EUROPEAN ACADEMIC RESEARCH
Vol. X, Issue 11/ February 2023

Impact Factor: 3.4546 (UIF)
DRJI Value: 5.9 (B+)

ISSN 2286-4822
www.euacademic.org

Production Inventory Model of Delayed Deteriorating Items under Trade Credit

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Abstract

This study analyzes the optimal production model for delayed deteriorating items. The study employed an analytical method to minimize total variable cost method for efficiency and also to determine best circle length for the purpose of the inventory. It is indicated that there were no shortages and production level is greater than the demand level. Hence, number example was utilized to show the application of the model. However, sensitivity examination was practically done to check the sensitivity of some system parameter of the model.

Keywords: Holding cost, Demand level, Production level, Trade credit, Delayed deterioration

1. INTRODUCTION

Generally, inventory refers to any item or property held in stock by a firm or any organization for smooth and functional running of business. Based on the assumption of traditional inventory model, items preserved their physical features in production center or kept in inventory. These assumptions are not generally true because items will continuous to lose as a result of damage, decay or spoilage. The phenomenon we called deterioration. Based on this, controlling and maintaining inventory of deteriorating items becomes a challenging problem for decision makers.

For several years, the control and maintenance of inventories of deteriorating items have attracted the attentions of many researchers. It all started with Ghare and Schrader [1] who developed a model of exponentially decaying inventory, with a constant rate of deterioration. After Ghare and Schrader [1], several reviews and models on deteriorating inventory has been developed. For example, Shah and Jaizwal [2] came up with an order level inventory model for a system with constant rate of deterioration. Aggarwal and Jaggi [3] extended the work of Shah and Jaizwal [2] by providing a note on an order level inventory model for a system with a constant rate of deterioration. Padmanabahna and Vrat provides an EOQ model for perishable items under stock dependent selling rate. The literatures mentioned above considered constant deterioration rates in their models. However, several researchers considered several scenarios of relationship between time and deteriorating rates. Some of these scenarios include: deteriorating rate as linear function of time, two-parameter Weibull distribution, three- parameter Weibull distribution and or deterioration rate as other function of time. Geol and Aggwarl [4] were the earlier researchers to consider Weibull deteriorating rate in their inventory models. Their research focused on maximization of profit rate function using the three parameter Weibull density function. Wu *et al.* [5] proposed an EOQ model for inventory of an item that deteriorates at a Weibull distribution rate where demand rate is a continuous function of time. In this model, the inventory started with an instant replenishment and ended with shortages. Wu [6] reconsidered the work of Wu *et al.* [5] by considering the model to start with zero inventories and ended without shortages.

In the above literature, the practitioners ignore or give little attention to items in the production centers. In the production circle, the production rate remains the primary key of production. Many production models considers constant production rate. A breakthrough to the production model of deteriorating items is that of [7] who came up with the economic production quantity (EPQ) model with probabilistic deteriorating rate. Kaliraman *et al.* [8] came up with an EPQ inventory model of deteriorating items with Weibull deterioration under stock dependent demand. Sakar [9] provided a production inventory model with probabilistic deterioration in two echelon supply chain management. Again, Sakar [10] came up with an economic manufacturing model with probabilistic deterioration in a production system.

The present study proposed a delayed (Non-instantaneous) deteriorating items production model. The model illustrates deteriorating level; holding cost and demand are regarded as stagnant fraction of the production items as well as the absence of shortages.

2. ASSUMPTIONS AND NOTATIONS

The model is built up based on the following assumptions and notations

Assumptions

(i) Production rate is constant
(ii) Demand rate is constant
(iii) Deterioration rate is constant
(iv) Shortages is not allowed
(v) Lead time is zero
(vi) There is no repair or replacement of items that deteriorate during a cycle.
(vii) The deterioration occurs only when the item are effectively in stock.

