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Non-Newtonian Fluid Flow through Cylindrical Surface in Presences of Porous Material

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Abstract

An investigation has been struggled, in order to examine the behaviour of boundary layer of viscous fluid which is subjected to stream through a modeled cylinder; having thin boundary wall, exponentially extended as much possible with the purpose of smooth flow of fluid, and finally kept at room temperature in a vertical position so that fluid can flow from the top of cylinder to its bottom easily in the existence of Porous material. The Navier-Stokes equations for this flow of fluid are obtained and elucidated by applying boundary layer approximations. In order to bring these equations appropriate for numerical solutions, a suitable transformation of dimensionless variables has been performed. The resulting nonlinear coupled ordinary differential equations are attempted to be solved by well-known method of Shooting. Finally, effect of the parameters involved in these systems of coupled ordinary differential equations have been scrutinized in tabular as well as graphical form.

Keywords: Cylindrical Surface, Variable Porosity, non-Newtonian flow, Shooting Method.

INTRODUCTION

In the past few decades, investigation of fluid flow through cylinder remained a field of interest for researcher. The importance of this geometrical shape caused by its application in automobile and industrial machinery. The behaviour of viscous fluid (Newtonian Fluid) can be completely defined by Navier-Stokes equations. Anwar and Amin [1] studied the case of boundary

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layer flow of a viscoelastic fluid in a model of cylinder placed horizontally, which is affected by mixed convection of heat transmission. He considered a steady stream of viscoelastic fluid moving vertically upward in the cylinder. Both aspects of temperature; keeping the cylinder cool as well as at intense heat, were examined. He invented that particular values of viscoelastic parameters and mixed convection at which the boundary layer separates from the surface of cylinder. Nadeem et al. [2] uncovered influence of thermophoresis and parameters of Brownian motions' over the profiles of hotness and concentration for flow of micropolar fluid through annular region between two concentric cylinders. Again, Nadeem et al. [3] investigated boundary layer flow and heat transfer of nanofluid through a vertical cylinder. He dissected the impact of coefficient of skin friction, Prandtl number, curvature parameter and buoyancy parameter on profiles of velocity, temperature and nanoparticles concentration. As for as the cases of stretched cylinders are concerned, Ishak et al. [4] discussed flow and heat transfer under pulling as well as blowing of fluid on the other end of a stretched cylinder. An enthusiastic contribution about natural convection of heart transfer has been made by Wang [5] for flowing of fluid through a vertical stretching cylinder. About one decade earlier, Abdul Rehman and Nadeem [6] analyzed heat transfer of boundary layer flow over a vertical cylinder which is subjected to exponential stretched along its axial axis. They obtained numerical solution of the Navier-Stokes equations for fluid's motion in that model. Further distribution of velocity as well as temperature profile for big range of Reynolds and Prandtl number were gained.

Chen et al. [7] decided to investigate the non-Darcian effects in a situation when mixed convection of heat transfer exist and viscous fluid is flowing through some porous medium in a vertical cylinder. They analyzed this project under the influence of no-slip boundary condition, inertial force of fluid flow, flexible porosity, and finally transvers thermal dispersion. Whereas Hossain et al. [8] discussed fluid flow of a special kind known as non-Darcy fluid in a modeled cylinder poured in a porous material vertically, in presence of natural convection of heat transfer as well as transfer of mass. Steady flow of viscous fluid through vertical cylinder poured in a porous material under natural convection was studied by Pop and Na [9], where the inner surface of cylinder is kept at constant temperature. They proved that flow of fluid under these circumstances raises the temperature of the outer surface of cylinder. The case of conduction of heat through thin wall of cylinder with high porosity parameter was debated by Kaya [10]. In this case, Newtonian fluid is flowing through a vertical cylinder enriched in a porous material under the influence of mixed convection of heat transfer. Few other related papers are cited in [11-30].

In this paper, the effects of porous material are being examined for boundary layer flow of viscous fluid over the surface of exponentially stretched cylinder. The Navier-Stokes equations for fluid flow through the model under consideration are described for boundary layer flow and then transformed these equations into dimensionless variables. These reduced nonlinear ordinary differential equations are then solved by a well-known numerical method, the shooting method. In the last, physical features of relevant parameters are also argued.

