

Fostering Mathematical Modeling Capacity through Teaching to Determine Mathematical Models for Real Problems of Circles and Three Conic Lines in Grade 10 in Vietnam

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Abstract

Mathematical modeling competence is one of the five core competencies that a student should be equipped with in Vietnam. Especially after the textbook program in Vietnam is written in the direction of capacity development, the teaching of practical problems is more and more concerned and therefore associated with real problems, mathematical modeling competence deserves special attention. Among the practical problems, the problems of circles and three conic lines clearly show the content of developing mathematical modeling capacity the most. In this article, we present the teaching process to develop mathematical modeling competence, and how to teach to develop mathematical modeling competence in our school. The actual teaching results show that this teaching method is an effective teaching method that meets the needs of students. Students showed enthusiasm and passion for this teaching method. Preliminary assessment shows that the quality of teaching is improved.

Keywords: Training, capacity, mathematical modeling capacity, practical problems, circle, three conic lines.

1. INTRODUCTION

Competence is what a person knows and can do effectively. Competence implies knowledge and abilities, while effectiveness depends on sentimental, motivational, attention and stylistic factors. As a result, students' abilities may not be adequately expressed in academic achievement or test results because of personal factors that are important to themselves or learners' behavior (Messick, 1984). The competence of professionals stems from the fact that they have for themselves relevant attributes such as knowledge, skills and attitudes. This set of attributes is often referred to as competence. Thus, competence is a combination of attributes that underlie professional success (Gonczi et al., 1993).

According to Mogens Niss (2003), "Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety of contexts and situations within and outside mathematics in which mathematics plays a role. A mathematical competence is the main clearly identifiable and distinct constituent element of mathematical competence. The author identifies 8 competencies that are made up, including: Mathematical thinking competency; problems tackling competency;

modelling competency; reasoning competency; representing competency; symbols and formalism competency; communicating competency; aids and tools competency" (Mogens, 2003).

In Vietnam, mathematical modeling competence is included in the 2018 General Education curriculum in Mathematics. Therefore, it can be said that mathematical modeling competence is of particular interest. In this article we focus on answering the following questions:

1. What is the view on mathematical modeling competence?
2. What is the teaching process to determine mathematical models for practical problems of circles and three conic lines in grade 10?
3. What is the method of organizing teaching to determine mathematical models for practical problems of circles and three conic lines in grade 10?

2. LITERATURE REVIEW

2.1. Perspectives on mathematical modeling competence

According to Anu Maria (1997), a model is a system that is built and set up for ease of handling. The model is similar to but simpler than the system it represents. The purpose of the model is to allow the analyst to predict the effect of changes on the system. On the one hand, a model should be an approximation of the real system and incorporate most of its salient features. On the other hand, it shouldn't be so complicated that it can't be understood or tested. A good model is a balance between realism and simplicity. Modeling is the process of producing a model. An important problem in modeling is model validity. Model validation techniques include simulating the model under known input conditions and comparing the model output with the system output. In general, a model for a simulation study is a mathematical model developed with the help of simulation software. Mathematical model classifications include deterministic (input and output variables are fixed values) or random (at least one of the input or output variables is a probability); static (no time taken into account) or dynamic (time-varying interactions between the variables taken into account). Usually, simulation models are random and dynamic (Maria, 1997).

According to Kwan Eu Leong and Jun You Tan (2020), mathematical modeling is a cyclical process of transforming a real-world problem into a mathematical problem by constructing and solving a mathematical model, interpreting and evaluating the solution in a real-world context, and refining or improving the model if the solution is unacceptable (Leong & Tan, 2020).

2.2. The teaching process that is oriented towards the development of mathematical modeling competence for students

According to Werner Blum and Rita Borrromeo Ferri (2009), the following four steps can be taken to solve a modeling task:

Step 1: Find out the mission

- Read the text correctly and visualize the situation clearly
- Draw normally

Step 2: Set up the model

- Search for the data you need. If necessary: make assumptions
- Searching for mathematical relations

Step 3: Use math

- Use proper procedures
- Write down your math results

Step 4: Interpret the results

- Round and link the results to the task. If necessary, return to step 1
- Write down your final answer.

