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Fostering the ability to solve math problems through teaching topics about three conic lines

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Abstract:

Today, Vietnam is transitioning from knowledge-centered teaching to capacity development-oriented teaching. Among the types of competencies, problem-solving is one of the most important. In particular, for the content of teaching about the three conic lines, teaching problem-solving skills proves to be very effective. Our research focuses on the perspectives of problem-solving capacity, the teaching process of fostering mathematical problem-solving capacity through teaching topics about three conic lines. Methods used in the research are investigation, observation, theoretical research, and statistical methods using SPSS software. Through the research, we found that this capacity development-oriented teaching is a potential research direction, demonstrating its positiveness in practice. In addition, the study also gives specific illustrative examples of teaching to foster the problem-solving capacity for some problems about three conic lines. From our preliminary surveys, we believe that this capacity development-oriented teaching really needs to be replicated and there are more studies to apply it in practice.

Keywords: Conic, Forstering, ability, math problems, structure.

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INTRODUCTION

Perspectives on Problem-Solving Ability

According to the 2018 General Education Program, “Creative and problem-solving capacity is an individual's ability to effectively use cognitive processes, actions, attitudes, motives and emotions to analyze, propose solutions, select solutions and implement solutions to situations, learning and practical problems where conventional processes, procedures, and solutions are not available; at the same time, the person evaluates solutions to solve problems to adjust and apply flexibly in new situations and tasks. The ability to detect and solve problems represents an individual's ability to think about problem situations, find and implement solutions to that problem when working alone or in groups. (Ministry of Education and Training, 2018)

The Program for International Student Assessment, abbreviated as PISA, defines problem-solving competence as follows: “Problem-solving competence is the ability of an individual or an organization to participate in the problem-solving process and analysis to shed light on problems for which there was no solution at the time” (OECD, 2013).

Author Agne Brilingaite and colleagues in the research paper with the topic "Competency assessment in problem-based learning projects of information technologies students" said that: "Problem-solving capacity is the capacity built on teaching activities. Learners can construct their own knowledge. Learners actively participate in the learning process, perform the tasks given by the teacher, solve real-life problems that are closely related to the knowledge being learned.” (Brilingaite et al., 2018)

In the study "Problem-Based Learning and Competency Development", two authors C.S. Chappell and P. Hager also stated their view that: “Problem-solving competence is the integration of knowledge, skills, and attitudes in order to solve a certain practical task in a particular context. Learners must analyze and express the problem in many different ways, thereby mobilizing knowledge to perform assigned tasks.” (Chappell & Hager, 1995)

According to Jean-Paul Reff and colleagues: “Problem-solving competence is the integration of thinking and acting in a particular situation. Learners cannot immediately “reach” the task they are assigned. Learners must step by step mobilize knowledge, ability, determination, and will to be able to perform the task.” (Reeff et al., 2007)

The trio of authors Wafiq Nurul Huda, Hardi Suyino, and Wiyanto mentioned the concept of problem-solving ability in the study "Analysis of Mathematical Problems Solving Abilities in Terms of Students' Motivation and Learning Styles" as follows: “Problem-solving is one of the essential competencies of mathematics. This capacity requires learners to actively participate in learning activities. The ability to solve problems helps students improve their learning efficiency and improve the skills they need for themselves.” (Huda & Suyitno, 2017)

In the research work "University students' learning styles and their ability to solve mathematical problems", the author said that: "Problem-solving capacity is the integration of knowledge, methods, ethics as well as analytical thinking, systems thinking, and creative thinking to implement an action plan that is effective and useful in practice. Problem-solving ability is a key issue that teachers need to train students to improve their thinking and develop problem-solving abilities. Math problem solving also plays an important role in becoming the goal and end result of the teaching and learning process; it is considered the right approach to thinking in general…” (Aljaberi, 2015)

In Vietnam, there have been many studies on problem-solving capacity. The doctoral thesis of Nguyen Ngoc Duy introduced the concept: “Problem solving capacity is the ability of an individual to think independently and creatively, effectively using the processes of cognition, action, attitude, motivation, and emotion to deal with learning and practical situations and problems for which conventional processes, procedures, and solutions are not available. At the same time, that individual can form and implement new ideas” (Nguyễn Ngọc Duy, 2020).

In the doctoral thesis with the topic "Teaching 8th-grade geometry exercises in middle schools for students in mountainous areas towards developing problem-solving and creative capacity", Hoang Thi Thanh said that problem-solving capacity is an individual attribute formed and developed by inherent qualities and the process of learning and training, allowing people to mobilize knowledge, skills, and attitudes to solve the problem posed when not knowing how to immediately solve the problem (Hoàng Thị Thanh, 2020).

Both Trang Quang Vinh and Khongvilay Volayuth agree with the concept of problem-solving capacity as follows: "problem-solving ability represents an individual's ability (when working alone or working in a group), to think about the problem situation, find, and implement the solution to that problem" (Trang Quang Vinh, 2020; Volayuth, 2019).

In this thesis, we use the perspective of the problem-solving capacity of Trang Quang Vinh and Khongvilay Volayuth: "Problem-solving capacity is an individual's ability that is formed from existing qualities and developed in the process of learning and training to think about a problem situation to find a solution to that problem."