Mathematical Formulation

Consider an exponentially stretched circular cylinder of radius b. Assuming free convection boundary layer flow of a viscous fluid is flowing in this cylinder embedded in a porous material. Let temperature of surface of this cylinder is kept at T_s and constant ambient temperature is retained as T_∞. The temperature of surface of cylinder is taken higher than its surrounding area, i.e., in case of assisting flow $T_s > T_\infty$, while in case of opposing flow the surface of cylinder is kept cooled, thus we have $T_s < T_\infty$.

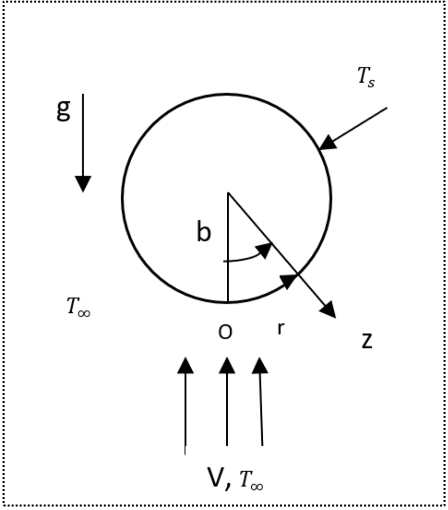


Figure (1) Two-dimensional motion of fluid in circular cylinder.

Figure (1) illustrates the geometry of the problem under consideration.

Under such circumstances we have Navier-Stokes equations in cylindrical coordinate for boundary layer flow is given as

In cylindrical coordinate system we have the velocity $V = \big(u(r,z), 0, w(r,z)\big)$.

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

$$u\,\frac{\partial w}{\partial r} + w\,\frac{\partial w}{\partial z} = \frac{-1}{\rho}\frac{\partial p}{\partial z} + v\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + g\beta(T - T_\infty) - \frac{v\phi_p w}{k_0} + \sqrt{2}\,v\Gamma\left[\left(\frac{\partial w}{\partial r}\right)\left(\frac{\partial^2 w}{\partial r^2}\right) + \frac{1}{2r}\left(\frac{\partial w}{\partial r}\right)^2\right] \qquad (2)$$

$$\left(u\,\frac{\partial T}{\partial r} + w\,\frac{\partial T}{\partial z}\right) = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad (3)$$

Equation (2) is the well-known equation for nonnewtonian Williamson fluid. The associated boundary conditions for boundary layer flow through exponentially stretched cylinder embedded in porous medium are

$$u(b,z) = 0, w(b,z) = V, w(r,z) \to 0 \; as \; r \to \infty \qquad (4)$$

$$T(b,z) = T_s(z), T(r,z) \rightarrow T_\infty \; as \; r \rightarrow \infty \qquad (5)$$

The details of the symbols defined in equation (1) – (3) are as under:

ρ	The mass density of fluid.
ν	$\nu = \frac{\mu}{\rho}$, kinematic viscosity of fluid.
p	pressure of the fluid.
g	the acceleration due to gravity acting along z-axis.
β	the coefficient of thermal expansion.
Γ	time rate constant.
W_e	$w_e = \frac{2\sqrt{2}\,\Gamma\, U_s}{b}$, *Weseissenberg number*.
T	the temperature of cylinder.
α	the thermal diffusivity.
ϕ_p	permeability of porous material.
U_s	fluid velocity at the surface of exponentially stretched cylinder. $U_s = 2bke^{z/b}$

Solution of Problem

Choosing a suitable dimensionless transformation:

$$u = -\frac{1}{2} U_s \frac{f(x)}{\sqrt{x}} , \quad w = U_s f'(x), \qquad (6)$$

$$\theta = \frac{T - T_\infty}{T_s - T_\infty} , \quad x = \frac{r^2}{b^2} \qquad (7)$$

Where T_s, is the surface temperature of cylinder and T_∞ is the ambient temperature, the characteristic temperature, i.e., $(T_s - T_\infty)$ is given by $(T_s - T_\infty) = c\, e^{z/b}$. By applying the transformation described in equation (6) and (7), the equations (1)-(3) will become as

$$x\,\theta'' + \theta' + \frac{1}{2} RePr\,(f\,\theta' - f'\theta) = 0 \qquad (8)$$

$$x\,f''' + f'' + Re\left(f\,f'' - f'^2 \right) + Re\,\lambda\theta - Kp\,f' + W_e\sqrt{x}\left(x\,f'''\,f'' + \frac{3}{4} f''^2 \right) = 0 \qquad (9)$$

