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Production Inventory Model for Non-Instantaneous Deteriorating Items with Constant Demand

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Abstract

This study developed optional delayed model for deteriorating items. The outcome shows that deterioration and production rates are constant in the event of high production with less demand rate as well as no shortages are allow at all. The study employed analytical method to reduce total variable cost of the organization in order to realize the finest inventory cycle length. Moreover, number illustrations were used for the function of the model development.

Keywords: Production Inventory Model, Non-Instantaneous Deteriorating Items

1. INTRODUCTION

For many decades, inventory of deteriorating items with regard to control and maintenance have been intensified and gain more emphasis by many researchers. It is originated from the work from of Ghare and Schrader [1], the study concerned with decaying stock model with a constant rate of deterioration. Hence, from this work several studies have critically analyzed deteriorating inventory models. For instance, Shah and Jaizwal [2] introduced order level inventory model with an up graded system of stable rate of fall. Similarly, Aggarwal [3] expanded the study by Shah and Jaiswal [2] that create an advancement on inventory model for a designed system. Nonetheless, many studies investigate the linkage between time and deteriorating rates. These may consist of declining rate of a normal function of time, deteriorating rate and other function of time. Maximization of profit rate, continuous demand rate function of time [Geol and Aggarwal 4; Wu *et al.*5; Wu 6].

Based on the reviewed literature, several have not put much emphasis on items in the production centers. However, Goyal and Giri [7] came up with initiatives on production inventories with time varying demand, production and deteriorating rates. Moreover, Kaliraman *et a*l. [8] introduced his new EPQ model of deteriorating time that has Weibull with in stock demand. Similarly, Sakar [10] studied probabilistic deteriorating model of economic production system. Palanivel and uthayakumar [11] proposed their initiatives on items deteriorating model with overseen production and holding cost, time as well as partial backlogging with in inflation period. Karthikeyan and Viji [13] investigates three level deteriorating model of economic production inventory item.

Recently, many studies were conducted on non-instantaneous inventory models. For instance, [Ahmad and Musa 16; Sani and Yakubu 17; Musa and Adam 18] developed their models on EOQ deteriorating items inventory consisting developed deterioration with discrete time, policies on delayed deteriorating stuff coming on linear trend of demand as well as shorfall. In this regard, the present study investigates a non-instantaneous deterioration items model with falling cost and steady production stuff with no shortages at all.

2. CHARACTERISTICS

This model is initiated on the below basic nature and notations

Characteristics

(i) Production rate is constant
(ii) Demand rate is constant
(iii) Deterioration rate is constant
(iv) Shortages is not allowed
(v) Pilot time is 0
(vi) No refurbish or alternate of stuff that reduced at a cycle.

Notations

$(i)\alpha$ = production rate
(ii) d = demand rate
$(iii)\vartheta$ = deterioration rate
(iv) T_1 = stop time of production stops and the beginning of falling.
(v) T = the cycle length time
$(vi)A_0$ = start-up cost
$(vii)c$= unit cost of production
$(viii)I_1(t)$= time on production.
$(ix)I_2(t)$= time on deterioration.
$(x)TVC(T)$ = total variable cost / unit time
$(xi)h$ = inventory charge.

3. Start-up and the model Solution

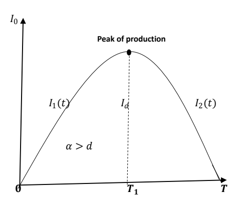

Figure 1: movement of inventory on delayed failing production structure

Level of production on time $[0, T_1]$ = production rate function$\alpha$and rate of demand d. Depletion of the inventory on time$[T_1, T]$is termed as deteriorating function rate$\boldsymbol{\vartheta}$and fixed demand rate d. Moreover, differential equations for inventory system are stated as follow:

$$\frac{dI_1}{dt} = \alpha - d, \qquad 0 \le t \le T_1 \qquad (1)$$

$$At\ t = 0,\ I_1(t) = I_0,\ at\ t = T_1,\ I_1(t) = I_d$$

$$\frac{dI_2}{dt} + \boldsymbol{\vartheta} I_2 = -d, T_1 \le t \le T \qquad (2)$$

