v_m (m/s)	E_d W/m^2	RMSE	χ^2	R^2
MLE	0.475	1.260	2.755	1590.851	0.846	0.716	0.995
MLM	1.403	6.445	5.816	370.048	0.154	0.023	0.996
MMLM	2.258	8.852	7.763	499.228	0.347	0.120	0.996
MoM	1.818	6.559	5.773	254.253	0.323	0.104	0.996
EM	1.362	6.367	5.773	379.955	0.157	0.024	0.996
EPFM	1.366	218.263	197.828	1.521×10^7	0.153	0.023	0.992

Table 5: Comparison between actual and estimated values of wind parameters for measured mean monthly k and c parameters using maximum likelihood method (MLM)

Month	2002-2011 (0000 hours)								2002-2011 (1200 hours)							
	k	c (m/s)	υ_{obs} m/s	σ_{obs}	υ_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2	k	c (m/s)	υ_{obs} m/s	σ_{obs}	υ_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2
Jan	1.98	3.59	1.98	0.73	3.15	1.70	4.754	37.65	2.13	6.78	5.48	1.17	5.95	3.02	100.797	236.67
Feb	1.70	4.30	2.2	0.86	3.80	2.34	6.522	78.36	2.27	8.05	7.13	0.63	7.06	3.39	222.011	374.12
Mar	1.90	4.54	2.62	0.65	3.99	2.23	11.016	79.85	3.32	8.80	7.85	0.82	7.82	2.73	296.289	398.45
Apr	2.04	5.10	3.58	0.89	4.47	2.35	28.103	104.63	3.54	10.11	9.13	1.07	9.01	2.98	466.142	592.35
May	2.39	7.18	6	1.62	6.30	2.89	132.300	254.55	4.74	12.02	11.04	1.07	10.89	2.86	824.163	945.55
Jun	2.52	7.71	6.52	1.61	6.78	2.98	169.765	304.76	3.35	11.60	10.63	1.35	10.31	3.57	735.709	908.63
Jul	2.62	8.35	7.02	2.00	7.35	3.12	211.893	377.86	3.84	11.67	10.52	1.87	10.45	3.24	713.105	893.49
Aug	2.39	7.47	6.32	1.26	6.56	3.01	154.617	287.53	3.47	10.49	9.37	1.23	9.34	3.14	503.877	664.92
Sept	2.12	6.17	4.99	0.77	5.41	2.75	76.104	178.71	3.22	9.12	8.18	1.41	8.09	2.89	335.248	447.50
Oct	2.09	3.97	1.79	0.70	3.49	1.79	3.513	48.33	3.09	7.03	6.28	0.68	6.22	2.31	151.700	208.47
Nov	1.97	3.06	1.17	0.71	2.69	1.46	0.981	23.56	2.47	5.74	4.91	0.75	5.04	2.25	72.502	127.10
Dec	1.87	3.79	1.86	0.59	3.33	1.89	3.941	47.38	2.04	5.55	4.36	0.59	4.87	2.57	50.765	135.50

Table 6: Comparison between actual and estimated values of wind parameters for measured mean monthly k and c parameters using modified maximum likelihood method (MMLM)

Month	2002-2011 (0000 hours)								2002-2011 (1200 hours)							
	k	c (m/s)	υ_{obs} m/s	σ_{obs}	υ_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2	k	c (m/s)	υ_{obs} m/s	σ_{obs}	υ_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2
Jan	2.34	3.05	1.98	0.73	2.68	1.25	4.754	19.88	2.65	6.31	5.48	1.17	5.55	2.34	100.797	161.95
Feb	2.52	3.78	2.2	0.86	3.33	1.46	6.522	36.00	2.96	7.90	7.13	0.63	6.98	2.68	222.011	301.06
Mar	3.06	4.22	2.62	0.65	3.73	1.39	11.016	45.07	4.18	8.42	7.85	0.82	7.58	2.20	296.289	330.60
Apr	2.46	4.75	3.58	0.89	4.17	1.87	28.103	72.40	4.94	9.71	9.13	1.07	8.82	2.24	466.142	496.95
May	2.84	6.89	6	1.62	6.08	2.41	132.300	203.11	5.86	11.60	11.04	1.07	10.64	2.38	824.163	838.92
Jun	3.22	7.27	6.52	1.61	6.45	2.31	169.765	227.03	4.24	11.14	10.63	1.35	10.03	2.87	735.709	764.16
Jul	3.04	7.85	7.02	2.00	6.94	2.60	211.893	291.60	4.89	11.20	10.52	1.87	10.16	2.60	713.105	761.87
Aug	2.99	7.19	6.32	1.26	6.36	2.42	154.617	225.76	4.55	10.05	9.37	1.23	9.08	2.46	503.877	554.61
Sept	2.61	5.54	4.99	0.77	4.87	2.08	76.104	110.54	4.29	8.81	8.18	1.41	7.94	2.25	335.248	376.52
Oct	2.54	3.38	1.79	0.70	2.97	1.30	3.513	25.48	3.75	6.56	6.28	0.68	5.87	1.86	151.700	159.82
Nov	1.73	2.25	1.17	0.71	1.99	1.21	0.981	11.03	3.39	5.36	4.91	0.75	4.77	1.64	72.502	89.58
Dec	2.93	3.49	1.86	0.59	3.08	1.19	3.941	26.08	2.62	5.44	4.36	0.59	4.79	2.03	50.765	104.38