Perspectives on the Ability to Solve Math Problems

In the world and in Vietnam, there are many different views on the ability to solve mathematical problems. "Mathematical problem-solving competence is the ability that students possess to solve mathematical tasks encountered. This capacity cannot be separated from activities in daily life. It helps learners give effective answers to problems that require high analysis." (Widodo et al., 2018)

In his thesis work, the author raised the point of view: "Students' problem-solving capacity in math is a combination of competencies revealed through activities in the problem-solving process." (Phan Anh Tài, 2014)

Mathematical problem-solving competence is the ability to use an organized set of mathematical knowledge, skills, and attitudes to successfully solve mathematical tasks that require methods of solving problems students do not know in advance. (Phạm Đức Tài, 2019)

According to Kuzle (2013), OECD (2004), Polya (1973), Szetela & Nicol (1992), "student's mathematical problem-solving ability can be understood as the student's ability to understand problems, plan problem-solving strategies, complete selected strategies and re-examine problem-solving, then offer alternative solutions or develop problems as students solve math problems." (quoted by Phạm Đức Tài, 2019)

From the points of view of the above authors, we also agree with the concept that mathematical problem-solving competence is a combination of abilities to successfully solve mathematical "problems" that students encounter.

Manifestations of Ability to Solve Mathematical Problems

The Mathematical General Education Program in 2018 clearly shows the manifestations of mathematical competence through:

> "- Recognizing and discovering problems that need to be solved mathematically.
> - Selecting and proposing ways and solutions to solve problems.
> - Using appropriate mathematical knowledge and skills (including tools and algorithms) to solve problems.
> - Evaluating proposed solutions and generalizing to similar problems.
>
> (Ministry of Education and Training, 2018a)

This General Education Program outlines the requirements for each level of study. However, within the scope of our study, we only consider high school students:

> "- Identify the problem situation; collect, organize, interpret and evaluate the reliability of information; share understanding with others.
> - Select and establish ways and procedures to solve problems.
> - Implement and present the solution to the problem.
> - Evaluate the implemented solution; reflect the value of the solution; generalizable to the same problems."
>
> (Ministry of Education and Training, 2018a)

The Structure of Mathematical Problem-Solving Competence

In the world, there have been many studies on the structure of problem-solving capacity. Mogen Niss, who is considered a leading expert in capacity development-oriented teaching, considers problem-solving capacity to include the following components. The first is to recognize the problem, pure or applied math problem. This is the first element of mathematical problem-solving capacity. The second is to propose and identify pure or applied mathematical problems. This is the element that requires higher thinking than the first element. The third is to solve different types of math problems. Learners must be able to apply theoretical knowledge to real life. Finally, they know how to propose a new problem or look at the problem in a different way. Learners develop closed-ended problems into open-ended problems. (Niss, 2003)

According to M. Wu, the structure of problem-solving competence includes the following elements. The first is reading comprehension. This is a fundamental skill that holds true not only for problem-solving abilities but also for other types of mathematical abilities. The second is mathematical reasoning skills. Learners must know how to apply the knowledge they have to deduce to find the crux of the problem. The third is computational skills. This is an important and characteristic skill of mathematical competence, not just problem-solving. Finally, it is a skill to apply theoretical knowledge to real life. This skill helps learners expand their knowledge, see math problems as "live" and real, not just academia. (Nguyễn Thị Ngọc Thẩm, 2021)

In Vietnam, there are also many research authors on mathematical problem-solving ability. Nguyen Thi Lan Phuong believes that the structure of problem-solving capacity includes the following components. The first is to find out the problem; The second is the problem space setting; The third is planning and implementing the solution; Finally, it is about evaluating and reflecting on the solution. (Nguyễn Thị Lan Phương, 2014) Ha Xuan Thanh, in his doctoral thesis, said that problem-solving capacity includes the following components. First, the ability to understand the problem, get information from the actual situation; Second, the skill of converting information from real situations to mathematical models; Third, the ability to find strategies to solve mathematical models; Fourth, the ability to implement strategies to find results; The fifth is the ability to switch from the results of solving mathematical models to the solutions of problems containing practical situations; Finally, the ability to come up with other problems. (Hà Xuân Thành, 2017)

From the factors constituting the capacity to detect and solve the above problems, we draw the structure of mathematical competence including the following components. The first component is reading and understanding the problem, that is, finding information related to the problem, identifying the problem to be solved. The second element is problem-solving, i.e. performing exact computational steps to find the result. The third element is problem critique, which is used to evaluate solutions. The fourth element is to expand knowledge, find multiple solutions, that is, generalize the problem to obtain new problems, search for other solutions, find similar problems as well as the "reverse" problem of the given problem.

LITERATURE REVIEW

The Teaching Process Is Oriented to Develop Problem-Solving Capacity for the Problems of Circles and Three Conic Lines

Step 1: Identify the problem to be solved
For the problems of circles and three conic lines, it is necessary to determine the relationship between the data, the condition, and the "output" of the problem. Are the conditions and data redundant or lacking? Are these data contradictory?

Step 2: Solve the problem
From the data of the problem, we need to choose and derive the equation of the circle and three conic lines according to the learned formula. Use knowledge of the equation of the circle and three conic lines to solve the problem posed.

Step 3: Evaluate and deepen the problem
We need to evaluate the pros and cons of the solution we use to solve the problem. We extend the problem by methods of generalization, analogy, and specialization to obtain new problems. Then we solve this new problem.

The teaching process to develop problem-solving capacity will be specifically applied to the measures developed in chapter 2, as well as in the experimental part in chapter 3.