Notations

(i) α Is the production rate
(ii) d Is the demand rate
(iii) ϑ is the deterioration rate
(iv) T_1 It is s the time the production stops and deterioration begins.
(v) T The length of time for the cycle
(vi) A_0 It is the set up cost
(vii) C_p It is the unit production cost of deteriorating items
$(viii)$ $I_1(t)$ Inventory level at any time t during the production period.
(ix) $I_2(t)$ Inventory level at any time t during the deterioration period.
(x) $TVC(T)$ It is the total variable cost per unit time.
(xi) I_e It is the interest that can be earned per naira investment in a year
(xii) m It is the permissible delay in payment (Trade credit)
$(xiii)$ E_1 It is the interest earned from the accrued sales
(xiv) I_p It is the interest paid per Naira investment in stock per year

(xv) S Unit selling price of the item (In Naira)
(xvi) C Unit cost of item (In Naira)

3. DIAGRAMMATICAL REPRESENTATION OF THE MODEL

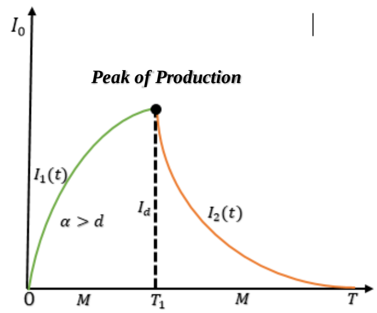

Figure I: Inventory movement in a delayed deteriorating production system

The production level during the production period $[0, T_1]$ is a function α (production rate) and d (demand rate)the depletion of the inventory during the period $[T_1, T]$ is function of deteriorating rate $\boldsymbol{\vartheta}$ and the constant demand rate d. The equation below, is the differential equation governing the inventory system:

$$\frac{dI_1(t)}{dt} = \alpha - d, \qquad 0 \le t \le T_1 \qquad (1)$$

$with\ the\ boundary\ conditions\ t = 0,\ I_1(t) = I_0,\ at\ t = T_1,\ I_1(t) = I_d$

$$\frac{dI_2(t)}{dt} + \boldsymbol{\vartheta} I_2(t) = -d, \qquad T_1 \le t \le T \qquad (2)$$

$with\ the\ boundary\ conditions\ t = T_1,\ I_2(t) = I_d.\quad at\ t = T,\ I_2(t) = 0$

We solve equation (24) using separation of variables

$\int dI_1(t)dt = (\alpha - d)\int dt$

$$\Longrightarrow I_1(t) = (\alpha - d)t + C_1 \qquad (3)$$

$at\ t = 0,\ I_1(t) = I_0$, equation (26) becomes:

$I_0 = C_1$

Substituting $I_0 = C_1$ into equation (3) gives;

$$I_1(t) = (\alpha - d)t + I_0 \qquad (4)$$

$at\ t = T, I_1(t) = I_d$

$$\therefore I_d = (\alpha - d)T + I_0 \qquad (5)$$

$$\Longrightarrow I_0 = I_d - (\alpha - d)T \qquad (6)$$

Using a suitable integrating factor $I = e^{\int \vartheta dt}$ the solution of equation (2) is obtained Thus;

$I = e^{\vartheta t}$

$I_2(t) = e^{-\vartheta t}(\frac{-d}{\vartheta} e^{\vartheta t} + C_2)$

$= \frac{-d}{\vartheta} + C_2 e^{-\vartheta t}$

$$I_2(t) = \frac{-d}{\vartheta} + C_2 e^{-\vartheta t} \qquad (7)$$

$at\ t = T_1, I_2(t) = I_d$

$\therefore I_d = \frac{-d}{\vartheta} + C_2 e^{-\vartheta T_1}$

$$\Longrightarrow C_2 = e^{\vartheta T_1}\left(I_d + \frac{d}{\vartheta}\right) \qquad (8)$$

Substituting C_2 in equation (7) yields,

$$I_2(t) = -\frac{d}{\vartheta}+e^{\vartheta(T_1-t)}(I_d + \frac{d}{\vartheta}) \tag{9}$$

$at\ t = T,\ I_2(t) = 0.$

We have from equation (9),

$$-\frac{d}{\vartheta}+e^{\vartheta(T_1-T)}(I_d + \frac{d}{\vartheta}) = 0$$

$$\Rightarrow I_d = \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1-T)}) \tag{10}$$