Here Kp is the porosity constant $Kp = \frac{R^2\,\phi_p}{4\,k_0}$, $Re = \frac{b\,U_s}{4\,\nu}$ is the Reynolds number, the natural convection parameter is denoted by $\lambda = g\beta b \frac{(T_s - T_\infty)}{{U_s}^2}$ and the Prandtl number is given by $\Pr = \frac{\nu}{\alpha}$. Further, the transformed boundary condition will take the form

$$\theta(1) = 1 , also\; \theta \rightarrow 0 , when\; x \rightarrow \infty \qquad (10)$$

$$f(1) = 0 , f'(1) = 1, also\; f' \rightarrow 0 , when\; x \rightarrow \infty \qquad (11)$$

Furthermore, the concerned physical quantities the skin friction coefficient c_f and the Nusselt number Nu are defined by the relations

$$c_f\, Re^{1/2} = f''(1) , \quad Nu\, Re^{1/2} = -\theta'(1) \qquad (12)$$

The coupled system of nonlinear D.E. (8) to (9) having boundary conditions (10) to (11) were transformed into initial boundary value problem by Numerical Shooting method.

Numerical Method

In order to solve the nonlinear partial D.E. (8) to (9) having boundary conditions (10) to (11) through MATLAB programing, we first of all transform these equations into first order ordinary differential equations.

$$\theta'' = -[\theta' + \frac{1}{2} RePr\,(f\,\theta' - f'\theta)]/x = 0 \quad (12)$$

$$f''' = -[3W_e f''^2 + 4f'' - 4\,Re\,f'^2 + 4Re\,\lambda\theta - 4Kp\,f' + 4Re\,f\,f'']/4(x + x^{\frac{3}{2}} W_e f'') \quad (13)$$

Now in order to transform these equations (12)-(13) into first order ordinary differential equations we assume the following substitutions

$$y_1 = \theta, y_2 = \theta', y_3 = f, y_4 = f', y_5 = f'' \quad (14)$$

The transformed first order ordinary differential equations will be

$$\frac{dy_1}{dx} = \theta' = y_2 \quad (15)$$

$$\frac{dy_2}{dx} = \theta'' = -[2y_2 - Re\,\mathrm{Pr}\,y_1\,y_4 + Re\,\mathrm{Pr}\,y_2 y_3]/2x \quad (16)$$

$$\frac{dy_3}{dx} = f' = y_4 \quad (17)$$

$$\frac{dy_4}{dx} = f'' = y_5 \quad (18)$$

$$\frac{dy_5}{dx} = f''' = -[3W_e y_5{}^2 + 4y_5 - 4\,Re\,y_4{}^2 + 4Re\,\lambda y_1 - 4Kp\,y_4 + 4Re\,y_3 y_5]/4(x + W_e x^{\frac{3}{2}} y_5) \quad (19)$$

Similarly, by assuming y_a as left boundary and y_b to be the right boundary the transformed boundary conditions will take the form

$$y_1(1) = 1, y_1(\infty) = 0, \quad (20)$$

$$y_3(1) = 0, y_4(1) = 1, y_4(\infty) = 0 \quad (21)$$

Next, we will solve these systems of ordinary differential equations (14)-(19) by Shooting method, which will transform these systems of Boundary Value Problem (BVP) into an Initial Value Problem (IVP) by supposition of two additional conditions

$$y_2(1) = \theta'(1) = u(1) \quad and \quad y_5(1) = f''(1) = u(2) \ . \quad (22)$$

The transformed IVP is then solved by Runge-Kutta method of order four.

RESULT AND DISCUSSION

Solution of a differential equation by Numerical methods is nothing but a table of discrete values of independent variables and dependent variables. Table (1) is that entity which represents solution of equation (15) to (19) with its given and guessed conditions (20) to (22) along with specific values of five parameters. Table (2) gives the effect of Reynolds number on shear stress, heat flux, skin friction coefficient C_f and finally Nusselt number Nu at the surface of exponentially stretched cylinder. Figure (1) and (2) describes this impact of table (2) graphically on skin friction coefficient C_f and Nusselt number Nu respectively. It is explored that skin friction coefficient is a parabolically increasing function of Reynolds number. Whereas, Nusselt number is a parabolically decreasing function of Reynolds number.