Table 7: Comparison between actual and estimated values of wind parameters for measured mean monthly k and c parameters using method of moments (MoM)

Month	2002-2011 (0000 hours)								2002-2011 (1200 hours)							
	k	c (m/s)	υ_{obs} m/s	σ_{obs}	υ_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2	k	c (m/s)	υ_{obs} m/s	σ_{obs}	υ_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2
Jan	1.05	2.00	1.98	0.73	1.94	1.87	4.75	24.37	0.73	4.46	5.48	1.17	5.37	7.54	100.78	1513.50
Feb	1.25	2.12	2.2	0.86	1.96	1.59	6.52	17.21	0.71	5.17	7.13	0.63	6.43	9.40	222.01	2963.38
Mar	1.04	2.67	2.62	0.65	2.60	2.51	11.02	59.13	0.54	4.53	7.85	0.82	7.77	15.58	296.29	16652.56
Apr	0.88	3.24	3.58	0.89	3.41	3.90	28.10	207.92	0.53	5.06	9.13	1.07	8.92	18.29	466.14	27514.58
May	0.69	4.37	6	1.62	5.58	8.40	132.30	2145.84	0.46	4.63	11.04	1.07	11.01	28.30	824.16	130575.28
Jun	0.69	4.67	6.52	1.61	5.94	8.92	169.77	2561.46	0.58	6.24	10.63	1.35	9.67	17.72	735.71	22684.43
Jul	0.66	4.82	7.02	2.00	6.40	10.07	211.90	3779.90	0.51	5.33	10.52	1.87	10.13	22.09	713.11	51494.00
Aug	0.66	4.59	6.32	1.26	6.14	9.72	154.62	3403.66	0.54	5.27	9.37	1.23	9.08	18.27	503.88	26955.23
Sept	0.76	4.03	4.99	0.77	4.72	6.36	76.10	900.89	0.57	4.89	8.18	1.41	7.73	14.42	335.25	12417.24
Oct	1.18	1.86	1.79	0.70	1.74	1.50	3.51	13.63	0.55	3.68	6.28	0.68	6.19	12.20	151.70	7866.16
Nov	1.50	1.18	1.17	0.71	1.05	0.72	0.98	1.97	0.66	3.54	4.91	0.75	4.70	7.38	72.50	1483.38
Dec	1.17	1.90	1.86	0.59	1.78	1.55	3.94	14.90	0.76	3.72	4.36	0.59	4.33	5.81	50.77	685.23

Table 8: Comparison between actual and estimated values of wind parameters for measured mean monthly k and c parameters using empirical method (EM)