The Role of Math Problem-Solving Ability

The role of problem-solving capacity has been mentioned by many authors around the world in their studies. For example, "Problem-solving is an essential competency to be possessed and improved upon by high school students. Mathematical problem-solving ability is a fundamental competency in learning mathematics, it helps individuals develop analytical thinking, helps students to be critical and creative, and at the same time to improve other mathematical competencies. " (Hendriana et al., 2018).

Cockcroft (1970) put his point of view on the role of problem-solving skills in a nutshell: "Problem-solving is central to mathematics. Mathematics is only useful to the extent that it can be applied to real-life situations." (quoted by Ernest, 1988)

Many students are not motivated to study math. Students do not see that mathematics is closely related to life. By giving them confidence and problem-solving abilities, students will see this close connection. From there, students will be interested in math. Teaching to develop problem-solving abilities is the best way to prepare students to cope with difficulties in mathematics in particular and in life in general. In addition, teaching to develop problem-solving abilities improves logical reasoning, which many consider important. Through teaching to develop problem-solving abilities, students are more creative and productive. (Ernest, 1988)

Problem-solving skills play an important role in the development of general mathematical competence. Thus, it plays a prominent role in mathematics education. Problem-solving enables students to adapt to intellectual challenges to improve the effectiveness of math learning. In addition, problem-solving ability also helps students connect mathematical knowledge with daily life (Novita, 2012).

This ability to solve mathematical problems should not be overlooked because this capacity to solve problems is one of the core competencies of science and technology. It is very important for education in particular and life in general (Balım, 2009).

Pinter says that problem-solving is very important. With this problem-solving competence, students will be able to solve real-life situations in mathematical models (Pintér, 2012).

Mathematical problem-solving ability is a mathematical ability that in itself is not only a goal in learning mathematics but also something very meaningful in daily life, at work. People who have the ability to solve math problems will be helpful in solving math problems. In addition, strengthening and fostering math problem-solving abilities helps students to improve their learning outcomes and improve the quality of math education (Simamora et al., 2017).

Mathematical problem-solving ability is one of the mathematical competencies that students need to have in learning math. However, students feel that math is something very scary, especially problem-solving. In fact, solving math problems has become a potential field in math teaching (Widada et al., 2019).

The formation and development of the ability to solve mathematical problems help students understand and master the basic content of the lesson. Students can expand and improve their mathematical and social knowledge. The formation and development of mathematical problem-solving ability help students to apply mathematical and social knowledge in real life. This formation and development of mathematical problem-solving ability help students to form communication skills, organization, thinking ability, cooperative attitude, and community integration (Nguyễn Thị Hồng Luyến, 2016). Thanks to the ability to solve math problems, an individual can understand and solve mathematical situations that have problems or mathematical situations that do not have a solution (Trần Doãn Vinh, 2017).

From the roles of mathematical problem-solving capacity above, we also agree with the view that: Mathematical problem-solving capacity plays a very important role in the process of learning mathematics. It helps students gain confidence in their thinking and reasoning abilities, and the ability to apply mathematical knowledge in real life. It supports the development of other core math competencies.

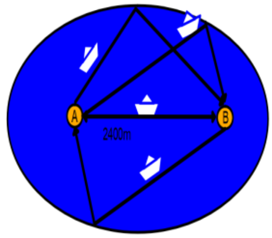
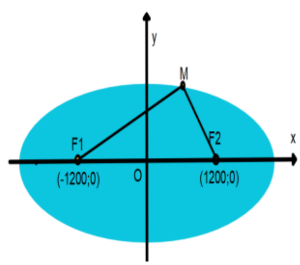
METHODOLOGY

Example 1

A tourist boat moves between two islands at positions A and B. The two islands are 2400m apart. The boat only holds enough fuel to travel 4000m for trips between islands. Write an equation that describes the limit line for the boat to move safely between the two islands. (Dương Đức Cường, 2018)

Table 1. Example on Conic problem teaching

Teacher	Students
Step 1: Identify the problem to be solved	
What important facts does the topic already provide?	Fact 1: The distance between the two islands is 2400m. Fact 2: The boat can go up to 4000m per trip.

Draw a picture depicting how the ship moves from island A to island B and vice versa?	 **Figure 1. Illustration of the Ship's Path (Source: Personal Collection)**
What is the problem to be solved?	The ship has many ways to move from island A to island B and vice versa. However, the ship can only go up to 4000m per trip while the distance between the two islands is 2400m which means that from A (or from B) the ship can move to a certain position M and then go to B (or A) such that the AM+MB distance is not more than 4000m. Therefore, the limit line of the ship's movement is an ellipse with the focal point being the position of islands A and B. Write the equation of the ellipse.
Step 2: Solve the problem	
What factors are needed to write an ellipse? What do islands A and B at the two focal points indicate? What does a boat traveling up to 4000m indicate? Choose the position of the Oxy coordinate axis, draw an illustration?	We need to know a and b. We find c because $F_1F_2 = 2c$ We find a because: $MF_1 + MF_2 = 2a$  **Figure 2. Ellipse** **(Source: Personal collection)**
Present the solution to the above problem.	Choose the Oxy axis system so that the midpoint of the two islands coincides with the origin O(0,0), with island A being the focal point F_1, B the focal point F_2. Equation (E) has the form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b > 0)$ We have:

<table>
<tr><td>Comments and adjustments (if any)</td><td>$F_1F_2 = 2c = 2400 \Rightarrow c = 1200$
$MF_1 + MF_2 = 2a = 4000 \Rightarrow a = 2000$
and
$b^2 = a^2 - c^2 = 2000^2 - 1200^2 = 2560000$
$\Rightarrow b = 1600$

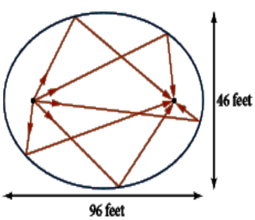
We infer:
$(E): \frac{x^2}{2000^2} + \frac{y^2}{1600^2} = 1$
So the train can only move within the range limited by a path
$(E): \frac{x^2}{2000^2} + \frac{y^2}{1600^2} = 1$</td></tr>
<tr><td colspan="2">Step 3: Evaluate and deepen the problem</td></tr>
<tr><td>Example 2
The Statuary Hall at the Capitol in Washington, D.C. is a whispering hall. The width of the hall is 46 feet, 96 feet long as shown in the figure. (Source: Nguyen Ngoc Giang (2020))


Figure 3. Hall
(Source: Nguyen Ngoc Giang (2020))

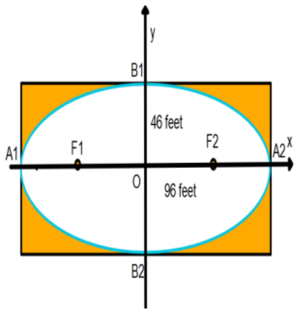
a) Write the equation of the canonical form of the ellipse representing the outline of the hall?
b) If two senators standing at two focal points of the hall could hear each other's whispers, how far apart should the senators stand?
Set the coordinate axis Oxy and draw an illustration</td><td>Fact 1: Width 46 feet
Fact 2: Length 96 feet
Illustrations:


Figure 4. Illustrations
(Source: Personal collection)

a) Equation (E)?
b) What is the distance between the two MPs?</td></tr>
<tr><td>Through example 2, how do we know to find (E)?

Analyze the requirements of example 3.

Analyze the relationship between example 2 and example 3.</td><td>In example 2, we already know how to find (E) by finding a, b. In which a, c are found by the formula $MF_1 + MF_2 = 2a$ and $F_1F_2 = 2c$, then b is found by the formula $b^2 = a^2 - c^2$.
Example 3:
a) Write equation (E) i.e. find a, b?
b) Find the distance of the two MPs i.e. find the length F_1F_2.
The requirement in question a) of example 3 is the same as that of example 2, but the way to find a and b is not the same because the two examples have given those two factors through</td></tr>
</table>

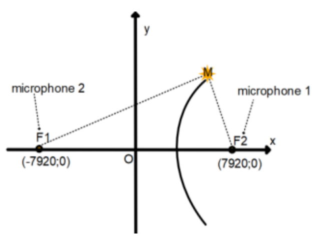
	the length and width of the base rectangle. The requirement in question b) of example 3 is the reverse of example 2, i.e. given c (found from a)), find F_1F_2 .
Present the solution to the problem Comments and adjustments (if any).	The elliptical hall is surrounded by a rectangle that is 96 feet long and 46 feet wide. Set the coordinate system Oxy as shown. a) We have: $a = 48, b = 23$ So $(E): \frac{x^2}{48^2} + \frac{y^2}{23^2} = 1$ b) Two congressmen whisper to each other, if they want to hear clearly, both must stand in two positions as the focus of (E). So the distance between the two MPs is the distance of F_1F_2 . We have: $c^2 = a^2 - b^2 = 48^2 - 23^2 = 1775$ $\Rightarrow c = 5\sqrt{71}$ We infer: $F_1F_2 = 2c = 10\sqrt{71}$ So the distance between the two congressmen is approximately 84.3 feet.

Through the above example, students have developed skills in collecting, interpreting and analyzing data. Students are trained in their ability to consider problems from different perspectives, evaluate solutions, and increase their confidence when faced with complex situations in learning as well as in practice. From there, it contributes to the development of students' ability to solve mathematical problems.

Example 4

Hyperbola can be used to model and solve many types of problems in life, especially for determining the location of explosions. Solve the problem: An explosion is recorded by two microphones placed 3 miles apart. The first microphone received the explosion sound 6 seconds before the second microphone. Assume that sound travels at 1100 feet/second. Please determine the location where the explosion occurred? (1 mile = 5280feet) (quoted from ((Thomas W. Hungerford, 2008))

Table 1. Hyperbola teaching

Teacher	Students
Step 1: Identify the problem to be solved	
What important facts does the topic already provide? What is the requirement of the problem? Interpret the data obtained. Sequence related facts about the hyperbolic equation to choose the appropriate Oxy axis and describe the data graphically. Withdraw the problem to be solved.	Fact 1: Two microphones are placed 3 miles apart Fact 2: The first microphone receives sound 6 seconds before the second microphone. Fact 3: The speed of sound is 1100feet/second and 1 mile = 5280feet. Determine the location of the explosion. Locating the explosion would require the use of a hyperbola. Two microphones are fixed in two positions ie consider the position of each microphone as a focal point. The distance from the explosion to the first microphone is 6600 feet closer than the second microphone. Convert: 3 miles = 15840feet  **Figure 5. Explosion** **(Source: Personal collection)** Write down the hyperbolic equation.
Step 2: Solve the problem	
What additional facts can we get from the figure? Choose a solution. What is the conclusion of the problem?	Choose the coordinate system Oxy whose origin is the midpoint of the two microphones. Place the first microphone at focus F_2. Second microphone at focus F_1. $F_1F_2 = 15840\ feet = 2c$ Let M be the location of the explosion. We have: $MF_2 - MF_1 = 6600\ feet = 2a$ We have the equation of the hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > 0, b > 0)$ Find a, b. We have : $2a = 6600 \Rightarrow a = 3300$ $2c = 15840 \Rightarrow c = 7920$ Apply the formula: $b^2 = c^2 - a^2$

