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Change in Space-time Geometry and the Tests of General Relativity

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Abstract:

This literature review casts light on tests of general theory of relativity considering non-Euclidian geometry of spacetime, and the resulting effects. Expanding special theory of relativity, Albert Einstein in 1915 published his geometric theory of gravitation, named general theory of relativity [GR]. GR is the absolute genius of Einstein providing the current profile of gravitation as the geometric property of spacetime continuum. The theory describes gravity as the effect of curvature of spacetime. This curvature—geometrical distortion in the hypothetical fabric of spacetime is associated with energy and momentum in the presence of matter or radiation. For verification, Albert Einstein predicted the following three ways to put the theory on test; predicting the value for perihelion advance of Mercury, deflection of light by the sun, the gravitational redshift of light. The validity of theory by such proofs continued to series of observations like shapiro time delay or gravitational time delay, detection of gravitational waves [GW], and direct observation of a blackhole.

Keywords: general relativity, spacetime curvature, non-Euclidean geometry, time delay, redshift

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INTRODUCTION

Nature's laws under Galilean relativity were considered absolute, until Einstein analyzed Galileo's principle of inertia (Simon & Schuster, 2007). This principle, in which Newtonian laws hold, refers to an absolute space and absolute time. Einstein' special theory of relativity on the other hand introduces non-absolute length, mass and time in perspective of speed. Special theory in the absence of gravity, defines how experience of time for an observer in a rest frame is not same as that of an observer in moving frame (Goldstein, 1980), (Miller, 2010). Special relativity or restricted relativity is used for special case and named after the general relativity was developed [4] , (Lanczos, 1970).

By generalizing special relativity, General relativity incorporate gravity in the theory where the geometry of space is no more Euclid. Einstein presents a new profile of gravity which unlike Newton's theory of gravity, accepted for more than two hundred years, does not treat gravity as a force. Basically, Newton's universal law of gravitation, which is mathematically formulated as,

$$F = G\frac{m_1 m_2}{r^2},$$

describes that gravity is a force between two masses which is directly proportional to the product of masses and inversely proportional to the square of distance between them (Cohen, et al, 1999). The theory after 71 years of Newton's death passed its first lab-based test by Cavendish in 1798. The experiment resulted the first accurate calculation of 'G', the gravitational constant which turned out to be 6.67408×10^{-11} m3 kg−1 s−2 (Lee, Jennifer Lauren, 2016).

The theory is known as the first great unification for unifying the gravitational phenomena on earth with astronomical behavior (Fritz Rohrlich, 1989), (Klaus Mainzer, 2013). The astronomical significance of Newton's gravity explains many known celestial observations, like, planetary motion which makes sense to Kepler's laws. But, there are effects that it doesn't take in to account like minor anomalies in orbits of planets like Mercury in particular. Planets don't revolve around perfect cirlcular path, as predicted by Kepler's laws, Earth and Venus have almost circular orbits, and the paths of Mercury and Mars are noticeably more elliptical. Kepler's laws of planetary motion aren't perfect, as they are only for a system of small mass orbiting a massive one in the absence of any other mass. Completing one revolution, planet is expected to reach the same spot in space, but it doesn't go this way, there are shifts or advances in orbits which are unexplained by Newton or Kepler's laws.

The general theory of relativity relate gravity with the geometry of space and predict the solutions to such unresolved phenomena. GR, instead of treating gravity as a force between the planetary masses, predicts it as the curvature of spacetime fabric in the presence of mass. Greater the mass, greater will be the curvature (A. pe'er, 2014). Matter determine how much space to be curved or bent while the curvature govern the motion of the matter by the equation, the left-hand side shows the curvature of spacetime and the right-hand side shows the energy and mass (Grøn, Øyvind; Hervik, Sigbjorn, 2007), (Einstein, 1916). So far, GR has been proved successful whenever put to the tests.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}.$$

Perihelion Precession of Mercury:

All the planets obey the Kepler's law of orbit and revolve around the sun in an elliptical path, followed by the mathematics of the relation,

$$r = \frac{p}{1+\varepsilon\ \cos\theta},$$

'r' is the distance between the sun and the planet, 'P' is the semi-latus rectum, ε is the eccentricity, and θ is the angle drawn by the planet (Russell, J. L. ,1964). Moving in an elliptical path the planet approach two extreme positions aphelion, when the planet is farthest away from the sun and the perihelion, when the planet is closest to the sun (Russell, J. L. ,1964).