Substituting equation (10) into (6), we get

$$I_0 = \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1-T)}) - (\alpha - d)T_1 \tag{11}$$

Substituting equation (11) into (4) we have,

$$I_1(t) = (\alpha - d)(t - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1-T)}) \tag{12}$$

Substituting equation (10) into (9), we get final expression for $I_2(t)$ as,

$$I_2(t) = -\frac{d}{\vartheta}+e^{\vartheta(T_1-t)}(\frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1-T)}) + \frac{d}{\vartheta})$$

$$\Rightarrow I_2(t) = \frac{d}{\vartheta}(-1 + e^{\vartheta(T-t)}) \tag{13}$$

In order to satisfy the various position of the permissible delay in payment 'm' the following cases are considered.

$case\ 1\text{:}\ 0 \leq m \leq T_1 < T$

Here, the customer is given the permissible delay from the beginning of the cycle up to the period of permissible delay, which is in the interval [0, T_1]. Beyond this interest is paid and will continue to be paid up to the end of the cycle.

3.2.1 Computation of the interest payable.

In this case the interest payable, p is computed thus:

$$P = cI_p \int_m^{T_1} I_1(t)dt + cI_p \int_{T_1}^{T} I_2(t)dt \qquad 0 \leq m \leq T_1 < T \tag{14}$$

$$P = cI_p \int_m^{T_1}((\alpha - d)(t - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1-T)}))dt + cI_p \int_{T_1}^{T} \left(\frac{d}{\vartheta}\left(\begin{matrix}-1 + \\ e^{\vartheta(T-t)}\end{matrix}\right)\right) dt \tag{15}$$

Integrating (15) yields

$$P = cI_p\left(\left((\alpha - d)(t - T_1)T_1 - m(m - T_1) + \frac{d}{\vartheta}\left(-1 + e^{-\vartheta(T_1-T)}\right)(T_1 - m)\right) - \frac{d}{\vartheta}((T_1 - T) - \frac{1}{\vartheta}(1 - e^{\vartheta(T-T_1)})\right) \tag{16}$$

3.2.2 Computation of the interest earned per cycle.

The interest earned per cycle E_1 is given by

$$E_1 = I_e \int_0^m (td)dt \tag{17}$$

Integrating (17) with respect to t yields.

$$E_1 = I_e\left(\frac{dm^2}{2}\right) \tag{18}$$

The total demand over deteriorating period $[T_1, T]$ is denoted by D_T

$$D_T = d(T - T_1). \tag{19}$$

The amount of item that deteriorate over the period $[T_1, T]$ is denoted by A_d

$$A_d = I_d - D_T$$

$$\Rightarrow A_d = d(\frac{1}{\vartheta}\left(-1 + e^{\vartheta(T-T_1)}\right) - (T_1 - T)) \tag{20}$$

The total cost of items that deteriorate over the period $[T_1, T]$ is given by

$$D_c = CA_d$$

$$\Rightarrow D_c = Cd(\frac{1}{\vartheta}\left(-1 + e^{\vartheta(T-T_1)}\right) - (T_1 - T)) \tag{21}$$

3.2.3 Computation of the inventory holding cost

The inventory holding cost H_{c1} is computed thus:

$H_c = i\int_0^{T_1} I_1(t)dt$

$$H_c = h\int_0^{T_1}(\alpha - d)(t - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})dt \tag{22}$$

Integrating equation (45) gives:

$$H_c = hT_1\left((\alpha - d)(t - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})\right) \tag{23}$$

3.2.4 Computation of total inventory cost

The total variable cost TV$C_1$is computed as follows:

TVC_1= inventory set up cost + inventory holding cost + total cost of items that deteriorated + interest payable - interest earned per cycle.