Is this conclusion satisfactory when the hyperbola has two left and right branches while the explosion site has only one?	We deduce $b^2 = (7920)^2 - (3300)^2 = 51836400$ The hyperbolic equation is: $\frac{x^2}{10890000} - \frac{y^2}{51836400} = 1$ (*) The explosion lies on the hyperbola whose equation is (*). The explosion is on the right branch of the hyperbola whose equation is (*), because we initially placed the first microphone at the focal position F_2.
Step 3: Evaluate and deepen the problem	
State the inverse problem of the above problem	An explosion is recorded by two microphones placed far apart. The explosion is located according to the equation of a hyperbola (H): $\frac{x^2}{10890000} - \frac{y^2}{51836400} = 1$. How far apart are the two microphones and determine the position of the two microphones.
Present the solution	The solution We have (H): $\frac{x^2}{10890000} - \frac{y^2}{51836400} = 1$ So $a^2 = 10890000$ $b^2 = 51836400$ We deduce $c^2 = a^2 + b^2 = 62726400$ We deduce $c = 7920$ Conclusion: Two microphones are placed at the 2 focal positions of the hyperbola, so they will be spaced $2c = 15840$ (measurement unit) apart and microphones 1, 2 are located at (15840;0), (-15840;0) respectively or vice versa.
From the results of the inverse problem, comment on the accuracy of the solution.	Based on the results of the inverse problem, the solution given to solve the original problem is correct.

In the example above, at a low level of application, students use the knowledge and skills they have learned to solve problems in life. Specifically, students are able to collect and interpret given data, organize it, and use it appropriately. Students use their imagination to realize that it takes more than 2 pairs of microphones to pinpoint the exact location of an explosion to create the intersection of two parabolas. Moreover, students are also trained in flexible thinking ability to turn from a stated problem into an inverse problem and come up with a solution by themselves. This shows that, through this example, students are trained to think positively, analyze situations, and apply knowledge, leading to the ability to successfully solve math problems.

Example 5

A solar hot dog grill is made of cardboard lined with aluminum foil and shaped like a parabolic trough. The grill is 4 inches deep and the mouth of the grill is 12 inches wide. The grill is depicted as shown in figure 6. In what position should two holes be made at both ends of the parabolic trough for the best skewers? (Applications, 2009)

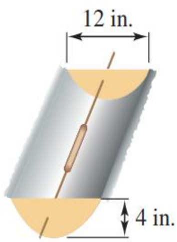


Figure 6. Description of the energy grill (Applications, 2009)

Table 1. Energy Grill

Teacher	Students
Step 1: Identify the problem to be solved	
What important facts does the topic already provide?	Fact 1: The grill has a parabolic shape. Fact 2: The depth of the grill is 4 inches, the mouth width is 12 inches.
Analyze and interpret data.	Sunlight will shine on the concave surface of the grill and reflect through the parabolic focus, so to get the best solar energy, we must make two holes at the focal point of the parabola at the ends of the trough.
What is the problem to be solved?	Find the hole punching position at both ends of the trough. That means determining the focal point of the parabola at the two ends of the trough.
Step 2: Solve the problem	
Have you seen this topic before?	Not yet.
Let's put the parabola in the Oxy coordinate system along with the given data.	The grill has a depth of 4 inches and a mouth width of 12 inches. The concave side of the grill faces up, so we will choose to describe the concave surface of the upward parabola and choose the vertex of the parabola to coincide with the origin O.

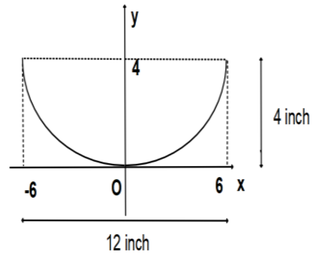
<table>
<tr><td>From the drawing, can we find any more elements?</td><td>

Figure 7. Grill
(Source: Personal collection)

Two points on the parabola have been found, A(6;4) and B(-6,4).</td></tr>
<tr><td>Please provide the solution

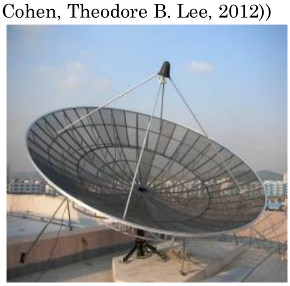
Conclusion of the problem</td><td>Select the system of axes Oxy so that the parabolic vertex coincides with the origin O(0,0), the concave surface is upward as shown in the figure.
The parabolic equation has the form: $x^2 = 2py$
Point A(6,4) and point B(-6,4) belong to the parabola.
We have: A(6;4) belongs to the parabola
We infer: $p = \frac{9}{2}$
We infer: the focal point of the parabola has coordinates $(0;\frac{9}{4})$
We will punch 2 holes at the ends of the trough at a position 2.25 inches from the top of the parabola.</td></tr>
<tr><td colspan="2">Step 3: Evaluate and deepen the problem</td></tr>
<tr><td>Example 6
The pan parabolic antenna (as shown in Figure 8) has a diameter of 4.9 m and the radiator is placed at the focal point at a distance of 1960 mm from the basin. Find the depth of the antenna basin. (quoted by (David Cohen, Theodore B. Lee, 2012))