Based on observations, orbits of planets showed precession per century. Precession was explainable using Newtonian laws by considering the effect of gravitational pull of planets on each other. The problem lied with the mysterious rate of precession of Mercury's orbit, first observed by Urbain Le Verrier, the one who predicted the existence of Neptune using mathematics merely, in 1859. He showed that the actual rate of precession differed from that of Newtonian prediction by 38″ (arcseconds) per tropical century, and the amount was later corrected to be 43″ by Simon Newcomb in 1882 (U. Le Verrier, 1859).

The gravitational forces of other planets on Mercury are not strong enough to be accounted for this large shift in rotation of its major axis or orbital precession. The calculated precession using Newtonian equations was 5557″ per century (1.5436°/century) but the observed precession was 5600″ per century (1.5436 °/ century). There is a 43″ per century discrepancy. The unexplained 43″ was proved by Albert Einstein by presenting the first confirmation of his General theory of relativity (Einstein, 1916). GR geometrizes gravity as the curvature or warping of spacetime (Bleeker, et al, 2001). Warping of spacetime is grater near the mass. As Mercury is the closest planet to the sun, its path around the sun is more ellipse than other planets. According to GR, at aphelion, gravity is weaker (less warping) and time is faster, and at perihelion, gravity is stronger (more warping) and slower will be the time which causes more precession (Einstein, 1916). The amount of orbital rotation by Mercury, using general relativity equation was found,

$$\sigma = \frac{24\pi^3 L^2}{T^2 c^2 \,(1 - e^2)} \ ,$$

Resulting 42.98 arcsecond/century (Bleeker, et al, 2001). 'T' is the orbital period, 'e' is the eccentricity, 'L' is the semi-major axis and 'c' is the speed of light. It was the first confirmation of Einstein's theory of relativity and when he found it, he became speechless with excitement for days (Bleeker, et al, 2001).

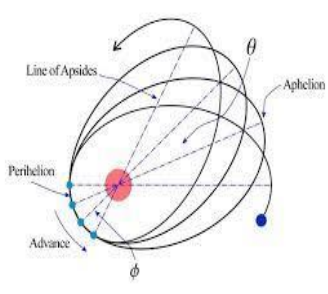

Figure 1: The semi-major axis rotates by the angle after each precession of the perihelion. Illustration by E. Asmodelle (Asmodelle, E. ,2017).

Deflection of Light by the Sun:

Light travels in a straight path unless impeded. What if the path is bent? Space is no more Euclidean in the vicinity of matter. And the effect is even more prominent in the presence of massive bodies, like celestial objects.

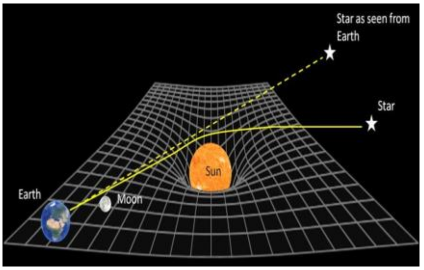

Figure 2: Apparent change in position of the star, as light coming from the star is deflected due to gravity or curved space around the sun

Einstein proposed that his theory could be tested if the deflection of star light by the sun was observed (Einstein, 1911b). He calculated the amount of deflection using the equation of deflection from GR.

$$\delta = \frac{4GM}{R_0 c^2}$$

This resulted in ~1".75 arcseconds (Will, C.M., December 2014). The first experimental proof was witnessed in 1919 by Sir Frank Dyson and Arthur Eddington during a total solar eclipse on May 29 (Dyson, F., 1920). Eddington with collaborators had already measured the true position of constellation (Taurus) near the sun (Dyson, F., 1920). On 29th of May during total solar eclipse, they measured the apparent change in position of the stars. The displacements they calculated were like, 1".98 ± 0".12 and 1".61 ± 0".30, which found to be compatible with the predictions of GR (Dyson, F., 1920).