Figure (3) describes effect of Reynolds number Re on velocity profile $f'(x)$. It has been observed that velocity profile $f'(x)$ decreases with increase in Reynold number. Figure (4) represents behaviour of velocity profile $f''(x)$ with distinct Reynolds number and having fixed other parameters. The graph depicts dual nature, i.e., the velocity profile $f''(x)$ decreases with increase in Reynolds number on one side of a critical point whereas it behaves reversal impact on other side of critical point. Figure (5) shows effect of λ on velocity profile $f'(x)$ having some fixed values of four other parameters. It can be observed in the graph that velocity profile $f'(x)$ decreases with increasing values of λ. Dual nature of velocity profile $\theta'(x)$ can be seen in figure (6) by different three values of Reynolds number Re and fixed values of other four parameters. Figure (7) depicts influence of Weissenberg number on velocity profile $f'(x)$. It is observed that by increasing W_e the velocity profile $f'(x)$ also increases. Similarly enlarging of W_e increase velocity profile $f(x)$, with fixed values of other four parameters. This can be demonstrated in figure (8). Impact of three different Prandtl numbers on temperature profile $\theta(x)$, by keeping other four parameters fixed is displayed in figure (9). It is found that enlarging values of Prandtl number Pr decreases velocity profile of temperature.

Table (1): Distribution of dimensionless variables $\theta, \theta', f, f', and\ f''$ in the variation $1 \leq x \leq 2$, having certain values of Parameters shown above the table.

We= 0.35, Re=3.00, λ =1.25, Prandtl number =7.00, Kp= 0.50

x	θ	θ'	f	f'	f''
1.00000	1.00000	-4.538	0.00000	1.00000	-2.1046
1.05000	0.79142	-3.8152	4.7436	0.89877	-1.9465
1.10000	0.61741	-3.1562	9.0004	0.80516	-1.7997
1.15000	0.47462	-2.5675	1.2807	0.71862	-1.6634
1.20000	0.35943	-2.0528	0.16198	0.63866	-1.5366
1.25000	0.2681	-1.6127	0.19204	0.56482	-1.4182
1.30000	0.19695	-1.2447	0.21855	0.49672	-1.3071
1.35000	0.1425	-0.9439	0.2418	0.43401	-1.2023
1.40000	0.10155	-0.7035	0.26204	0.3764	-1.1032
1.45000	0.071276	-0.5155	0.27952	0.32361	-1.009
1.50000	0.049266	-0.3715	0.29448	0.27543	-0.91918
1.55000	0.033525	-0.2634	0.30714	0.23162	-0.83356
1.60000	0.022442	-0.18401	0.31771	0.192	-0.75196
1.65000	0.014756	-0.12661	0.3264	0.15636	-0.67439
1.70000	0.009503	-0.08589	0.33341	0.12449	-0.60096
1.75000	0.0059618	-0.05749	0.33891	0.096193	-0.53186
1.80000	0.0036059	-0.03798	0.34308	0.071233	-0.46731
1.85000	0.0020582	-0.02479	0.34609	0.049382	-0.40755
1.90000	0.0010535	-0.01599	0.34807	0.030395	-0.35276
1.95000	0.00040892	-0.01019	0.34917	0.01402	-0.30309
2.00000	-7.3924E-16	-0.00643	0.34951	2.1901E-16	-0.25858

Table (2): Behavior of shear stress $f''(1)$, heat flux $-\theta'(1)$, Skin friction coefficient C_f and Nusselt number Nu at the surface of the cylinder for different values Reynolds number.

Re	$f''(1)$	$-\theta'(1)$	$C_f = \frac{f''(1)}{\sqrt{Re}}$	$Nu = -\frac{\theta'(1)}{\sqrt{Re}}$
0.5	-1.6206	2.4549	-2.2918	3.4718
1.0	-1.7466	2.9950	-1.7466	2.9950
1.5	-1.8525	3.4518	-1.5126	2.8184
2.0	-1.9452	3.8518	-1.3755	2.7236
2.5	-2.0284	4.2104	-1.2829	2.6629
3.0	-2.1045	4.5376	-1.2150	2.6198

Figure (2): Behaviour of Skin friction coefficient Cf with different Reynolds numbers.

