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A Graphical Comparison of Continous Fourier transform and Discrete Fourier transform of Sinusoidal Signals

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Abstract

Fourier analysis is a family of mathematical techniques, all based on decomposing signals into sinusoids and very powerful tool for time-varying signal analysis. The Fourier transform along with the discrete Fourier transform is most important tools in digital signal processing, these transforms can calculate a frequency signal's frequency spectrum and applied for a direct examination of information encoded in the, phase, and amplitude of the component sinusoids. In this article we have presented a graphical comparison by applying the continuous Fourier transform and discrete Fourier transform to the sinusoidal signals.

Keywords: Fourier transform, Discrete Fourier transform, sinusoidal signals, Matlab

1. INTRODUCTION

One of the most useful transform in the family of Fourier analysis is the Discrete Fourier transform (DFT). DFT plays a very important role in various applications in case of digital processing the function or signal that varies with respect to time.

In digital signal processing, the function or signal that varies with respect to time, for example daily temperature readings, the pressure of a sound wave, or a radio signal experimented with respect to time. In image processing, the models can be the standards of pixels along a row or column of a two dimensional image. The DFT is also used to solve partial differential equations, and to perform other operations such as convolutions or multiplying large integers. (Olver, 2018).

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Quantum Fourier Transform (QFT) is the quantum analogue of the Discrete Fourier Transform (DFT). QFT has applications in quantum computation and information, for a detailed discussion in this area see (Weinstein, 2001). The discrete Fourier transform method has proven to be an effective way of solving a wide class of electromagnetic scattering problems. The Discrete Fourier transform is most powerful tools in digital signal processing and digital filtering which open the doors in order to find the spectrum of a finite-duration signal (Oppenheim, 2001; Bagchi, 2012; Hammani,2019).
. In this article we have applied the continuous and discrete Fourier presented to sinusoidal signals that is sine and cosine function and presented graphically using matlab software.

1.1 Methodology and Preliminary concept

1.1.1 Continuous Fourier transform:

Fourier is a mathematical model which helps to transform the signals between two different domains, such as transforming signal from frequency domain to time domain or vice versa. Continuous Fourier transform or simply Fourier transform (FT) has many applications in Engineering and Physics, such as signal processing, Radar, and so on. In this article, we are going to discuss the formula of Fourier transform, properties, tables, Fourier cosine transform, Fourier sine transform with complete explanations. The generalisation of the complex Fourier series is known as the Fourier transform. The term "Fourier transform" can be used in the mathematical function, and it is also used in the representation of the frequency domain. The Fourier transform helps to extend the Fourier series to the non-periodic functions, which helps us to view any functions in terms of the sum of simple sinusoids. Three are four version of expressing the Fourier transform for detail see (Bracewell, 1986). The FT is denoted and defined as

$$\mathcal{F}[f(t)] = F(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-ikt}\,dt \qquad (1)$$

$$\mathcal{F}^{-1}[F(k)] = f(kt) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(k)e^{ikt}\,dk \quad (2)$$

1.1.2 Discrete Fourier transform (DFT)

Discrete Fourier transforms (DFTs) are extremely useful because they reveal periodicities in input data as well as the relative strengths of any periodic components. There are however a few subtleties in the interpretation of discrete Fourier transforms. In general, the discrete Fourier transform of a *real* sequence of numbers will be a sequence of *complex* numbers of the same length. Now consider generalization to the case of a discrete function

$f(t) \to f(t_k)$ by letting $f_k \equiv f(t_k)$ where $t_k \equiv k\Delta$, with $k = 0,1,2,\ldots,N-1$

Writing this out gives the discrete Fourier transform

$$F_n = \mathcal{F}_k[\{f_k\}_{n=0}^{N-1}](n) \quad as$$

$$F_n = \sum_{k=0}^{N-1} f_k e^{\frac{-2\pi i n k}{N}} \qquad (3)$$

The inverse transform

$$f_k = \mathcal{F}^{-1}{}_k[\{F_n\}_{n=0}^{N-1}](k)$$

is then written as

$$f_k = \frac{1}{N}\sum_{k=0}^{N-1} F_n e^{\frac{2\pi i n k}{N}} \qquad (4)$$

For detail studies see (Weisstein, 2002).

2. RESULTS AND DISCUSSIONS

The Sinusoidal Signals are periodic functions add can be based on the sine and cosine functions.

The general form of a Sinusoidal Signal based on cosine function is given below

$$x(t) = A\,cos(\omega_0 t + \theta) = A\,cos(2\pi f_0 t + \theta)\,) \qquad (5)$$

$\omega_0 = 2\pi f_0$ is the radian frequency of the sinusoidal signal,θ is the phase shift or phase angle A is the amplitude of the signal and is the scaling factor that determines how large the signal will be.