(Blum & Borromeo, 2009)

From Werner Blum and Rita Borromeo Ferri's teaching process oriented to developing mathematical modeling competence for students (2009), we propose a teaching process to identify mathematical models for real problems of circles and three conic lines in grade 10 as follows:

- Step 1: Analyze the practical problem

We must know the direction of finding solutions. Real-life problems contain many real-life facts. We need to be able to refine and convert the language of life into the language of mathematics. We must know how to set unknown numbers and find the relationship between objects to convert them into explicit mathematical language.

- Step 2: Set up the mathematical model

From the actual problem, after setting the unknowns, and converting the data of the actual problems to the math problem, we can set up and come up with equations, diagrams, tables, graphs, formulas, etc. This is the model setting step.

2.3 How to organize teaching to determine mathematical models for practical problems of circles and three conic lines in grade 10

2.3.1. The purpose of teaching

The purpose of teaching is to help students establish mathematical models (including formulas, equations, diagrams, drawings, tables, graphs, ...) to describe the situation posed in the practical math problem. This is an important step in the mathematical modeling process. Through building mathematical models for practical problems, students will practice the ability to read the problem, recognize the structure of the problem, the relationship of the elements in the problem, and realize the relationship between these elements and mathematical objects. Finally, it is possible to connect these mathematical objects to create a mathematical model for a real-world problem.

2.3.2. Scientific basis

Modeling is an interactive, trial-and-error process. Model building is usually defined in a sequence of steps that increase in complexity until the resulting model is capable of describing all aspects of the problem, the real problem that the specifier is interested in. (Muthuri, 2009).

„Modeling competence” is the ability to build models by performing various steps appropriately as well as to analyze or compare given models (Cheng, 2001).

2.3.3. Illustration

Example 1

Many railway viaducts are built in the form of a semicircle. A stone arch railway viaduct in Rockville, Pennsylvania, spanning the Susquehanna River is made of 48 semicircular

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arches, each with a span of 70 feet. Write equations to model the first two semicircular arches.

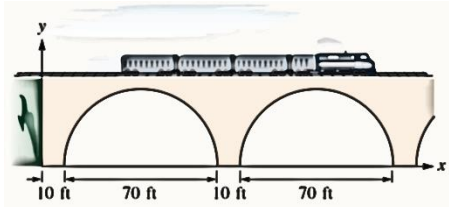


Figure 2.1. Viaduct (Source: Nguyen Ngoc Giang et al., 2020)

| Teacher's activities | Student activities |
|---|---|
| Step 1: Analyze the practical problem | |
| - What is the requirement of the problem? | - Write the equation of the circle. |
| - What do you know about the circle above? | - Its diameter length is 70 feet. |
| - What do you need to know to write the equation of the circle? | - The coordinates of the center are $(a; b)$, the length of the radius is R . Then the equation is: $(x - a)^2 + (y - b)^2 = R^2$ |
| Step 2: Set up the mathematical model | |
| - Representing the first two semicircular arches on the Oxy -axis system? | <p>Figure 2.2. Coordinates of two semicircular arches (Source: Nguyen Ngoc Giang et al., 2020)</p> |
| - Determine the coordinates of the center and the length of the radius of the first semicircular arch? | - The coordinates of the center are $(45; 0)$, the radius length is 35. |
| - Write the equation of the first semicircular arch? | - The equation of the first semicircular arch is: $(x - 45)^2 + y^2 = 35^2, y \geq 0$. |
| - Determine the coordinates of the center and the length of the radius of the second semicircular arch? | - The coordinates of the center are $(125; 0)$, the radius length is 35. |
| - Write the equation of the second semicircular arch? | The equation of the first semicircular arch is: $(x - 125)^2 + y^2 = 35^2, y \geq 0$. |

Example 2

We know that the Moon moves around the Earth in an ellipse of which the Earth is one focus. That ellipse has $A_1A_2 = 768\,800$ km and $B_1B_2 = 767\,619$ km. Write the canonical equation of that ellipse (Do Duc Thai et al., 2022).

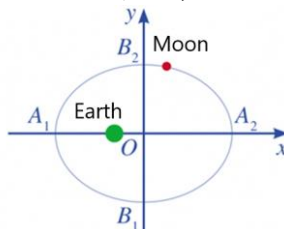
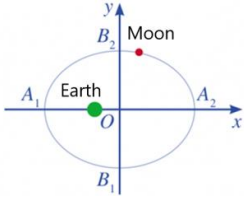


Figure 2.3. Lunar orbit (Source: Do Duc Thai et al., 2022).