Gravitational Redshift of Light:

There is a change in frequency of light as the distance between source and observer changes. This phenomenon is well explained as doppler redshift. In Astronomy, as the star moves towards observer, the wavelength of emitted electromagnetic wave compresses, star looks bluer, and as a star moves away from observer, the wavelength stretches, and star appears slightly redder. Mathematically this shift is calculated as,

$$f_{observed} = f_{source}\sqrt{\frac{1-\beta}{1+\beta}}$$

here, $\beta = \frac{v}{c}$ which shows this redshift is the function of velocity.

In case of gravitational redshift, the shift in observed frequency is the function of position, 'r'.

$$f_\infty = f_{source}\sqrt{1 - \frac{2GM}{c^2 r}}$$

GR predicts gravitational redshift in light coming out of a gravitational well. Emerging from a region of strong gravitational field—where space is more warped, electromagnetic wave experiences slowdown of time and lengthened wavelength (Hetherington, 1980), (Holberg, 2010), (Joyce, et al, 2018). The first experimental confirmation became possible by observing non-doppler redshift of Sirius B[16] by an American Astronomer Walter Sydney Adams in 1925, (Adams, 1925). The value measured was one fourth of the actual value. Later, the accurate redshift was measured, 80.4±4.8 km/s by Hubble Space Telescope (Holberg, 2010).

Beside astronomical experiment, Robert Pound, and his graduate student Rebka at Harvard university performed a ground-based experiment to prove gravitational redshift in 1959 (Pound and Rebka, 1959). The experiment was conducted in 22.5m high Jefferson tower.

Figure 3: experiment performed in Jefferson laboratory, in the left Tower at Harvard University. Picture by Lubos Motl.
http://en.wikipedia.org/wiki/Image:JeffersonLeft.jpg

The idea was to measure change in wavelength of photon (gamma ray) emitted by an excited nucleus as a source at the bottom of the tower and absorbed by the nucleus as a detector at the top of the tower (Pound and Rebka, 1959), (Pound and Rebka, 1960). Since the recoil of nucleus would cause some loss of gamma ray energy by emitting and absorber nuclei to conserve momentum. However, in 1958, Rudolf Mossbauer discovered that the energy loss by gamma ray due to recoil of nuclei can be made negligible when the effect is taken up by the entire solid if the emitting and absorber atoms are embedded in lattice (Nave,2005). Applying Mossbauer effect, Pound and Rebka used Fe^{57} and conducted the experiment. They measured redshift when gamma ray was detected at the top (weaker gravitational field) and blueshift when setup was reversed, and the radiation was detected at the bottom (stronger gravitational field).

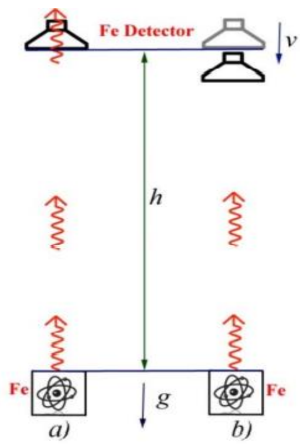

Figure 4: Illustration of Pound-Rebka experiment. a) shows wavelength is not absorbed in stationary detector. b) wavelength absorbed in moving detector (Asmodelle, E. ,2017).