We= 0.35 λ=1.25 Prandtl number =7.00 Kp= 0.50

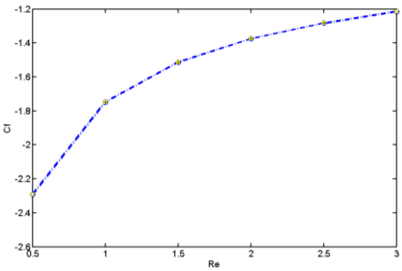


Figure (3): Behaviour of Nusselt number Nu with different Reynolds numbers.

We= 0.35 L=1.25 Prandtl number =7.00 Kp= 0.50

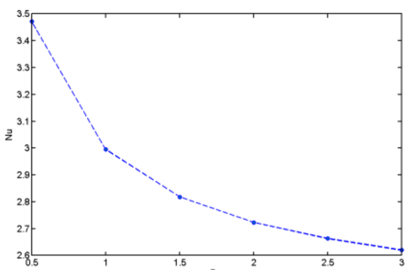


Figure (4) Effect of Reynolds number on velocity profile $f'(x)$.

We= 0.30, λ =0.25 Prandtl number =7.00 Kp= 2.00:

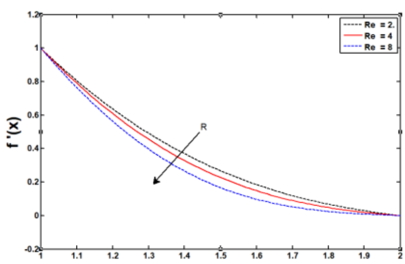


Figure (5) Effect of Reynolds number on velocity profile $f''(x)$.
We= 0.30, λ =0.25 Prandtl number =7.00 Kp= 2.00:

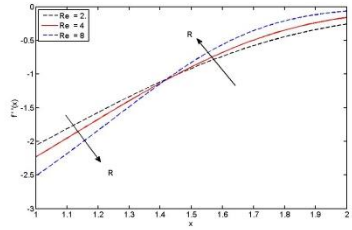


Figure (6): Effect of λ on velocity profile $f'(x)$.
We= 0.30, Re=3.00, Prandtl number =7.00, Kp= 2.00

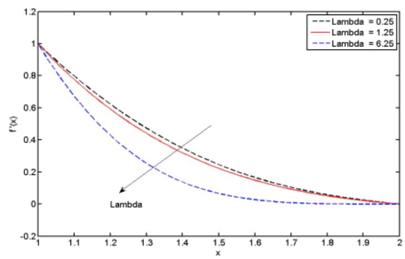


Figure (7): Effect of Re on velocity profile $\theta'(x)$.
We= 0.30, λ=0.25 Prandtl number =7.00 Kp= 2.00

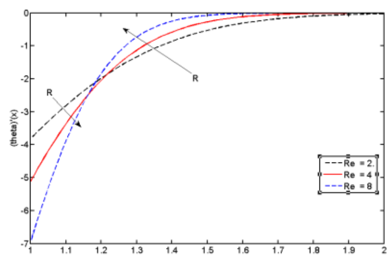


Figure (8): Effect of We on velocity profile $f'(x)$.
Re=2.00 λ =0.25 Prandtl number =7.00 Kp= 2.00

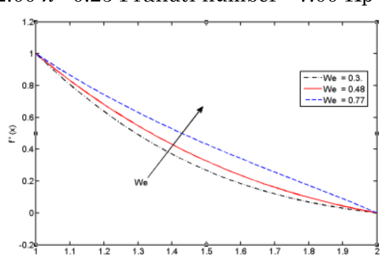


Figure (9): Effect of We on velocity profile $f(x)$.

Re=2.00 λ =0.25 Prandtl number =7.00 Kp= 2.00

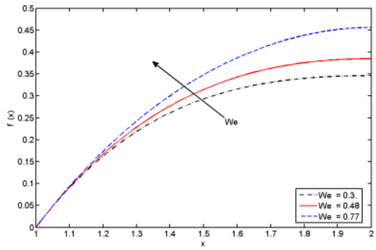
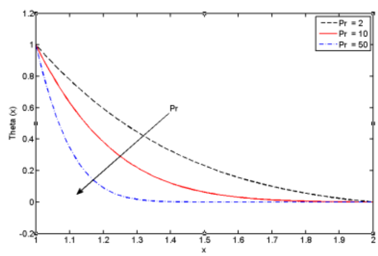


Figure (10): Distribution of Temperature Profile θ for three different values of Prandtl number.