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| Teacher's activities | Student activities |
|--|---|
| Step 1: Analyze the practical problem | |
| - What is the requirement of the problem? | - Set up the equation of the ellipse. |
| - What do you know about the upper ellipse? | - That ellipse has: $A_1A_2 = 768\ 800\ km$ $B_1B_2 = 767\ 619\ km$ And the Earth is a focus. |
| - To write the equation of the ellipse, what knowledge do you use? | - The length of the major axis is $2a$, the length of the minor axis is $2b$. Then the ellipse equation is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ - Note: Focal length $2c$ must satisfy the following condition: $b^2 = a^2 - c^2.$ |
| Step 2: Set up the mathematical model | |
| - Draw an illustration. |  <p>Lunar orbit (Source: Do Duec Thai et al., 2022)</p> |
| - Find a . | - We have: $A_1A_2 = 2a = 768\ 800$ We infer: $a = 384\ 400$ |
| - Find b ? | - We have: $B_1B_2 = 2b = 767\ 619$ We infer: $b = 383\ 809,5$ |
| - Write the equation of the above ellipse? | - The ellipse has the equation: $\frac{x^2}{384400^2} + \frac{y^2}{383809,5^2} = 1.$ (Unit: km) |

Example 3

Before the global positioning system GPS, the long-range navigation radio navigation system (LORAN) was a system used to determine the position of ships, boats, and aircraft. Two radio stations located at locations A and B simultaneously send signals to the ship. The onboard receiver converts the time difference in receiving these signals into the distance difference $PA - PB$, and by the definition of hyperbola, the ship's position is on a hyperbolic branch with A and B as the two focal points. The second pair of radio stations located at C and D at the same time also send signals to the ship. Similarly, the receiver on the ship converts the time difference in receiving these signals into the distance difference $PC - PD$, and by definition of the hyperbola, the position of the ship is on a hyperbolic branch where C and D are the two focal points. Point P, where the ship is located, is the intersection of the two hyperbolic branches just identified.

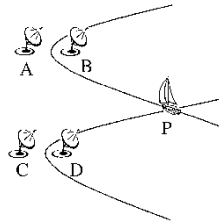


Figure 2.4. LORAN (Source: Nguyen Ngoc Giang et al., 2020)

We assume that radio stations A and B are 150 km apart and send radio signals to the ship at the same time. The signal from B arrived 0.0003 seconds earlier than the signal from A. If the signal travels at 300000km/s, find the hyperbolic equation to determine the position of the ship.

(Nguyen Ngoc Giang et al., 2020)

| Teacher's activities | Student activities |
|--|--|
| Step 1: Analyze the practical problem | |
| - What is the requirement of the problem? | - Write the equation of the hyperbola. |
| - What do you know about the hyperbola above? | - Distance between two focal points: $AB = 2c = 150 \text{ km}$. - Point P belongs to the hyperbola and we get $ PA - PB = 2a$. |
| - What do you need to know to write the equation of the hyperbola? | - Distance between two focal points $F_1F_2 = 2c$, point M belongs to hyperbola satisfying $ MF_1 - MF_2 = 2a$ and $b^2 = c^2 - a^2$. Then the hyperbolic equation is: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. |
| Step 2: Set up the mathematical model | |
| - Find c. | - Two radio stations A and B are the focal points. The distance from the center to either focal point is: $c = \frac{150}{2} = 75 \text{ (km)}$. |
| - Find a. | - The hyperbolic equation that takes the origin (0;0) and the focal points ($\pm 75;0$) has the canonical equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. - The difference between the ship and the two radio stations is: $k = 300\,000(\text{km} / \text{s}) \cdot 0,0003\text{s} = 90 \text{ (km)}$. - Do $ PA - PB = 2a \Rightarrow a = \frac{90}{2} = 45 \text{ (km)}$. |
| - Find b. | - Substitute a and c into the equation $b^2 = a^2 - c^2$, We get $b^2 = 75^2 - 45^2 = 3600$. |
| - Write the hyperbolic equation. | - The canonical equation of the hyperbola is: $\frac{x^2}{45^2} - \frac{y^2}{3600} = 1 \text{ hay } \frac{x^2}{2025} - \frac{y^2}{3600} = 1$. |

3. Experimental organization

3.1. Time and experimental subjects

With the consent of the management board of Phu Tan High School, Phu Tan District, Ca Mau Province, we conducted a pedagogical experiment to test and evaluate the

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research results. We have conducted to find out the actual situation of students and the teaching situation to improve the mathematical modeling capacity of students.