The figure shows, in section (a) when the detector is stationary, there is no absorption or detection of photon because the wavelength is redshifted due to change in strength of gravitational field (Asmodelle, E. ,2017). For change in frequency v1, of gamma ray, in part (a),

$$v_1 \approx (1 - gh/c^2) v_0 \qquad 1)$$

for blueshift frequency v_2,

$$v_2 = \frac{v_o}{1 - \frac{gh}{c^2}} \qquad 2)$$

by plugging v2 in equation 1,

$$v_1 \approx (1 - \frac{gh}{c^2}) v_2 \quad = \quad \left(1 - \frac{gh}{c^2}\right) \frac{v_0}{1 - \frac{gh}{c^2}} = v_o$$

For the unchanged wavelength to be detected, the detector should be moving towards the source as shown in section (b). When detector moves towards or away from the source with a certain velocity, it creates Doppler effect of magnitude same as that of gravitational redshift, but in opposite direction. Thus the effect is cancelled and the unchanged wavelength is detected.

V2 for doppler effect is given as,

$$\nu_2 = \left(1 + \frac{\nu}{c}\right)\nu_0 \qquad 3)$$

Putting this value in equation 1,

$$\nu_1 \approx \left(1 - \frac{g_h}{C^2}\right)\left(1 + \frac{\nu}{\tau}\right)\nu_0$$

To make v1 = v2,

$$\left(1 - \frac{g_h}{C^2}\right)\left(1 + \frac{\nu}{\tau}\right) = 1$$

By rearranging we get,

$$\mathrm{V} = \left[\frac{\frac{gh}{c^2}}{1 - \frac{gh}{c^2}}\right]\mathrm{c}$$

The velocity in pound and Rebka experiment turned out to be 0.7 mm/sec.

Shapiro time delay:

Irwin Shapiro, an American Astrophysicist was first to predict gravitational time dilation in 1964. He proposed a test: hit the surface of Mercury and Venus with radio signal and measure the time delay in a round trip (Shapiro, 1964). He showed that when the Earth, Sun and Venus are aligned, the signal travelling from Earth to Venus and bouncing back would be subject to time delay of 200 microseconds (Shapiro, 1964). In 1966 and 1967, the first tests performed using MIT Haystack radar antenna, were successful. Due to gravitational potential well of sun in between Earth and Venus, it takes the signal longer to travel to Venus and return to Earth than in the absence of sun (Shapiro, et al).

From Schwarzschild solution, Shapiro derived the relation of time dilation for the system he considered for test.

$$\Delta t \approx \frac{4GM}{c^3}\left(\ln\left[\frac{x_p + (x_p^2 + d^2)^{1/2}}{-x_e + (x_e^2 + d^2)^{1/2}}\right] - \frac{1}{2}\left[\frac{x_p}{(x_p^2 + d^2)^{1/2}} + \frac{x_e}{(x_e^2 + d^2)^{1/2}}\right]\right) + \mathcal{O}\left(\frac{G^2M^2}{c^6}\right),$$

Here 'd' is the closest approach to the radius of sun, 'x_e', Earth's orbital radius, 'x_p', radius of the orbit of reflecting planet.

To improve the results, in November 1976, a Viking spacecraft was set to operate at Mars and send a radio wave on Earth. When planet Mars went behind the sun, the wave was sent which travelled very close to the sun. The time delay observed, confirmed GR within 0.1% (Rindler, 2006).

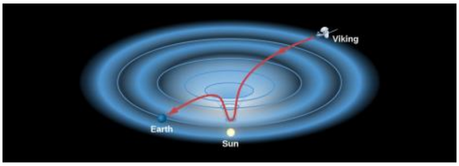

Figure 5: Illustration of radio signal sent by Viking spacecraft from Mars to Earth

https://courses.lumenlearning.com/astronomy/chapter/time-in-general-relativity/

Gravitational waves:

Gravity is the distortion in the spacetime geometry in the presence of mass, and the distortion becomes immense in case of a massive astronomical mass which if spiral with very high speed in the fabric of spacetime, the distortion will emanate in the form of prominent waves, called gravitational waves (Einstein, 1916), (Einstein, 1918). GWs were introduced by Henri Poincare, later, predicted by Einstein in 1916 as consequence of GR (Einstein, 1916), (Einstein, 1918). Gravitational waves radiate energy as the electromagnetic waves in the fabric of spacetime and travel at the speed of light (Einstein and Rosen, 1937).