$TVC_1 = A_0 + H_c + D_c + P - E_1$

Thus;

$$TVC_1 = A_0 + iT_1\left((\alpha - d)(t - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})\right) + Cd\left(\frac{1}{\vartheta}(-1 + e^{\vartheta(T - T_1)}) - (T_1 - T)\right) + cI_p\left(\left((\alpha - d)(t - T_1)T_1 - m(m - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})(T_1 - m)\right) - \frac{d}{\vartheta}((T_1 - T) - \frac{1}{\vartheta}(1 - e^{\vartheta(T - T_1)})\right) - SI_e\left(\frac{dm^2}{2}\right) \tag{25}$$

The total inventory cost per unit time $TVC(T)$ is given as:

$$\boldsymbol{TV}C_1(\boldsymbol{T}) = \frac{A_0}{T} + \frac{iT_1\left((\alpha - d)(T - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})\right)}{T} + \frac{Cd}{T}\left(\frac{1}{\vartheta}(-1 + e^{\vartheta(T - T_1)}) - (T_1 - T)\right) + \frac{cI_p}{T}\left(\left((\alpha - d)(T - T_1)T_1 - m(m - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})(T_1 - m)\right) - \frac{d}{\vartheta}((T_1 - T) - \frac{1}{\vartheta}(1 - e^{\vartheta(T - T_1)})\right) - \frac{SI_e}{T}\left(\frac{dm^2}{2}\right) \tag{26}$$

Differentiating equation (48) with respect to T and multiplying the result through by T^2 and equating to zero yields;

$$A_0 + i\left((\alpha - d)(t - T_1) - \frac{d}{\vartheta}\left(1 + (1 - \frac{T}{\vartheta})e^{-\vartheta(T_1 - T)}\right)\right) + Cd\left(\left(1 + (1 - \frac{T}{\vartheta})e^{\vartheta(T - T_1)}\right) - T_1\right) + cI_p\left((\alpha - d)\left(\frac{T_1(T_1 - 2T) + -m(m - 2T)}{2}\right) + \left((T_1 - m) - \frac{T}{2\vartheta}\right)e^{2\vartheta T} + \frac{d}{\vartheta}\left(\frac{T^2}{2\vartheta} - T_1\left(1 - \frac{T}{2\vartheta}\right) - (2\vartheta - T)e^{2\vartheta T} - \frac{1}{2\vartheta}e^{2\vartheta T_1}\right)\right) + SI_e\left(\frac{dm^2}{2}\right) = 0 \tag{27}$$

The solution of equation (26) gives us the optimal cycle length $T = T^*$ The optimal total variable cost per unit time for the inventory system is obtained from equation (27) with $T = T^*$.

Case 2: $\boldsymbol{T_1 \leq m < T}$

In this case the permissible delay starts at the onset of deterioration and up to the allowed period, m but no up to the end of the cycle, T. the permissible period falls within the period [T_1,T]

3.2.5 The Interest payable P is given below as

$$P = cI_p\int_m^T I_2(t)dt \tag{28}$$

$$P = cI_p\int_m^T\left(\frac{d}{\vartheta}(-1 + e^{\vartheta(T - t)})\right) dt \tag{29}$$

Integrating equation (51) gives

$$P = \frac{cI_p d}{\vartheta}\left((T - m) + \frac{1}{\vartheta}(1 - e^{\vartheta(T - m)})\right) \tag{30}$$

3.2.6 Computation of the interest earned per cycle

The interest earned per cycle E_1 is computed as:

$$E_1 = SI_e \int_0^{T_1}(td)dt + SI_e \int_{T_1}^{m}(td)dt = SI_e\left(\frac{dm^2}{2}\right) \quad (31)$$

3.2.7 Computation of the inventory holding cost

The inventory holding cost is given by:

$$H_c = i\int_0^{T_1} I_1(t)dt + i\int_{T_1}^{T} I_2(t)dt$$

$$= i\int_0^{T_1} (\alpha - d)(t - T_1) + \frac{d}{\vartheta}(-1 + e^{-\vartheta(T_1 - T)})\, dt + i\int_{T_1}^{T}\frac{d}{\vartheta}(-1 + e^{\vartheta(T-t)})dt$$

$$H_c = i\left(\left(\frac{T_1 d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(\left(-T_1 - \frac{1}{\vartheta}e^{\vartheta(T-T_1)}\right) + \frac{1}{\vartheta}e^{\vartheta T}\right)\right) \quad (32)$$