- *Time to conduct the experiment:* From March 15, 2022, to May 23, 2022.
- *Location:* Phu Tan High School, Phu Tan District, Ca Mau Province.
- *Content:* According to the basic curriculum for the academic year 2021 - 2022.
- *Control class:* 10C2, including 40 students. Teacher: Doan Duy Mong. The class is studying according to the Basic textbook program.
- *Experimental class:* 10C3, including 42 students. Teacher: Le Hung Cuong. The class is studying according to the Basic textbook program.
- We design an experimental lesson plan and implement the teaching content of "Equation of a circle", "Exercise for equations of a circle", "Equation of an ellipse" and "exercise on an ellipse equation" for both classes. For the experimental class, the teacher will teach according to lesson plans to improve mathematical modeling capacity to solve practical problems. For the control class, the teacher will teach according to the program distribution.

3.2. Pedagogical experimental process

- The teacher collects the first semester transcripts of the experimental class and the control class to assess the student's learning situation.
- The teacher prepares experimental lesson plans and conducts teaching according to compiled lesson plans.
- The teacher gives the experimental class and the control class a test to analyze and evaluate the experimental results.

3.3. Analysis before conducting experiment

We have collected the learning results of the first term mathematics of the control class and the experimental class as follows:

Table 3.1. Frequency distribution table of average scores of math in the first semester of the school year 2021 - 2022 of two experimental and control classes (Source: Author)

| Control class | | Experimental class | |
|---------------|-----------|--------------------|-----------|
| Scores | Frequency | Scores | Frequency |
| 4.6 | 1 | 4 | 2 |
| 4.8 | 2 | 4.2 | 1 |
| 5 | 1 | 4.4 | 1 |
| 5.3 | 1 | 4.5 | 1 |
| 5.4 | 1 | 4.6 | 1 |
| 5.5 | 2 | 4.8 | 1 |
| 5.6 | 1 | 4.9 | 1 |
| 5.7 | 1 | 5 | 1 |
| 5.8 | 1 | 5.1 | 1 |
| 5.9 | 1 | 5.2 | 2 |
| 6 | 2 | 5.3 | 1 |
| 6.2 | 3 | 5.4 | 2 |
| 6.3 | 2 | 5.5 | 2 |
| 6.4 | 1 | 5.6 | 1 |
| 6.6 | 3 | 5.8 | 3 |
| 6.7 | 2 | 6 | 1 |
| 6.8 | 2 | 6.1 | 1 |
| 6.9 | 2 | 6.2 | 2 |
| 7 | 1 | 6.4 | 2 |
| 7.2 | 2 | 6.5 | 2 |
| 7.3 | 1 | 6.6 | 2 |
| 7.4 | 2 | 6.8 | 2 |
| 7.6 | 1 | 7.2 | 2 |
| 7.8 | 1 | 7.4 | 1 |
| 8.1 | 1 | 7.6 | 1 |
| 8.3 | 1 | 7.7 | 2 |
| 8.6 | 1 | 8 | 1 |
| | | 8.1 | 1 |
| | | 8.4 | 1 |
| Total | 40 | Total | 42 |

Through the frequency distribution table of the average score of math in the first semester and through the teaching practice as well as the teacher interviews, we found that:

- *In the control class:* Students actively participate in constructive speeches, have a spirit of learning, cooperate with each other to find a way to solve a problem, know how to divide the work, prepare new lessons before going to class, have a confident learning attitude, and have a good basic knowledge base.

- *In the experimental class:* Students are still timid, lack confidence, and rarely express their opinions in class. When the teacher gives a problem to solve, the class does not know how to divide the work among members. The basic knowledge base is at a good level.

We conduct a normal distribution test for the sample set which is the average score of the first semester of the experimental class and the control class. As a result, we get the following graph:

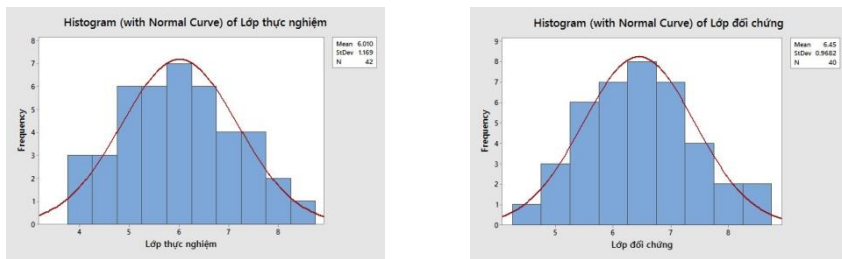


Figure 3.1. Distribution of math scores of the experimental class and the control class in the first semester of the school year 2021 - 2022 (Source: Author)

We notice that the distribution graph is bell-shaped (Figure 3.1). Distributions of this form are called normal distributions. Therefore, we continue to study this sample.