GWs are radiated by spherically asymmetric accelerating or rotating masses like binaries, the system of two bodies orbiting each other, and the waves emitted carry energy away from the source (peters and Mathews, 1963), (Peters, 1964). Spherically symmetric motion do not radiate gravitational waves, like spinning neutron stars, having perfectly spherical shape due to high density and very strong gravitational field. Sometimes, because of the deformities or bumps above the surface which extend about 4 inches, spinning becomes spherically asymmetric that provides quadruple moment to star, it will radiate GWs until bumps are smoothed out.

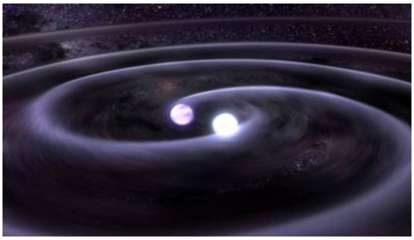

Figure 5: Illustration of a binary system of two white dwarf stars, orbiting each other and losing energy in the form of radiating GWs waves.
Image credit: NASA/Tod Strohmayer (GSFC)/Dana Berry (Chandra X-Ray Observatory

GWs were first confirmed on 14^{th} of September 2015, it was the first ever direct observation of gravitational waves by Laser Interferometer Gravitational Wave Observatory (LIGO) and Virgo detectors (Abbott, et al, 2016), (Castelvecchi, et al 2016). Direct detection of GWs is another significant verification of general relativity by verifying its predictions. The signal, GW150914, detected because of merging of two blackholes of approximately 36 and 29 solar masses of a binary system about 1.3 billion light years away. These ripples travelling from such huge distance are extremely weak to detect here on ground, as also doubted by Albert Einstein (Pais and Abraham, 1982). The cylindrical wave when hit LIGO, changed the length the of a 4km long arm of detector by thousandth of a size of proton (Naeye, 2016).

LIGO is designed to split a laser beam into to two halves that travel into the perpendicular arms of the detector and reflected by suspended mirrors placed at the ends of the arms (Abbott, et al, 2016). At the recombination of reflected beams, there is no detection by the photodetector because the beams are 180° out of phase in the absence of a GW. In the presence of a cylindrical gravitational ripple, the space is stretched and squeezed and hence lengthen one arm and shorten the other in one half cycle and reverse happens in others half cycle (Abbott, et al, 2016). Ligo is extremely sensitive that can detect a change in distance of 10^-19m (Abbott, et al, 2016).

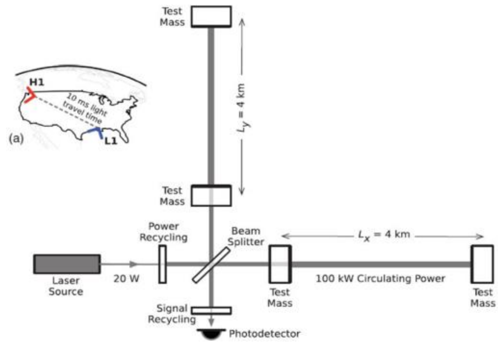

Figure 6: Simplified diagram of LIGO. (a) represents the two detectors, LIGO located at Hanford (H1) and LIGO at Livingston (L1), separated by the distance of 3000km.
https://www.ligo.caltech.edu/

Direct Observation of a Blackhole:

On April 10, 2019, the first ever image of a blackhole, captured by event horizon telescope [EHT], was made public (Doeleman, 2019). EHT have been observing the center of M87 galaxy since 2017. Capturing the evidence of existence of a blackhole in an image is one of the greatest breakthroughs in the history of Astrophysics.