We= 0.30 Re=2.00 λ =0.25 Kp= 5.00



CONCLUSION

We have examined boundary layer flow of a viscous fluid over surface of an exponentially stretched cylinder. The Navier Stokes equations for this flow was solved numerically by Shooting Method. Our concluding remarks are as under: -

1. Velocity profile of $f'(x)$ decreases with increase in Reynolds number or λ.
2. Dual nature of velocity profile of $f''(x)$ and $\theta'(x)$ is found with three different values of Reynolds number.
3. Increase in Weissenberg number increase velocity profiles of both $f(x)$ and $f'(x)$.
4. Temperature profile $\theta(x)$ decreases by enlarging Prandtl number.
5. C_f and Nu are respectively, parabolically increasing and decreasing functions of Re.

REFERENCES

[1]I. Anwar, N. Amin, I. Pop, Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder, Int. J. Non-Linear Mech. 43 (2008) 814-821

[2]S. Nadeem, Abdul Rehman, K. Vajravelu, J. Lee, C. Lee, Axisymmetric stagnation flow of a micropolar
nanofluid in a moving cylinder, Math. Prob. Eng. Volume 2012 (2012), Article ID 378259, 17 pages, doi:10.1155/2012/378259.

[3]S. Nadeem, Abdul Rehman, Mohamed E Ali, The boundary layer flow and heat transfer of a nanofluid over a vertical slender cylinder, *Journal of Nanoengineering and Nano systems (*2012) 226: DOI:10.1177/1740349912453806.

[4]Ishak A, Nazar R and Pop I. The effects of transpiration on the boundary layer flow and heat transfer over a vertical slender cylinder. Int J Non-Linear Mech 2007; 42: 1010–1017.

[5]C. Y. Wang, Natural convection on a vertical stretching cylinder, Commun. Nonlinear Sci. Numer. Simulat, 17 (2012) 1098-1103

[6]Abdul Rehman & S. Nadeem. Heat Transfer Analysis of the Boundary Layer Flow over a Vertical Exponentially Stretching Cylinder, Global Journal of Science Frontier Research
Mathematics and Decision Sciences Volume 13 Issue 11 Version 1.0 (2013).

[7]C. K. Chen, C. H. Chen, W. J. Minkowycz, U. S. Gill, Non-Darcian effects on mixed convection about a vertical cylinder embedded in a saturated porous medium, Int. J. Heat Mass Transfer 35 (1992) 3041-3046.

[8]Hossain MA, Vafai K and Khanafer KMN. Non-Darcy natural convection heat and mass transfer along a vertical permeable cylinder embedded in a porous medium. Int J Therm Sci 1999; 38: 854–862.

[9]Pop I and Na TY. Conjugate free convection over a vertical slender hollow cylinder embedded in a porous medium. Heat Mass Transfer 2003; 36: 375–379.

[10]Kaya A. Effects of buoyancy and conjugate heat transfer on non-Darcy mixed convection about a vertical slender hollow cylinder embedded in a porous medium with high porosity. DOI: 10.1016/j.ijheatmasstransfer.2010.10.021.

[11]S. Nadeem, Abdul Rehman, Changhoon Lee, Jinho Lee, Boundary layer flow of second grade fluid in a cylinder with heat transfer, Mathematical Problems in Engineering, Volume 2012, Article ID 640289.

[12]S. Nadeem, Abdul Rehman, Mohamed Ali, The boundary layer flow and heat transfer of a nanofluid over a vertical slender cylinder, J. NanoEngineering and NanoSystems (2012) 1-9.

[13] Abdul Rehman, S. Nadeem, M. Y. Malik, Stagnation flow of couple stress nanofluid over an exponentially stretching sheet through a porous medium, J. Power Tech. 93(2) (2013) 122-132.

[14]Abdul Rehman, S. Nadeem, M. Y. Malik, Boundary layer stagnation-point flow of a third grade fluid over an exponentially stretching sheet, Braz. J. Che. Eng. 30(3) (2013) 611-618.