Month	2002-2011 (0000 hours)								2002-2011 (1200 hours)							
	k	c (m/s)	v_{obs} m/s	σ_{obs}	v_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2	k	c (m/s)	v_{obs} m/s	σ_{obs}	v_m m/s	σ_m	P_{obs} W/m^2	P_m W/m^2
Jan	0.98	1.91	1.98	0.73	1.91	1.97	4.754	27.67	1.72	6.15	5.48	1.17	5.43	3.31	100.797	225.95
Feb	0.74	1.62	2.2	0.86	1.92	2.65	6.522	65.07	1.82	7.33	7.13	0.63	6.45	3.75	222.011	354.98
Mar	0.93	2.50	2.62	0.65	2.56	2.78	11.016	76.03	3.14	8.86	7.85	0.82	7.85	2.87	296.289	414.79
Apr	1.22	3.72	3.58	0.89	3.45	2.88	28.103	99.39	3.16	10.09	9.13	1.07	8.94	3.25	466.142	611.44
May	2.06	6.74	6	1.62	5.91	3.09	132.300	240.08	4.96	12.17	11.04	1.07	11.06	2.80	824.163	977.79
Jun	2.03	7.11	6.52	1.61	6.24	3.30	169.765	285.39	2.96	11.46	10.63	1.35	10.12	3.89	735.709	917.11
Jul	2.34	7.90	7.02	2.00	6.93	3.24	211.893	344.96	3.82	11.73	10.52	1.87	10.50	3.26	713.105	908.00
Aug	2.12	7.16	6.32	1.26	6.28	3.20	154.617	279.54	3.20	10.53	9.37	1.23	9.33	3.35	503.877	690.07
Sept	1.61	5.41	4.99	0.77	4.80	3.09	76.104	169.07	2.78	8.97	8.18	1.41	7.91	3.19	335.248	453.09
Oct	0.77	1.54	1.79	0.70	1.78	2.35	3.513	45.46	2.99	7.08	6.28	0.68	6.26	2.38	151.700	215.03
Nov	0.67	0.87	1.17	0.71	1.14	1.76	0.981	19.82	2.06	5.40	4.91	0.75	4.74	2.47	72.502	123.07
Dec	0.85	1.68	1.86	0.59	1.81	2.16	3.941	34.79	1.57	4.88	4.36	0.59	4.34	2.88	50.765	130.34

Table 9: Relative errors (r.e.) for Weibull parameters (k and c) between actual and simulated data, 100 data points repeated 100 times using Monte Carlo.

Actual Values		MLM		r.e. in k	r.e. in c	MMLM		r.e. in k	r.e. in c
K	c (m/s)	k	c (m/s)			k	c (m/s)		
2.24245	7.503619	2.611781	7.727896	0.1647	0.029889304	3.224495	7.874636	0.437934	0.049445
2.24245	8.503468	2.611418	8.748925	0.164538	0.028865557	3.224021	8.914375	0.437723	0.048322
2.24245	6.559257	2.612311	6.712974	0.164937	0.023435113	3.26565	6.841593	0.456287	0.043044
2.24245	6.367244	2.609937	6.591046	0.163878	0.035148813	3.2344	6.707923	0.442351	0.053505
2.611459	7.503619	2.689686	7.558001	0.029955	0.007247554	3.429514	7.686365	0.313256	0.024354
2.611459	8.503468	2.688356	8.514843	0.029446	0.001337743	3.436795	8.695123	0.316044	0.022539
2.611459	6.559257	2.694336	6.606199	0.031736	0.007156511	3.419146	6.729811	0.309286	0.026002
2.611459	6.367244	2.695967	6.416749	0.032361	0.007774896	3.408163	6.538092	0.30508	0.026832
1.818194	7.503619	2.489208	8.130331	0.369055	0.083521363	2.921081	8.211702	0.606584	0.094366
1.818194	8.503468	2.483928	9.30472	0.366151	0.094226567	2.930523	9.405939	0.611777	0.10613
1.818194	6.559257	2.482514	7.048452	0.365373	0.074580741	2.981245	7.179457	0.639674	0.094553
1.818194	6.367244	2.486759	6.855543	0.367708	0.076689113	2.97089	6.998606	0.633978	0.099158
1.3627	7.503619	2.263721	8.868932	0.661203	0.181954068	2.570201	8.945704	0.886109	0.192185
1.3627	8.503468	2.269059	10.15617	0.66512	0.194356758	2.578443	10.20206	0.892157	0.199752
1.3627	6.559257	2.269986	7.822706	0.6658	0.192620734	2.568575	7.871795	0.884916	0.200105
1.3627	6.367244	2.274899	7.612847	0.669405	0.19562655	2.566203	7.676235	0.883175	0.205582