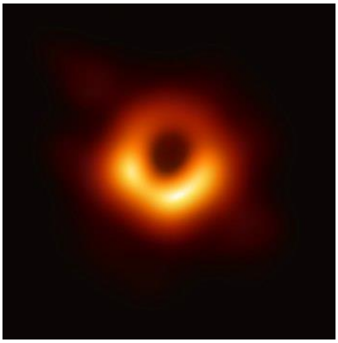

Figure 7: The first image of a blackhole captured by EHT. A bright ring surrounding the dark center confirms the predictions of GR.
https://www.eso.org/public/images/eso1907a/

The existence of blackhole have been assumed by the predictions of general relativity. The solution of Einstein's field equations presented by Karl Schwarzschild, provides the Schwarzschild radius which define the event horizon of a blackhole (Kutner and Marc, 2003), (Schwarzschild, 1916).

$$r_s = \frac{2GM}{c^2},$$

According to GR if mass is contained in a spherical volume of a Schwarzschild radius, the gravity will be immensely strong and so will be warping of space, eventually, lead all the paths of sphere to the center (Schwarzschild, 1916). Since blackholes emit only thermal radiation, it is not possible to detect it from Earth (Hawking, 1976). However, they can be identified by observing accretion discs, the well trapped gas and dust particles circling blackhole due to its strong gravitational pull (Clery, 2020). The collision of these particles when reach the event horizon, release high-energy radiation which can be detected.

Accretion discs are formed by highly dense astronomical bodies. To distinguish blackholes, the properties of radiations are studied which provide information how quickly the particles are moving around that object and thus by putting limit on its mass and size, can disclose is characteristics. Like, for its mass, if the size is less than Schwarzschild radius then it's a blackhole.

CONCLUSION

20th century changed the profile of gravity which had been treated as a force in Newtonian Physics. With the development of general theory of relativity by Albert Einstein, the perception about gravity evolved that gravity is an effect of distortion in spacetime geometry. Not just gravitational effect but this groundbreaking theory provides a new perspective of nature of space, the way its geometry changes in the presence of mass or energy. With this intuition about the laws of nature, GR explains the phenomena which are unexplained by Newtonian laws. General relativity has been verified by numerous tests with its testable predictions.

Mercury's mysterious 43 arcsecond orbital precession is calculated by mathematical relations developed from GR based on the concept that times slows down at perihelion, subject to more warping of spacetime and at aphelion, time runs fast because of less warping of spacetime. The orbit of Mercury precesses by 43 arcsecond consequently.

Einstein proposed that his theory could be tested if the deflection of star light by the sun was observed. The first experimental proof was witnessed in 1919 by Sir Frank Dyson and Arthur Eddington during a total solar eclipse on May 29. They observed change in position of star located behind the sun because of the deflection of star light passing through curved space around the sun.

GR predicts gravitational redshift that light emerging out of a gravitational potential well experiences shift in wavelength. The first experimental confirmation became possible by observing non-doppler redshift of Sirius B[16] by an American Astronomer Walter Sydney Adams in 1925. Gravitational redshift was also tested in lab by Pound and Rebka. They measured change in wavelength of photon (gamma ray) emitted by an excited nucleus as a source at the bottom of the tower and absorbed by the nucleus of similar element as a detector at the top of the tower.

A region of strong gravitational potential causes time delay for an electromagnetic wave passing through it. It was first proposed by Irwin Shapiro and therefore dubbed as Shapiro time delay. The radio signal sent to Venus and travelled back to Earth, took longer time in the presence of Sun.

Spherically asymmetric accelerating or rotating astronomical bodies generate ripples in the fabric of spacetime that carry energy away from the source. These gravitational waves were detected by LIGO and Virgo detectors in September 2015 from an event of merging of two blackholes about 1.3 billion light years away.

And the direct observation of a blackhole has confirmed existence of event horizon, the boundary of blackhole, as predicted by GR.

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