$$\frac{dI_1(t)}{dt} = \alpha - d$$

$$\int dI_1(t)dt = (\alpha - d)\int dt$$

$\Longrightarrow I_1(t) = (\alpha - d)t + C_1$ (3)
$at\ t = 0, I_1(t) = I_0.$

$$\therefore I_0 = C_1$$

Substituting for $I_0 = C_1$ in (3),
$I_1(t) = (\alpha - d)t + I_0$ (4) $at\ t = T, I_1(t) = I_d$
$\therefore I_d = (\alpha - d)T + I_0$ (5)
$\Longrightarrow I_0 = I_d - (\alpha - d)T$ (6)
From (2)
$\frac{dI_2(t)}{dt} + \vartheta I_2(t) = -d$
Using integrating factor
$I_2(t) = \frac{-d}{\vartheta}e^{2\vartheta t} + C_2$ (7)
$at\ t = T_1, I_2(t) = I_d$
$\therefore I_d = -\frac{d}{\vartheta}e^{2\vartheta T_1} + C_2$
$\Longrightarrow C_2 = I_d + \frac{d}{\vartheta}e^{2\vartheta T_1}$ (8)
Substituting for C_2 in (7)
$I_2(t) = -\frac{d}{\vartheta}e^{2\vartheta t} + I_d + \frac{d}{\vartheta}e^{2\vartheta T_1}$ (9)
$at\ t = T,\ I_2(t) = 0.$
From (9),
$-\frac{d}{\vartheta}e^{2\vartheta T} + I_d + \frac{d}{\vartheta}e^{2\vartheta T_1} = 0$
$\Longrightarrow I_d = \frac{d}{\vartheta}(e^{2\vartheta T} - e^{2\vartheta T_1})$ (10)
Substituting (10) in (6)
$I_0 = \frac{d}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (\alpha - d)T$ (11)
Substitute (11) into (4)
$I_1(t) = (\alpha - d)(t - T) + \frac{d}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right)$ (12)
Substitute (10) into (9) for I_2
$I_2(t) = \frac{d}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta t}\right)$ (13)
TD on deteriorating time $[T_1, T]$ is illustrated by D_T
$D_T = d(T - T_1).$ (14)
$[T_1, T]$ is illustrated by A_d
$A_d = I_d - D_T$
$\Longrightarrow A_d = d\left[\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right]$ (15) TC of the deteriorate stuff over time $[T_1, T]$ is
$D_c = CA_d$
$\Longrightarrow D_c = Cd\left[\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right]$ (16)

Holding cost of inventory is express as:

$H_c = \int_o^{T_1} hI_1(t)dt + \int_{T_1}^{T} hI_2(t)dt$
$= h\left[\left(\frac{Td}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - \frac{T_1^2(\alpha - d)}{2}\right) + \frac{d}{\vartheta}\left(e^{2\vartheta T}\left(T - T_1 - \frac{1}{2\vartheta}\right) + \frac{1}{2\vartheta}e^{2\vartheta T_1}\right)\right]$ (17)

Therefore;

$$TVC = A_0 + h\left[\left(\frac{Td}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - \frac{T_1^2(\alpha-d)}{2}\right) + \frac{d}{\vartheta}\left(e^{2\vartheta T}\left(T - T_1 - \frac{1}{2\vartheta}\right) + \frac{1}{2\vartheta}e^{2\vartheta T_1}\right)\right] + Cd\left[\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right] \quad (18)$$

$$TVC(T) = \frac{A_0}{T} + \frac{h}{T}\left[\left(\frac{Td}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - \frac{T_1^2(\alpha-d)}{2}\right) + \frac{d}{\vartheta}\left(e^{2\vartheta T}\left(T - T_1 - \frac{1}{2\vartheta}\right) + \frac{1}{2\vartheta}e^{2\vartheta T_1}\right)\right] + \frac{cd}{T}\left[\frac{1}{\vartheta}\left(e^{2\vartheta T} - e^{2\vartheta T_1}\right) - (T - T_1)\right] \quad (19)$$

Differentiating (19) with T we get,

$$\frac{dTVC(T)}{dT} = \frac{A_0}{T^2} + \frac{h}{\vartheta T^2}\left(dT_1 + \frac{d}{2\vartheta^2} - 2e^{-2\vartheta T_1}\right) + \frac{hd}{\vartheta^3}e^{2\vartheta T} - \frac{h(\alpha-d)}{2} + \frac{1}{\vartheta T^2}\left(e^{2\vartheta T}(cd\vartheta - Tcd) + e^{2\vartheta T_1}(cd) - cT_1 d\vartheta\right) \quad (20)$$

Multiplying (20) through by T^2 and equate to zero yields;

$$A_0 + \frac{h}{\vartheta}\left(dT_1 + \frac{d}{2\vartheta^2} - 2e^{-2\vartheta T_1}\right) + \left(\frac{hd}{\vartheta^3}e^{2\vartheta T} - \frac{h(\alpha-d)}{2}\right)T^2 + \frac{1}{\vartheta}\left(e^{2\vartheta T}(cd\vartheta - Tcd) + cde^{2\vartheta T_1} - cdT_1\vartheta\right) = 0 \quad (21)$$

EPQ = sum of demand before declining + sum of demand after declining + sum of stuffs that declined.

. (22)

4. NUMERICAL EXAMPLES

Below are five number samples with dissimilar factor values. The result found that the finest cycle length $T = T^*$, $TVC(T)$ and EPQ.

Table 1: EPQ, TVC and OCL of the production model

S/N	A_0	C	ϑ	d	h	α	T_1	$T(days)$	TVC	EPQ
1	100	500	0.30	250	0.06	100	0.0023	324	20,510	808
2	500	100	0.50	250	0.02	200	0.005	331	56,935	965
3	300	500	0.40	150	0.02	150	0.002	325	14,401	523
4	1000	500	0.15	300	0.01	500	0.003	333	19,553	902
5	700	100	0.75	500	0.01	500	0.003	358	17,892	2,731

5. CONCLUSION

The study came up with production inventory items model that has delayed deterioration system with constant rate of production and demand as well as no shortages at all. The model obtained finest solution for total variable cost and EPQ. The result of the study shows that higher demand rate lead to higher EPQ. This become essential for the rise in production that came with increase in demand rate which avoid stocking that lead to shortage cost and backordering.

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