Actual Values		MoM		r.e. in k	r.e. in c	EM		r.e. in k	r.e. c
k	c (m/s)	k	c (m/s)			k	c (m/s)		
2.24245	7.503619	1.620364	7.503919	0.277413	4.00927E-05	2.288209	7.581964	0.020406	0.010441
2.24245	8.503468	1.619	8.506588	0.278022	0.000366973	2.294098	8.596674	0.023032	0.010961
2.24245	6.559257	1.621334	6.513726	0.276981	0.006941599	2.278733	6.581885	0.01618	0.00345
2.24245	6.367244	1.62216	6.393249	0.276613	0.004084161	2.272942	6.459881	0.013598	0.014549
2.611459	7.503619	1.577751	7.528458	0.395836	0.003310354	2.659693	7.603312	0.01847	0.013286
2.611459	8.503468	1.577853	8.482419	0.395796	0.002475236	2.649904	8.568639	0.014722	0.007664
2.611459	6.559257	1.576839	6.582984	0.396185	0.003617322	2.660157	6.649511	0.018648	0.01376
2.611459	6.367244	1.576761	6.391897	0.396214	0.00387175	2.66563	6.455866	0.020743	0.013918
1.818194	7.503619	1.686672	7.587433	0.072337	0.01116981	1.877537	7.619209	0.032638	0.015405
1.818194	8.503468	1.685612	8.700245	0.072919	0.023140854	1.878525	8.740608	0.033182	0.027887
1.818194	6.559257	1.688196	6.578998	0.071499	0.003009573	1.867542	6.606052	0.027141	0.007134
1.818194	6.367244	1.687633	6.395071	0.071808	0.004370245	1.870257	6.421998	0.028635	0.008599
1.3627	7.503619	1.807054	7.770077	0.326084	0.035510673	1.407884	7.562837	0.033158	0.007892
1.3627	8.503468	1.805166	8.901002	0.324698	0.046749664	1.415668	8.667839	0.03887	0.01933
1.3627	6.559257	1.80579	6.851829	0.325156	0.044604396	1.412319	6.671845	0.036412	0.017165
1.3627	6.367244	1.805672	6.656972	0.325069	0.045502805	1.4083	6.484773	0.033463	0.018458

Table 10: Relative errors (r.e.) for Weibull parameters (k and c) between actual and simulated data, 1000 data points repeated 100 times using Monte Carlo.

Actual Values		MLM		r.e. in k	r.e. in c	MMLM		r.e. in k	r.e. in c
k	c (m/s)	k	c (m/s)			k	c (m/s)		
2.24245	7.503619	2.607247	7.740904	0.162678	0.031623	3.174985	8.03088	0.415856	0.070268
2.24245	8.503468	2.608535	8.762946	0.163252	0.030514	3.187177	9.103881	0.421293	0.070608
2.24245	6.559257	2.606571	6.758569	0.162377	0.030386	3.157941	6.977713	0.408255	0.063796
2.24245	6.367244	2.609165	6.550803	0.163533	0.028829	3.160128	6.765084	0.40923	0.062482
2.611459	7.503619	2.687926	7.540931	0.029281	0.004973	3.35096	7.771411	0.283176	0.035688
2.611459	8.503468	2.686877	8.542957	0.02888	0.004644	3.357406	8.841938	0.285644	0.039804
2.611459	6.559257	2.6912	6.583993	0.030535	0.003771	3.375537	6.795254	0.292587	0.035979
2.611459	6.367244	2.694147	6.413311	0.031664	0.007235	3.381973	6.620575	0.295051	0.039786
1.818194	7.503619	2.474849	8.113624	0.361158	0.081295	2.914011	8.426097	0.602695	0.122938
1.818194	8.503468	2.474581	9.180481	0.361011	0.079616	2.839637	9.422909	0.56179	0.108125
1.818194	6.559257	2.474452	7.087885	0.360939	0.080593	2.918214	7.398013	0.605007	0.127874
1.818194	6.367244	2.474763	6.892204	0.36111	0.082447	2.918942	7.1801	0.605407	0.127662
1.3627	7.503619	2.264812	8.978577	0.662004	0.196566	2.509127	9.299138	0.841291	0.239287
1.3627	8.503468	2.259366	10.16766	0.658007	0.195708	2.510745	10.54009	0.842478	0.239505
1.3627	6.559257	2.261178	7.794833	0.659337	0.188371	2.460395	8.042169	0.80553	0.226079
1.3627	6.367244	2.267466	7.552501	0.663951	0.186149	2.456892	7.75981	0.802959	0.218708