3.2.8 Computation of the total cost of deteriorated items

The total demand over deteriorating period $[T_1, T]$ is denoted by D_T

$$D_T = d(T - T_1). \quad (33)$$

The amount of item that deteriorate over the period$[T_1, T]$ is denoted by A_d

$$A_d = I_d - D_T$$

$$\Rightarrow A_d = \frac{d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - d(T - T_1) \quad (34)$$

The total cost of items that deteriorate over the period $[T_1, T]$ is given by

$$D_c = CA_d$$

$$\Rightarrow D_c = C\left[\frac{d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - d(T - T_1)\right] \quad (35)$$

3.2.9 Computation of total inventory cost.

The Total variable cost TVC_2 is given by the expression,

TVC_2= inventory set up cost + inventory holding cost + total cost of items that deteriorated + interest payable - interest earned per cycle.

$$TVC_2 = A_0 + H_c + D_c + P - E_1$$

Thus;

$$TVC_2 = A_0 + i\left(\left(\frac{T_1 d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(\left(-T_1 - \frac{1}{\vartheta}e^{\vartheta(T-T_1)}\right) + \frac{1}{\vartheta}e^{\vartheta T}\right)\right) + C\left[\frac{d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - d(T - T_1)\right] + \frac{cI_p d}{\vartheta}\left((T - m) + \frac{1}{\vartheta}(1 - e^{\vartheta(T-m)})\right) - SI_e\left(\frac{dm^2}{2}\right) \quad (36)$$

The total inventory cost per unit time $TVC_2(T)$ is given as:

$$TVC_2(\boldsymbol{T}) = \frac{A_0}{T} + \frac{i}{T}\left(\left(\frac{T_1 d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(-T_1 - \frac{1}{\vartheta}e^{\vartheta(T-T_1)} + \frac{1}{\vartheta}e^{\vartheta T}\right)\right) + \frac{C}{T}\left[\frac{d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - d(T - T_1)\right] + \frac{cI_p d}{T\vartheta}\left((T - m) + \frac{1}{\vartheta}(1 - e^{\vartheta(T-m)})\right) - \frac{SI_e}{T}\left(\frac{dm^2}{2}\right) \quad (37)$$

Differentiating equation (37) with respect to T and multiplying the result through by T^2 and equating to zero yields;

$$A_0 - i\left(\frac{T_1 d}{\vartheta}\left(1 + \left(1 - \frac{T}{\vartheta}\right)e^{-\vartheta(T_1 - T)} + \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(T_1 - (\vartheta + T)(e^{\vartheta(T-T_1)} + e^{\vartheta T}\right)\right) - \frac{cd}{\vartheta}\left((1 + d) + \left(1 - \frac{T}{\vartheta}\right)e^{-\vartheta(T_1 - T)}\right) + \frac{cI_p d}{\vartheta}\left(m - \frac{1}{\vartheta}\left(1 + \left(1 - \frac{T}{\vartheta}\right)e^{\vartheta(T-m)}\right)\right) + SI_e\left(\frac{dm^2}{2}\right) = 0 \quad (38)$$

3.2.10 Computation of the EPQ

The economic production quantity (EPQ) is computed thus:

The Economic production Quantity EPQ = Total demand before deterioration + total demand after deterioration + total items that deteriorated.

$$\text{EPQ} = dT + d(T - T_1) + \frac{d}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right) - d(T - T_1)$$

$$= d\left[T + \frac{1}{\vartheta}\left(-1 + e^{-\vartheta(T_1 - T)}\right)\right].$$

4. NUMERICAL EXAMPLES

The following ten numerical examples with different parameter values. The output obtained (using Maple 2017 Mathematical software) gives the optimal cycle length$T = T^*$, and the optimal production quantity EPQ.

Table 3: EPQ, Optimal cycle length and total inventory or variable cost per unit time for production model with constant production rate under permissible delay in payment.