From table 3.1, we obtain the graph of the frequency distribution of math scores of two experimental and control classes in the first semester of the school year 2021 - 2022 as follows:

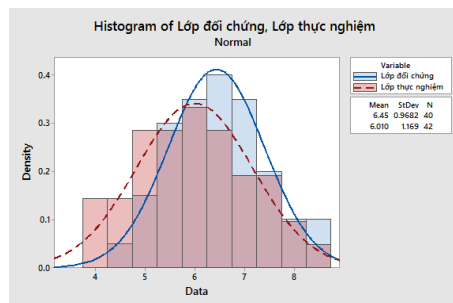


Figure 3.2. The graph shows the frequency distribution of math scores of two experimental and control classes in the first semester of the school year 2021 - 2022 (Source: Author)¹

¹ Lớp đối chứng: Control Class
Lớp thực nghiệm: Experiment Class

From the chart in Figure 3.2, it can be seen that the heights of the score columns and the distribution of points of the two classes are different. The scores of the experimental class are distributed widely in most of the score frames and most of them are in the range of 5.25 - 6.75 points. The score of the control class mainly concentrates from 5.75 to 7.25 points.

| Variable | Mean | StDev | Variance | Minimum | Median | Maximum | Skewness |
|------------------|-------|-------|----------|---------|--------|---------|----------|
| Control Class | 6.450 | 0.968 | 0.937 | 4.600 | 6.500 | 8.600 | 0.12 |
| Experiment Class | 6.010 | 1.169 | 1.366 | 4.000 | 5.900 | 8.400 | 0.20 |

Figure 3.3. The specific parameters of statistics about the average score of math in the first semester of the school year 2021 - 2022 of two classes (Source: Author)

From Figure 3.3, we have:

- The average score in math of the experimental class in the first semester is 6,010 points.
- The average math score of the control class in the first semester is 6,450 points.
- The standard deviation and variance of the experimental class are higher than that of the control class, showing that the dispersion of scores around the mean of the experimental class is higher than that of the control class. We conduct a T-test (table 3.1) to accurately assess the difference or equivalence between the mean scores of the two classes. We test hypothesis H_0 : "The average math score of the first semester of the experimental class and the control class is similar" with the significance level $\alpha = 0,05$.

| | N | Mean | StDev | SE Mean |
|------------------|----|-------|-------|---------|
| Control Class | 40 | 6.450 | 0.968 | 0.15 |
| Experiment Class | 42 | 6.01 | 1.17 | 0.18 |

Difference = μ (control Class) - μ (Experiment Class)
 Estimate for difference: 0.440
 95% lower bound for difference: 0.045
 T-Test of difference = 0 (vs >): T-Value = 1.85 P-Value = 0.034
 Both use Pooled StDev = 1.0756

Figure 3.4. The average T-test of the average math score of the first semester of the school year 2021 - 2022 of the experimental and control classes (Source: Author)

We have the following results: Because $P = 0,034 < \alpha = 0,05$, the hypothesis H_0 is not accepted, therefore, the average score in math in the first semester of the experimental class is lower than that of the control class.

3.4. Analysis of the results after conducting the experiment

Through conducting experimental teaching, monitoring, observing the classroom, and interviewing teachers, we draw the following comments:

3.4.1. Before conducting the experiment:

- Students of both classes have mastered the basics of the topics of circles and ellipses. However, the ability to solve practical problems is still not flexible and limited.
- The experimental class was still timid, not confident, spoke little, gave little constructive ideas, and was not very excited about the lessons. Compared with the control class, students lack initiative and sensitivity in analyzing and solving problems.

3.4.2. While conducting the experiment:

During the experiment, we found:

- ***In the first experimental period:*** Although the students in the experimental class were shy at first, after being clearly analyzed by the teacher every step of the way when solving the problem, the students more actively raised their hands to speak and participated in constructive activities. Compared to before the experiment, we found that the students had a change in their learning style and learning attitude. Compared with the control class, the problem-solving speed is still not equal, but the number of students who can do the test has increased much compared to before.