Actual Values		MoM		r.e. in k	r.e. in c	EM		r.e. in k	r.e. in c
k	c (m/s)	k	c (m/s)			k	c (m/s)		
2.24245	7.503619	1.620801	7.507464	0.277219	0.000513	2.264386	7.59138	0.009782	0.011696
2.24245	8.503468	1.619564	8.505782	0.27777	0.000272	2.27346	8.601184	0.013829	0.011491
2.24245	6.559257	1.622	6.549812	0.276684	0.00144	2.25532	6.622846	0.00574	0.009695
2.24245	6.367244	1.621544	6.348267	0.276887	0.00298	2.258613	6.419156	0.007208	0.008153
2.611459	7.503619	1.577894	7.504491	0.395781	0.000116	2.632006	7.584397	0.007868	0.010765
2.611459	8.503468	1.578435	8.498111	0.395574	0.00063	2.62642	8.588897	0.005729	0.010046
2.611459	6.559257	1.577652	6.551408	0.395873	0.001197	2.633975	6.621156	0.008622	0.009437
2.611459	6.367244	1.576294	6.386398	0.396393	0.003008	2.647419	6.453931	0.01377	0.013614
1.818194	7.503619	1.690219	7.548957	0.070386	0.006042	1.841764	7.584766	0.012963	0.010814
1.818194	8.503468	1.689402	8.547801	0.070835	0.005214	1.84577	8.589443	0.015166	0.010111
1.818194	6.559257	1.690982	6.59167	0.069966	0.004942	1.838226	6.622054	0.011017	0.009574
1.818194	6.367244	1.690901	6.410296	0.070011	0.006761	1.838527	6.439957	0.011183	0.01142
1.3627	7.503619	1.807044	7.836977	0.326076	0.044426	1.39533	7.63876	0.023945	0.01801
1.3627	8.503468	1.809837	8.862953	0.328126	0.042275	1.387194	8.629643	0.017975	0.014838
1.3627	6.559257	1.809619	6.794292	0.327966	0.035833	1.388063	6.616059	0.018613	0.00866
1.3627	6.367244	1.806235	6.593094	0.325483	0.035471	1.397863	6.428124	0.025804	0.009561

Table 11: Relative errors (r.e.) for Weibull parameters (k and c) between actual and simulated data, 10000 data points repeated 100 times using Monte Carlo.

Actual Values		MLM		r.e. in k	r.e. in c	MMLM		r.e. in k	r.e. in c
k	c (m/s)	k	c (m/s)			k	c (m/s)		
2.24245	7.503619	2.607559	7.733197	0.162817	0.030596	3.163209	8.051731	0.410604	0.073046
2.24245	8.503468	2.606216	8.763413	0.162218	0.030569	3.162876	9.126128	0.410456	0.073224
2.24245	6.559257	2.608633	6.761192	0.163296	0.030786	3.160727	7.03921	0.409498	0.073172
2.24245	6.367244	2.609556	6.557391	0.163708	0.029863	3.160531	6.828086	0.40941	0.072377
2.611459	7.503619	2.686937	7.539137	0.028903	0.004733	3.358902	7.83416	0.286216	0.044051
2.611459	8.503468	2.686053	8.545293	0.028564	0.004919	3.359728	8.880248	0.286533	0.044309
2.611459	6.559257	2.688752	6.592153	0.029598	0.005015	3.358547	6.849594	0.286081	0.044264

2.611459	6.367244	2.689218	6.396995	0.029776	0.004672	3.358803	6.646466	0.286179	0.043853
1.818194	7.503619	2.47443	8.109489	0.360927	0.080744	2.855737	8.434634	0.570645	0.124076
1.818194	8.503468	2.473158	9.182419	0.360228	0.079844	2.855558	9.551758	0.570546	0.123278
1.818194	6.559257	2.476066	7.089611	0.361827	0.080856	2.855833	7.37261	0.570697	0.124001
1.818194	6.367244	2.476345	6.877256	0.361981	0.080099	2.855377	7.15179	0.570447	0.123216
1.3627	7.503619	2.257541	8.963197	0.656668	0.194517	2.437924	9.231691	0.78904	0.230299
1.3627	8.503468	2.257345	10.15133	0.656524	0.193787	2.438641	10.45648	0.789566	0.229672
1.3627	6.559257	2.258127	7.831833	0.657097	0.194012	2.437938	8.064243	0.78905	0.229445
1.3627	6.367244	2.257738	7.600787	0.656813	0.193733	2.436084	7.825456	0.787689	0.229018