S/N	A_0	C	Θ	d	M	α	T_1	I_p	I_e	TVC_1	TVC_2	T	EPQ
1	1000	100	.05	500	.24	500	.003	.28	.5	71207	90252	299	928
2	1500	100	.05	600	.24	800	.003	.28	.5	79913	108895	321	271
3	1500	100	.03	300	.24	700	.003	.018	.6	112659	56476	236	478
4	7000	100	0.2	500	0.16	600	0.03	0.05	.07	584205	117810	141	397
5	7000	100	0.1	200	.12	250	.02	.15	.35	102534	77696	65	72
6	1000	150	0.1	200	.12	250	.025	.25	.45	167,361	38296	60	228
7	5000	150	.09	450	.12	500	.02	.35	.45	400466	165258	53	297
8	5000	150	.08	800	.02	950	.025	.25	.45	445366	229479	43	92
9	5000	100	.07	800	.02	850	.02	.45	0.5	762917	224804	26	212
10	1000	100	.06	800	.02	850	0.3	0.5	.75	186238	210895	13	945

4.1 Sensitivity analysis

The result of the sensitivity analysis carried out to test the effects of changes in the parameter values on the cycle length T, the total variable cost per unit time, $TVC_1(t)$, $TVC_2(t)$, the economic production quantity EPQ is given in table 4 below.

Table 4: Result of sensitivity analysis on some selected parameter values

S/N	Changing parameter	%	T	TVC_1	TVC_2	EPQ
01.	A_0	-50	0.9755	844213	57037	412
		-25	0.9755	844514	57535	412
		+25	0.9755	845116	58137	412
		+50	0.9755	845416	58438	412
02.	m	-50	0.9755	883668	53037	412
		-25	0.9755	864242	57456	412
		+25	0.9755	825386	58178	412
		+50	0.9755	805956	58482	412
03.	ϑ	-50	0.9730	984226	504788	401
		-25	0.9742	666310	332642	406
		+25	0.9768	412410	198807	417
		+50	0.9780	349119	165937	423
04.	d	-50	0.9755	423210	29519	206
		-25	0.9755	634013	43678	309
		+25	0.9755	1055617	71994	515
		+50	0.9755	1266419	86153	618
05.	i	-50	0.9755	507531	245561	412
		-25	0.9755	507531	247066	412
		+25	0.9755	507531	250077	412
		+50	0.9755	507531	251582	412

4.2 Discussion on the result of sensitivity analysis

Here we study the effects of changes in the system parameters α, h, d, m, ϑ, A_0 . The results of the sensitivity analysis are obtained by changing each of the parameter values by +50%, +25%, -25% and -50%, taking one parameter at a time and keeping the remaining parameters unchanged.

Under listed are the observed effects of changes in the parameter values on the cycle length 'T', the inventory or variable cost per unit time $TVC_1(t)$ (in case 1), $TVC_2(t)$ (in case 2) and the Economic Production Quantity EPQ.

(a) Increase in the parameter m will result in an decrease of TVC_1 and increase of TVC_2 & has no effect on T & EPQ.

(b) Increase in the parameter ϑ will result in an decrease of TVC_1, TVC_2 & increase of EPQ.

(c) Increase in the parameter d will result in an increase of TVC_1, TVC_2 & EPQ.

(d) Decrease in the parameter A_0 will result in increase of TVC_1, TVC_2. But it has no impact on EPQ & T.

(e) Increase in the parameter i has less or no impact T, TVC_1 & EPQ.

The conclusion from the above discussions on the sensitivity analysis on the model developed inn shows that the changes in the parameter values 'ϑ, m and d' have effects significantly on the model, whereas percentage increase or decrease in the parameter 'i' has less or no effect on the model.

5. CONCLUSION

In this study derived an inventory production model for items that exhibit delay in deterioration under Trade credit (permissible delay in payment). Both the production rate and demand rate were taken as constant, shortages were not allowed in the model and the optimal solution of total cycle length and EPQ were obtained. The outcome of the result reveals that the higher the demand rate the higher the EPQ. This is reasonable as the production must rise with rise in the demand to avoid under stocking which will result in the shortage cost. The model can be extended by considering shortages and backordering.

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