2.1. A graphical comparison of FT and DFT of sinusoidal signal

Here we have consider the sinusoidal signals $\cos t$, $\cos 2t$, $\cos 3t$, $\cos 4t$, $\cos 5t$ and figures 1,3,5,7,9 are representing the graphs of Fourier transform (FT) of sinusoidal signals whereas figures 2,4,6,8 10 representing the graphs of discrete Fourier transform (DFT)

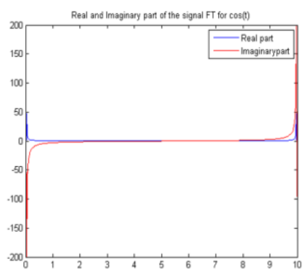

Fig.1

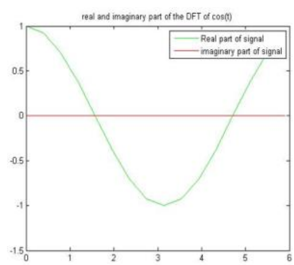

Fig.2

The graphical representation of the cost function with its magnitude and phase spectra is shown in Fig.1with respect to Fourier Transform and in Fig. 2 with respect to Discrete Fourier Transform.

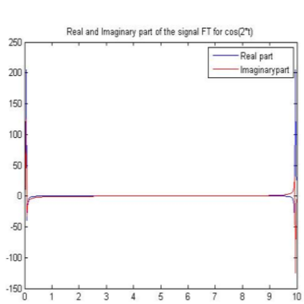

Fig. 3

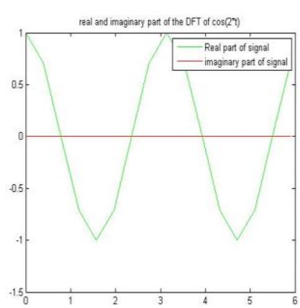

Fig. 4

The graphical representation of the cost function with its magnitude and phase spectra is shown in Fig. 3 with respect to Fourier Transform and in Fig. 4 with respect to Discrete Fourier Transform.

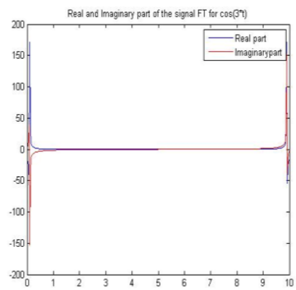

Fig. 5

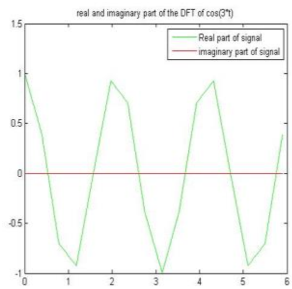

Fig. 6

The graphical representation of the cost function with its magnitude and phase spectra is shown in Fig. 5 with respect to Fourier Transform and in Fig.6 with respect to Discrete Fourier Transform.

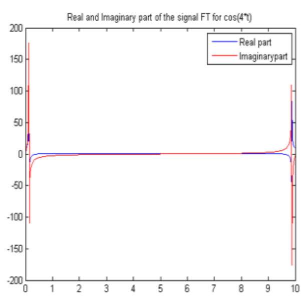

Fig. 7

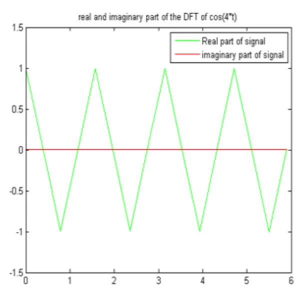

Fig. 8

The graphical representation of the cos4t function with its magnitude and phase spectra is shown in Fig. 7 with respect to Fourier Transform and in Fig.8 with respect to Discrete Fourier Transform.

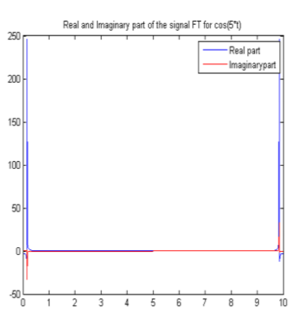

Fig. 9

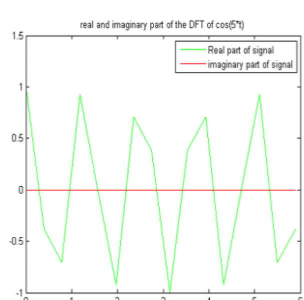

Fig.10

2.1.3 A graphically comparison of FT and DFT for real and imaginary sinusoidal signals

The Fourier transform produces a complex number valued output image which can be displayed with two images, either with the real and imaginary part or with magnitude and phase. In image processing, often only the magnitude of the Fourier Transform is displayed, as it contains most of the information of the geometric structure of the spatial domain image. However, if we want to re-transform the Fourier image into the correct spatial domain after some processing in the frequency domain, we must make sure to preserve both magnitude and phase of the Fourier image. So the real and imaginary numbers can be negative numbers, which in digital image processing cannot be displayed. From the comments above and other sources it came to conclusion that real and imaginary components of the Fourier Transform comprise from sinusoidal signal of cosine form.