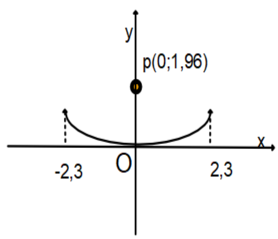
Figure 8. Antten parabol (thietbitruyenhinh.tv, 2021)</td><td>Read and analyze the math problem
Fact 1: Diameter 4.9m
Fact 2: $\frac{p}{2} = 1,96m$
Finding the depth of the basin means finding y.


Figure 9. Antenna description in the coordinate plane (Source: Personal collection)</td></tr>
</table>

What data has the topic given? Draw a description based on the data found.	
Through example 5, how do we find p? Find the similarity of example 5 and example 6. Can the method in example 5 be applied to example 6?	In example 6, we learned how to find p by substituting the coordinates of a parabolic point into an equation of the form $x^2 = 2py$ (parabolic concave upward, p>0). Example 7 We see that the concave surface of the parabola is pointing up, so the parabolic equation also has the form: $x^2 = 2py$ (p>0). Find the y -coordinate of a point on the parabola when we know the x -coordinate and p. To find the y -coordinate, we also put x and p into the equation $x^2 = 2py$.
Present the solution	Select the system of axes Oxy so that the parabolic vertex coincides with the origin O(0,0), the concave surface is upward as shown in the figure. Parabolic equation: $x^2 = 2py$ (p>0) (*) Let A (2,45;y) belong to the parabola (1) We have: p = 3.92 m (2) From (1) and (2), we put them in (*), we get y = 0.765625 We infer $A(2,45;0,765625)$, So the depth of the basin is: 0.765625m

To solve the problem in the above case, students must collect, analyze and interpret the given data to understand how the homemade solar cooker works, thereby detecting the problem to be solved. Next, students find a solution to determine the position of the focal point by simulating the parabola and attaching the coordinate system and identifying the parabolic equation to use. Finally, they find a solution for a similar problem based on the method of the original problem. Through those activities, students are trained in their ability to find solutions, the ability to apply learned knowledge to real life, and the ability to recognize similar problems. These will help improve problem-solving capacity.

RESULTS

Time and Objects

The experiment was conducted at Binh Chanh High School (Binh Chanh District, Ho Chi Minh City) for the 2020-2021 school year.

Experimental class 10A5 includes 41 students. Math teacher: Le Thi Thanh Phuong.

The control class 10A9 consists of 41 students. Math Teacher: Le Thi Thanh Phuong.

We designed a lesson plan for lessons 1, 2 on "Practice on the topic of circles" and the third lesson "Practice on elliptical topics", the fourth lesson on "Practice on topics of hyperbola". For the experimental class, the teacher will teach in the direction of fostering the ability to solve mathematical problems. For the control class, the teacher will follow the lesson plan according to the current program distribution.

Experimental Process

Find out the learning situation of students in the experimental class and the control class to have a preliminary assessment of the absorptive ability of the two classes.

Compile experimental lesson plans according to selected topics.

Teachers prepare necessary materials and conduct teaching according to the compiled lesson plan.

Always observe to monitor the learning attitude, test-taking skills, ability to absorb knowledge of both classes.

After completing the experimental lessons, the teacher will organize a test for the experimental class and the control class.

Collect, analyze and evaluate the test results of the experimental class and the control class by using SPSS software.

Evaluation Methods

Observe the class

In the process of conducting lessons, consider the level as well as the ability to interact between students with teachers, students with students to be able to assess a positive learning attitude, interest, and ability to get knowledge. In addition, observing students' notebooks also tells a bit about the learning process and attitude.

Interview

Student Interview: We conducted a short conversation to be able to more accurately assess the students' interest and ability to absorb and apply knowledge about the topics. From there, the effectiveness of the measure can be seen.

Teacher interview: We also had an interview with the classroom teacher to get objective assessments and comments about the students' interest and cognitive ability in the experiment.

Essay test

To assess students' ability to acquire knowledge through lessons on each topic, there is a knowledge test for each student of the experimental class and the control class. The content of the test will be based on the goals and requirements of the lessons combined with exercises to evaluate the effectiveness of the student's ability to solve math problems. The test is graded on a 10-point scale.

Mathematical statistics

To ensure accuracy and science, we use more mathematical statistical methods. The purpose is to calculate the mean, variance, and standard deviation to understand the meaning of the collected data. We choose the statistical software SPSS. This software is often used in economic statistics, statistics in education because of its high reliability, fast data processing time, and almost all features to meet all user needs.

We performed a three-pronged assessment. The first is the average score of the experimental class and the control class in the first semester. The second is the midterm test score of the second semester in math. This is also the latest test. The third is post-empirical test scores on the circular and three-conic topics. The evaluations are shown through the following test tables:

i) Score frequency table of the experimental class and the control class.

ii) Score chart of the experimental class and the control class.

iii) Table of characteristic values including mean, variance, standard deviation,...

iv) Table of independent tests T-test in the type of Independent Samples T-Test tests whether there is a statistically significant difference between the scores of the experimental class and the control class.