- ***In the second experimental period:*** Students started to familiarize themselves with the use of the mathematical modeling process to solve real-life problems. Students actively raised their hands to express their opinions, and the results were right or wrong, but the atmosphere in the class changed markedly. After the class, some students spoke and raised questions about the problem to discuss in front of the class. We see the positivity of the class. It can be seen that students are interested in the way of teaching in the direction of developing mathematical modeling competence for students. However, the answer to the question of the actual problem is still incomplete.

- ***In the third experimental period:*** Students actively participated in lesson-constructive activities. Students actively divided study groups, and assign tasks to members to perform assigned tasks. Students are more confident when participating in giving constructive feedback and contributing ideas to find problems when solving problems.

- ***In the fourth experimental period:*** Students were able to use the teaching process oriented to develop mathematical modeling competence to solve practical problems. Students are confident when participating in expressing their opinions about the lesson and answering questions posed to real problems.

3.4.3. After conducting the experiment:

For the control class: Students have basic knowledge of circles and ellipses. Students can write equations of circles and write equations of ellipses in basic forms. They can apply their knowledge of circles and ellipses to solve the requirements of the problem. However, learning the applications of circles and ellipses in real life has not been done, so the answer to the question for the practical problem is still incomplete. Most of the students did not have an answer; only a few had an answer but could not explain the reason.

For the experimental class: After conducting the experiment, through teacher interviews, student interviews, and classroom observations, we found that experimental class students had the following changes:

- Students know the sequence of steps to take when solving a real-world problem by the modeling process: analyzing the real-world problem, setting up a mathematical model, solving a mathematical problem in the model that is set up (if any), and deepening the solution (if any).

- Students actively solve the problem under the guidance of the teacher or without the teacher's suggestion. Students confidently give constructive ideas and participate in discussions with class members and teachers.

- After each lesson, some students themselves found practical problems with the same format as the ones being taught. Some students with good academic performance can find a different solution on their own or develop a new problem for the original problem.

To test the feasibility and evaluate the effectiveness of the cases, we gave the experimental class and the control class a 45-minute test in essay form. The quantitative analysis is based on test results.

Table 3.2. The frequency distribution table of the average scores of the 45-minute test after the experiment of the experimental and control classes (Source: Author)

| Control class | | Experimental class | |
|---------------|-----------|--------------------|-----------|
| Scores | Frequency | Scores | Frequency |
| 4 | 1 | 5 | 2 |
| 4.5 | 1 | 5.5 | 3 |
| 5 | 2 | 6 | 3 |
| 5.5 | 2 | 6.5 | 4 |
| 6 | 3 | 7 | 4 |
| 6.5 | 4 | 7.5 | 6 |
| 7 | 6 | 8 | 6 |
| 7.5 | 6 | 8.5 | 5 |
| 8 | 5 | 9 | 4 |
| 8.5 | 5 | 9.5 | 3 |
| 9 | 3 | 10 | 2 |
| 9.5 | 2 | | |
| Total | 40 | Total | 42 |

We conduct the test of normal distribution for the sample set which is the average score of math in the first semester of the experimental and control classes. As a result, we get the following graph:

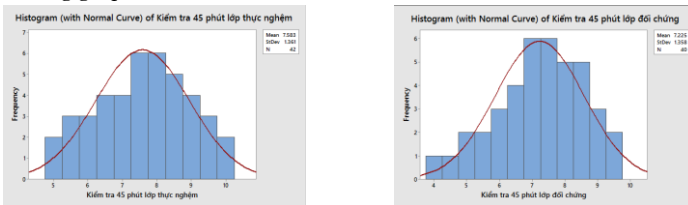


Figure 3.5. Distribution of scores of the 45-minute test after experimenting with two experimental and control classes (Source: Author)

We find that the distribution graph is bell-shaped (Figure 3.5). Distributions of this form are called normal distributions. Therefore, we continue to study this sample.