Actual Values		MoM		r.e. in k	r.e. in c	EM		r.e. in k	r.e. in c
k	c (m/s)	k	c (m/s)			k	c (m/s)		
2.24245	7.503619	1.620515	7.500672	0.277346	0.000393	2.265451	7.584887	0.010257	0.010831
2.24245	8.503468	1.620522	8.500671	0.277343	0.000329	2.265296	8.596144	0.010188	0.010899
2.24245	6.559257	1.621024	6.55471	0.277119	0.000693	2.261605	6.628227	0.008542	0.010515
2.24245	6.367244	1.620972	6.356754	0.277142	0.001648	2.261946	6.428073	0.008694	0.009553
2.611459	7.503619	1.57874	7.497464	0.395457	0.00082	2.622355	7.577897	0.004172	0.009899
2.611459	8.503468	1.578553	8.499585	0.395528	0.000457	2.624223	8.590679	0.004888	0.010256
2.611459	6.559257	1.578791	6.554773	0.395437	0.000684	2.621822	6.625115	0.003968	0.01004
2.611459	6.367244	1.578737	6.36083	0.395458	0.001007	2.62233	6.429078	0.004163	0.009711
1.818194	7.503619	1.689957	7.54617	0.07053	0.005671	1.842286	7.582641	0.013251	0.010531
1.818194	8.503468	1.690048	8.544944	0.07048	0.004878	1.841836	8.58612	0.013003	0.00972
1.818194	6.559257	1.68992	6.596077	0.07055	0.005613	1.842481	6.627991	0.013358	0.010479
1.818194	6.367244	1.690016	6.397864	0.070497	0.004809	1.842003	6.428722	0.013095	0.009655
1.3627	7.503619	1.810884	7.806965	0.328894	0.040427	1.383487	7.59918	0.015254	0.012735
1.3627	8.503468	1.810966	8.841407	0.328954	0.039741	1.383365	8.60575	0.015165	0.012028
1.3627	6.559257	1.811117	6.8196	0.329065	0.039691	1.382936	6.637436	0.01485	0.011919
1.3627	6.367244	1.811272	6.618481	0.329179	0.039458	1.382478	6.441339	0.014514	0.011637

For the estimation methods MLM, MMLM, MoM and EM, R^2-test gives a common value of 0.996 indicating a good fit between the measured wind speed distribution and calculated Weibull distribution (table 4). More specifically the fit explains on the average 99.6 % of the total variation in the measured data. Weibull distribution calculated using MLE and EPFM show slightly less goodness of fit values of R^2-test. RMSE for the four estimation methods, i.e. MLM, MMLM, MoM and EM give values which are less than calculated using MLE and EPFM. In case of χ^2-test often used significance level is 0.05 i.e. the null hypothesis is rejected for $\chi^2 >$ p-value. In the present case with a bin size of 30, the calculated degree of freedom is 27 and the p-value from χ^2-table is 40.113. χ^2-test for four estimation methods i.e. MLM, MMLM, MoM and EM, give values ranging from 0.023 to 0.120. Therefore χ^2(0.023 to 0.120) $<$ 40.113 the decision is not to reject the null hypothesis, i.e. there is sufficient evidence to conclude that the measured wind speed data is in good agreement with the wind speed data computed using the theoretical model. From the statistical analysis the values of RMSE, χ^2 and R^2 are consistent for the estimation methods MLM, MMLM, MoM and EM. For MLE both RMSE and χ^2-test give comparatively large values, i.e. 0.846 and 0.716, respectively, indicating a less probable fit. Although for EPFM both RMSE and χ^2-test gives small values, i.e., 0.153 and 0.023, respectively but the mean Weibull speed is much higher than monthly mean measured wind speeds. Inspection of table 5 to 8 indicate that the estimated mean monthly wind speeds are in good agreement with the actual mean monthly wind speeds.

4. Conclusions

In this paper wind speed data for Karachi is analyzed for a period of 10 years, i.e. from 2002 to 2011. Using the measured wind speed data wind energy potential for Karachi is calculated. For wind energy applications the actual measured wind speed data is modeled to Weibull function. To have the best estimate of Weibull shape and scale parameters, various statistical methods are employed. Systematic analysis includes the comparison through Monte Carlo simulation and statistical error analysis. It is concluded through simulation test that for the given measured wind speed data EM, MoM, MLM and MMLM for estimating Weibull parameters show better performance in order of suitability. The best estimates are for MLM with RMSE = 0.154, χ^2 = 0.023 and R^2 = 0.995 with calculated Weibull energy density of 370.048 W/m^2. The average value of Weibull energy density is from four methods of best estimates is c.a. 375 W/m^2 and the highest is 499.228 W/m^2 calculated using MMLM. Best values of power density, based on the estimated k and c values are obtained for MLM, MMlM, EM. In these cases the estimated power densities are comparable to the values measured using actual wind speed.

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