Analysis of Pedagogical Experiments

Pre-experiment analysis

Qualitative: through the investigation of the experimental class and the control class between us, the math teacher and the homeroom teacher, both teachers thought that the math performance of the two classes was similar.

Quantitative: We use SPSS software to process the data obtained as the result of the average score in math in the first semester of the experimental class and the control class.

We evaluate the average score in math in the first semester of the experimental class and the control class.

Table 1. Frequency Distribution Table of Average Scores in Mathematics in the First Semester of Two Classes (Source: Personal Collection)

SCORES* Class Crosstabulation				
Count				
		Class		
		Control	Experimental	Total
SCORES	4,6	1	1	2
	4,7	1	3	4
	4,8	3	2	5
	4,9	1	0	1
	5,0	1	0	1
	5,5	0	1	1
	5,6	0	1	1
	5,7	1	0	1
	5,8	1	1	2
	5,9	2	2	4
	6,0	0	1	1
	6,1	0	3	3
	6,2	0	1	1
	6,3	3	0	3
	6,4	0	2	2
	6,5	2	0	2
	6,6	2	0	2
	6,8	0	2	2
	6,9	1	1	2
	7,0	1	1	2
	7,1	0	1	1
	7,2	0	2	2
	7,3	0	2	2
	7,4	2	0	2
	7,5	1	0	1
	7,6	1	0	1
	7,7	1	0	1
	7,8	1	0	1
	7,9	1	0	1
	8,0	3	1	4
	8,1	2	0	2
	8,2	0	1	1
	8,4	1	0	1
	8,5	1	2	3

	8,6	1	1	2
	8,8	0	2	2
	8,9	1	1	2
	9,0	1	0	1
	9,1	1	2	3
	9,2	1	1	2
	9,3	0	2	2
	9,5	1	1	2
	9,6	1	0	1
Total		41	41	82

Next, we proceed to draw the column chart from table 1 above and obtain the characteristic parameters of the statistics.

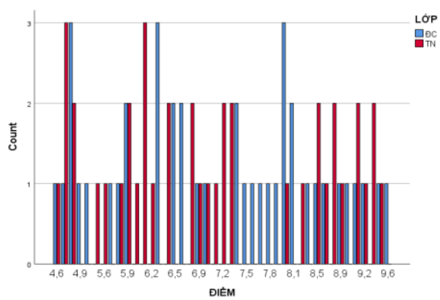


Chart 1. Column chart comparing the average score of mathematics in the first semester of the experimental class and the control class (Source: Personal collection)

From the image of column chart 1, we observe that the number of students in the score range from 4.5 to 6.5 of both classes is similar. The number of students in the score range from 6.9 to 7.2 in the experimental class is more than that of the control class. Meanwhile, the control class has many students in the score range of 7.3 to 7.9. In the range of 8.1 to 9.6, the number of students in the experimental class is higher. In general, the number of students achieving four levels of the excellent, good, pass, and fail in the experimental class and the control class is similar.

Table 2. Table of Typical Statistical Parameters of the Average Scores of Math in the First Semester of the Experimental Class and the Control Class (Source: Personal Collection)

Descriptive Statistics							
	N	Minimum	Maximum	Mean		Std. Deviation	Variance
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic
TN	41	4,6	9,5	7,017	,2402	1,5378	2,365
DC	41	4,6	9,6	7,078	,2297	1,4706	2,163
Valid N (listwise)	41						

From Table 2 we see that:

Average score: the average score of the experimental class is 7,017. Meanwhile, the mean score of the control class is 7,078. That shows that the score of the control class is slightly higher than that of the experimental class.

Standard deviation and variance: the standard deviation and variance of the experimental class are larger than the control class. That shows that the math ability of students in the experimental class is relatively higher than that of the control class.

To assess more accurately whether or not there is a difference in academic performance of the two experimental and control classes, we conduct a T-Test with the hypothesis H_0: "The average score in mathematics in the first semester of the experimental class and the control class is equivalent" with significance level $\alpha = 0,05$. The results are in the following table:

Table 3. The Average T-Test of the Average Score of Math in the First Semester of the Experimental Class and the Control Class (Source: Personal collection)

Independent Samples Test										
	Levene's Test for Equality of Variances			t-test for Equality of Means						
	F		Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
ĐIỂM	Equal variances assumed	068	,795	,183	80	855	-,0610	,3323	-,7223	,6003
	Equal variances not assumed			- ,183	79,841	855	,0610	,3323	,7223	,6004

The Levene test has a value of $Sig. = 0,795 > \alpha = 0,05$, so the variance of the experimental class and the control class has no significant difference, so it is considered equivalent, using the results of Independent-samples T-test for the case where the equal variances of the two classes are assumed.

Independent-samples T-test, $Sig.(2-tailed)=0,855>\alpha=0,05$, so we accept the hypothesis H_0, the average score of math in the first semester of the experimental class and the control class is equivalent.

Quantitative assessment

To see how feasible and effective the pedagogical measures proposed in Chapter II are, we analyze and evaluate post-empirical tests. These tests were administered by subject teachers after each experimental lesson. Because of the limitation of the study, we would like to evaluate the test of the 4th experimental period.