From Table 3.2, we obtain the graph of the frequency distribution of math scores of two experimental and control classes for the 45-minute test as follows:

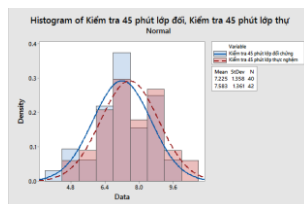


Figure 3.6. The graph shows the frequency distribution of scores of the 45-minute test after the experiment of two experimental and control classes (Source: Author)²

² (Kiểm tra 45 phút lớp đối chứng: The 45-minute test of the control class
Kiểm tra 45 phút lớp thực nghiệm: The 45-minute test of the experimental class)

From the chart (Figure 3.6), it can be seen that the heights of the score columns and the distribution of scores of the two classes do not have much difference. The scores of the experimental class range from 5.0 to 10 points and focus mainly from 7.5 to 8.5 points. The scores of the control class range from 4.0 to 9.5 points and most of them are in the range of 6.5-8.5 points.

| Variable | Mean | StDev | Variance | Minimum | Median | Maximum | Skewness |
|---------------------------|-------|-------|----------|---------|--------|---------|----------|
| Kiểm tra 45 phút lớp đối | 7.225 | 1.358 | 1.846 | 4.000 | 7.500 | 9.500 | -0.43 |
| Kiểm tra 45 phút lớp thực | 7.583 | 1.361 | 1.853 | 5.000 | 7.500 | 10.000 | -0.15 |

Histogram (with Normal Curve) of Kiểm tra 45 phút lớp đối chứng

Histogram (with Normal Curve) of Kiểm tra 45 phút lớp thực nghiệm

Figure 3.7. Statistical-specific parameters of the average score of the 45-minute test after the experiment of two classes (Source: Author)³

From Figure 3.7, we have:

- The average score of the 45-minute test of the experimental class is 7,583 points.
- The average score of the 45-minute test of the control class is 7,225 points.
- Standard deviation and variance of the control class are slightly lower than that of the experimental class, showing that the dispersion of scores around the mean of the control class is higher than that of the experimental class. We conduct a T-test (Figure 3.8) to accurately assess the difference or equivalence between the mean scores of the two classes. We test the following two hypotheses with significance level $\alpha = 0,05$.

H_0 : "The average score of the 45-minute test of the experimental class and the control class is similar".

H_1 : "The average score of the 45-minute test of the experimental class is higher than that of the control class".

| | N | Mean | StDev | SE Mean |
|---------------------------|----|------|-------|---------|
| Kiểm tra 45 phút lớp đối | 40 | 7.22 | 1.36 | 0.21 |
| Kiểm tra 45 phút lớp thực | 42 | 7.58 | 1.36 | 0.21 |

Difference = μ (Kiểm tra 45 phút lớp đối chứng) - μ (Kiểm tra 45 phút lớp thực nghiệm)
 Estimate for difference: -0.358
 95% lower bound for difference: -0.858
 T-Test of difference = 0 (vs >): T-Value = -1.19 P-Value = 0.882 DF = 80
 Both use Pooled StDev = 1.3598

Figure 3.8. T-test of average scores of 45-minute test of two experimental and control classes in Minitab (Source: Author)

We have the following result:

- Since the variances of the two classes have a difference but are not significant, they are considered equivalent, so the T-test results should be used for the case where the variances of the two classes are the same.

³ (Kiểm tra 45 phút lớp đối chứng: The 45-minute test of the control class
 Kiểm tra 45 phút lớp thực nghiệm: The 45-minute test of the experimental class)

- Testing 2 T-test samples, we have $P_Value = 0,882 > \alpha = 0,05$, so we accept hypothesis H_0 , reject hypothesis H_1 . The mean score of the experimental class is similar to the control class at the 5% level of significance.

Thus, by the test method between classes with lower academic ability, we found that in the experimental class after being taught with the experimental lesson plan to develop mathematical modeling ability, the test results are better and students' performance in the learning process has changed in a positive direction. It can be seen that the experimental situations applied to the experimental class are completely feasible and achieve certain effectiveness in teaching.

4. CONCLUSION

Teaching to foster mathematical modeling competence through teaching to identify mathematical models for real-life problems of circles and three conic lines in grade 10 is an appropriate capacity development-oriented teaching method with current teaching trends. The article has given some views on teaching mathematical modeling, the teaching process to foster mathematical modeling competence as well as how to teach in practice about fostering mathematical modeling competence. In particular, the process of pedagogical experimentation shows the effectiveness of teaching to foster mathematical modeling competence. Students showed interest in the real problems that the teacher gave. Students showed enthusiasm to express their opinions and complete their homework. The quantitative evaluation shows that the experimental class is more effective than the control class. Although before the experiment, the ground of the control class proved to be slightly better than that of the experimental class, after the experiment, the quality of the experimental class was better.

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