The DFT can transform a sequence of evenly spaced signal to the information about the frequency of all the sine waves that needed to sum to the time domain signal.

The main issue with the above DFT implementation is that it is not efficient if we have a signal with many data points. It may take a long time to compute the DFT if the signal is large.

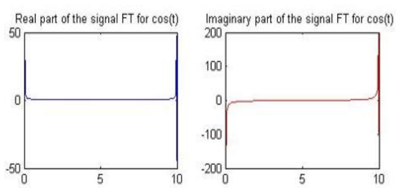

Fig.11

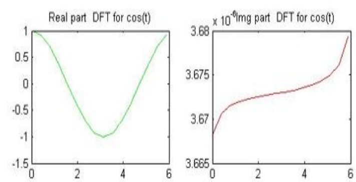

Fig.12

We can see from the above graphical representation of Fig. 11 and Fig. 12 here that the output of the FT and DFT of real and imaginary parts of cost is symmetric at half of the sampling rate.

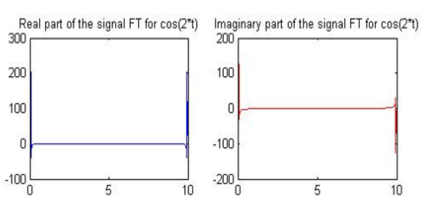

Fig.13

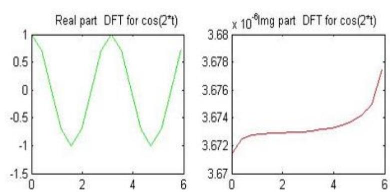

Fig.14

We can see from the above graphical representation of Fig. 13 and Fig. 14 here that the output of the FT and DFT of real and imaginary parts of cos2t is symmetric at half of the sampling rate.

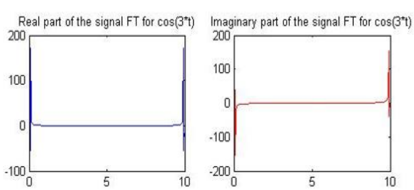

Fig.13

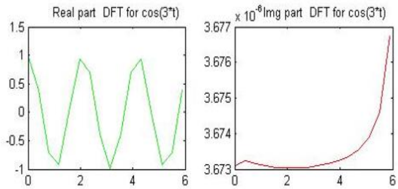

Fig.14

We can see from the above graphical representation of Fig. 13 and Fig.14 here that the output of the FT and DFT of real and imaginary parts of cos3t is symmetric at half of the sampling rate.

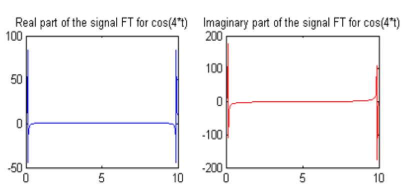

Fig.15

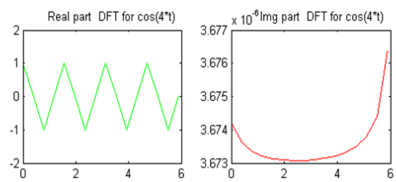

Fig.16

We can see from the above graphical representation of Fig. 15 and Fig. 16 here that the output of the FT and DFT of real and imaginary parts of cost is symmetric at half of the sampling rate.

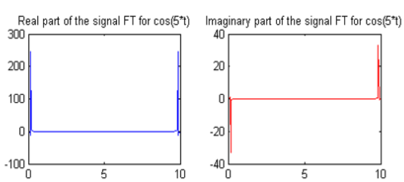

Fig.17

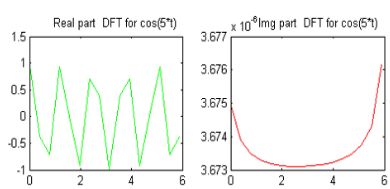

Fig.18

We can see from the above graphical representation of Fig. 17 and Fig.18 here that the output of the FT and DFT of real and imaginary parts of cost is symmetric at half of the sampling rate.

CONCLUSION:

We have represented a comparison of FT and DFT of sinusoidal signals of cosine form and also presented their real and imaginary behavior graphically which will help to understand such phenomena in order to apply in many applications of signal processing

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