Table 4. Frequency Table of Experimental Math Test Scores of the Two Classes (Source: Personal collection)

SCORES * CLASS Crosstabulation				
Count				
		CLASS		Total
		Control	Experimental	
SCORES	4,0	1	0	1
	4,3	1	0	1
	4,5	2	0	2
	4,8	1	2	3
	5,0	2	1	3
	5,3	2	0	2
	5,5	1	1	2
	5,8	2	2	4
	6,0	4	1	5
	6,3	3	3	6
	6,5	4	3	7
	6,8	4	3	7
	7,0	1	2	3
	7,3	2	1	3
	7,5	2	0	2
	7,8	0	4	4
	8,0	2	3	5
	8,3	2	3	5
	8,5	1	0	1
	8,8	0	3	3
	9,0	2	3	5
	9,3	1	2	3
	9,5	0	2	2
	9,8	1	0	1
	10,0	0	2	2
Total		41	41	82

From Table 4, we draw a column chart and obtain the characteristic parameters of statistics as follows:

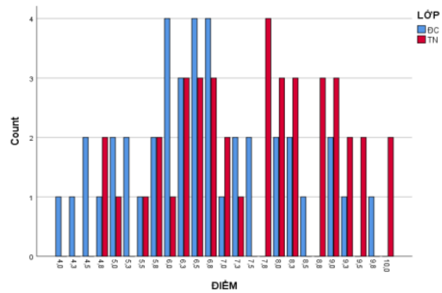


Chart 2. Column chart comparing math test scores of the experimental class and the control class. (Source: Personal collection)

Observing the column chart, we notice that the scores of the experimental class are no longer distributed much in the range of scores from 4.0 to 4.8 compared to the control class. The score range from 7.8 to 10 of the experimental class is also relatively higher than that of the control class. From those things, we can see the effective signal of the proposed pedagogical measure. In order to have more convincing parameters about the experimental results, we tabulate the characteristic parameters of statistics.

Table 5. Table of Typical Parameters of Statistics on Math Test Scores of the Experimental Class and the Control Class (Source: Personal collection)

Descriptive Statistics						
	N	Minimum	Maximum	Mean	Std. Deviation	Variance
TN	41	4,8	10,0	7,534	1,4489	2,099
DC	41	4,0	9,8	6,602	1,4329	2,053
Valid N(listwise)	41					

Average score: the average score of the experimental class is 7,534. Meanwhile, the mean score of the control class is 6.602. That shows that the mean score of the experimental class is higher than that of the control class.

Standard deviation and variance: the standard deviation and variance of the experimental class are still higher than that of the control class, but not significantly. Furthermore, the experimental class has variance and standard deviation that in this test are closer to the mean score than in the second-semester midterm exam. That shows that in the experimental class, the difference in math ability is closer in a positive direction.

In order to more accurately assess the equivalence of math ability of the two experimental and control classes, we conduct a T-Test with the hypothesis H_0: “The average score of the experimental class and the control class in math is equivalent" and H_1: "The average score in math of the experimental class is higher than that of the control class" with significance level $\alpha = 0,05$. The results are in the following table:

Table 6. The Average T-Test Table of the Math Test Scores of the Experimental Class and the Control Class (Source: Personal collection)

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
SCORES	Equal variances assumed	367	546	,928	0	,004	9317	,3182	2984	1,5650
	Equal variances not assumed			,928	9,990	,004	,9317	,3182	2984	1,5650

We make the following comments from the test table:

The Levene test has a value of $Sig. = 0.546 > 0.05$, so the Independent Sample T-Test results should be used for the case where the equal variances of the two samples are assumed.

Independent Sample T-Test: The test scores of the two classes give $p = 0.004 < 0.05$, so we reject the H_0 hypothesis and accept the H_1 hypothesis. So the average score of the math test of the experimental class is higher than that of the control class.

From the initial qualitative test, the math performance of the experimental class and the control class is similar. However, after the experimental teaching process, it initially shows that the results of the experimental class are better than that of the control class.

DISCUSSION

From the experiment, we draw the following. First, the experimental class and the control class have similar math abilities. This result was identified through a qualitative and quantitative mass survey using the SPSS statistical software when comparing the results of the first semester and the most recent test before the experiment. Second, the experimental class has more positive results than the control class. Quantitative analysis by the mathematical statistics of the tests after the experimental periods shows that the average score of the experimental class in math is higher than that of the control class; Qualitative analysis by observing learning attitudes and behaviors in experimental classes also shows that students in the experimental class were more active, more excited and better at interacting than students in the control class. Third, the students in the experimental analysis have markedly improved their results through the post-experiment tests, and their attitudes, behaviors, and expressions are more positive than before. These conclusions prove that the teaching method is effective and has achieved the goal of fostering students' problem-solving capacity through three-conic problems.

CONCLUSIONS

Teaching in the direction of developing problem-solving capacity for students through problems about three conic lines is a teaching method that proves to be more effective than traditional teaching. Students are more interested in math class. They not only find a way to solve a problem, but also actively search for other solutions, initially evaluate the advantages and disadvantages of the solutions, and find problems for similar problems. In addition, students show confidence and do not make mistakes in solving math problems. The test results of the experimental class are higher than that of the control class. That shows that the measures we propose are feasible and effective in fostering mathematical problem-solving capacity. Besides the obtained results, we also see some limitations of the experimental process such as we only experimented with one class in a high school; Not fully exploiting the features of the statistical software SPSS. The above limitations have reduced the effectiveness of the study because it has only been tested on a small scale, and the evaluation tool is still quite simple.

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