[15]Abdul Rehman, S. Nadeem, Heat transfer analysis of the boundary layer flow over a vertical exponentially stretching cylinder, Global J. Sci. Fron. Res. 13(11) (2013) 73-85.

[16]M. Y. Malik, M. Naseer, S. Nadeem, Abdul Rehman, The boundary layer flow of Casson nanofluid over a vertical exponentially stretching cylinder, Appl. NanoSci. DOI: 10.1007/s13204-012-0267-0

[17]Abdul Rehman, S. Nadeem, S. Iqbal, M. Y. Malik, M. Naseer, Nanoparticle effect over the boundary layer flow over an exponentially stretching cylinder, J. NanoEngineering and NanoSystems (2014) 1-6.

[18]M. Y. Malik, M. Naseer, S. Nadeem, Abdul Rehman, The boundary layer flow of hyperbolic tangent fluid over a vertical exponentially stretching cylinder, Alexandria Engineering J.,53 (2014) 747-750

[19]Y. Malik, M. Naseer, Abdul Rehman, Numerical study of convective heat transfer on the Power Law fluid over a vertical exponentially stretching cylinder, Applied and Computational Mathematics, 4(5), (2015) 346-350.

[20]Abdul Rehman, Razmak Bazai, Sallahuddin Achakzai, Saleem Iqbal, Muhammad Naseer, Boundary Layer Flow and Heat Transfer of Micropolar Fluid over a Vertical Exponentially Stretched Cylinder, Applied and Computational Mathematics, 4(6) (2015) 424-430.

[21]Abdul Rehman, Ghulam Farooq, Israr Ahmed, Muhammad Naseer, Muhammad Zulfiqar, Boundary Layer Stagnation-Point Flow of Second Grade Fluid over an Exponentially Stretching Sheet, American Journal of Applied Mathematics and Statistics, 3(6) (2015) 211-219.

[22]Abdul Rehmana, Sallahuddin Achakzai, Sohail Nadeem, Saleem Iqbal, Stagnation point flow of Eyring Powell fluid in a vertical cylinder with heat transfer, Journal of Power Technologies 96 (1) (2016) 57–62

[23]Abdul Rehman, Saleem Iqbal, Syed Mohsin Raza, Axisymmetric Stagnation Flow of a Micropolar Fluid in a Moving Cylinder: An Analytical Solution, Fluid Mechanics, 2(1) (2016) 1-7

[24]Naheeda Iftikhar, Abdul Rehman, Peristaltic flow of an Eyring Prandtl fluid in a diverging tube with heat and mass transfer, International Journal of Heat and Mass Transfer 111 (2017) 667–676

[25]Abdul Rehman, Naveed Sheikh, Boundary Layer Stagnation-Point Flow of Micropolar Fluid over an Exponentially Stretching Sheet, International Journal of Fluid Mechanics & Thermal Sciences, 2017; 3 (3): 25-31.

[26]Haroon Rasheed, Abdul Rehman, Naveed Sheikh, Saleem Iqbal, MHD Boundary Layer Flow of Nanofluid over a Continuously Moving Stretching Surface, Applied and Computational Mathematics, 2017; 6 (6): 265-270.

[27]Najeeb Alam Khan, Umair Bin Saeed, Faqiha Sultan, Saif Ullah, and Abdul Rehman, Study of velocity and temperature distributions in boundary layer flow of fourth grade fluid over an exponential stretching sheet, AIP ADVANCES 8, 025011 (2018).

[28]Naheeda Iftikhar, Abdul Rehman, Hina Sadaf, Muhammad Najam Khan, Impact of wall properties on the peristaltic flow of Cu-water nano fluid in a non-uniform inclined tube, International Journal of Heat and Mass Transfer 125 (2018) 772–779.

[29]Naheeda Iftikhar, Abdul Rehman and Muhammad Najam Khan, Features of Convective heat transfer on MHD peristaltic movement of Williamson fluid with the presence of Joule heating, IOP Conf. Series: Materials Science and Engineering 414 (2018) 012010

[30]Amina Panezai, Abdul Rehman, Naveed Sheikh, Saleem Iqbal, Israr Ahmed, Manzoor Iqbal, Muhammad Zulfiqar, Mixed Convective Magnetohydrodynamic Heat Transfer Flow of Williamson Fluid Over a Porous Wedge, American Journal of Mathematical and Computer Modelling, 4(3) (2019) 66-73