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Finite Element Simulation of Simple Bending Problem and Code Development in C++

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Abstract:

In this study, the simulation of simple bending problem is performed using an example of cantilever beam which is an important structural member. The author executed numerical simulations for simple and cracked cantilever beams using finite element stiffness method, analytical beam theory, finite element package (ANSYS) and then verified results with code generated in C++ language. In conclusions, the comparison of results is tabulated and the graphs suggested that the finite element method give less deflection as compared with the results obtained using beam theory for any specific location along span length. MD-Solid software is used to draw bending moment and shear force diagram.

Key words: Cantilever Beam, Crack UDL, Beam Theory, FEM Simulation, C++ , coating, Plasma

1 Introduction

Cantilever beams are fixed from one end. They carry loads to the fixed support where shear stresses, deflections and bending moments are opposed. (Hool and Johnson 1920) Cantilever beams are used in manufacturing of micro and macro overhanging structures e.g. trusses, slabs balconies and bridges.

Cracks have unfavorable and negative effects that might appear on a structure during its utilization. They should be detected as soon as possible, as they may significantly decrease the global stiffness over the local reduction of flexural rigidity. (Skrinar and Plibersk 2008) Since in inverse identification of crack only a limited amount of measured data is available, it is thus usually not possible to obtain the information of crack directly but as a result of some gradual and systematic computational model modifications. In order to make the computational model more adaptable and a faster analysis process, it is reasonable to neglect all the irrelevant information from the computational model. Therefore, in simplified model a non-propagating crack beam can be analyzed for static conditions.

Cantilevers beams are commonly used in bridge structures, where the cantilevers are made in form of pairs for supporting the central section. The Elland Road Stadium in Leeds and Forth Bridge in Scotland are famous examples of cantilever structures. The largest cantilever in Europe is located at St James' Park in Newcastle-Upon-Tyne and the home stadium of Newcastle United F.C. (Cf. “James's Park” 1999 and “Existing stadiums: St James' Park, Newcastle” 2005) In micro structures, the topic is focused due to use of small cantilever beam component near high temperature devices as plasma and laser beam sources. The author consider a cantilever beam whose modulus of elasticity (E) is 200,000 N-mm^{-2}, moment of inertia (I) is 6666.667 mm^{4}, effective moment of inertia (Ie) is 6368.104mm^{4} and load per unit area (w) is 1 N-mm^{-2}, for cracked cantilever beam (CCB) a crack of 20*1*1mm is considered. Fig.1 shows nodal deflection of simple cantilever beam (SCB).

2 Calculations and Discussion

2.1 Simple Cantilever Beam

2.1.1 FEM simulation

Using work equivalence method [5] the following equations were obtained where $\hat{\emptyset}_1, \hat{\emptyset}2, \hat{d}_{1y}, \hat{d}2y, \hat{m}_1, \hat{m}_2, \hat{f}_{1y}, \hat{f}_{2y}$ are slops, displacements, moments and forces respectively at two Consecutive nodes 1 and 2 as shown in fig. 2 (a and b).

$$\begin{Bmatrix} d_{2y} \\ \emptyset_2 \end{Bmatrix} = \begin{Bmatrix} \frac{-wL^4}{8EI} \\ \frac{-wL^3}{6EI} \end{Bmatrix} \tag{2.1}$$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{Bmatrix} = \begin{Bmatrix} \frac{wL}{2} \\ \frac{5wL^2}{12} \\ \frac{-wL}{8} \\ \frac{wL^2}{2} \end{Bmatrix} = \begin{Bmatrix} \frac{wL}{2} \\ \frac{5wL^2}{12} \\ \frac{-wL}{8} \\ \frac{wL^2}{2} \end{Bmatrix} - \begin{Bmatrix} \frac{-wL}{2} \\ \frac{-wL^2}{12} \\ \frac{-wL}{2} \\ \frac{wL^2}{2} \end{Bmatrix} = \begin{Bmatrix} wl \\ \frac{wL^2}{2} \\ 0 \\ 0 \end{Bmatrix} \tag{2.2}$$

$\hat{f}_{1y}$ is the vertical force reaction and $\hat{m}_1$ is the moment reaction as applied by the clamp support at node 1. The results for displacement given by eq. 2.1 and the global nodal forces given by the above eq. 2.2 are sufficient to complete the solution for the cantilever problem.

The shear force (F), bending moment (M), displacement (d), slop ($\emptyset$) were calculated by using following equations

$$F = w * L \tag{2.4}$$

$$M = ½ [w * L^2] \tag{2.5}$$

$$d = \left[\frac{-wL^4}{8EI}\right] \tag{2.6}$$

$$\emptyset = \frac{-wL^3}{6EI}$$

(2.7)

2.1.2 Beam Theory

To obtain the solution from beam theory, double integration method (Logan 2007) is used. Therefore, the moment curvature equation is given below,

$$y'' = \frac{M(x)}{EI}$$

(2.8)

Where the double prime superscript indicates differentiation with respect to *x* and M is expressed as a function of *x* by using a section of the beam as described below,

$$\sum F_y = 0; \quad V(x) = wL - wx$$

(2.9)

$$\sum M_2 = 0; \quad M(x) = \frac{-wL^2}{2} + wLx - (wx)\left(\frac{x}{2}\right)$$

(2.10)

Solving the above equations yield following relation,

$$y = \frac{1}{EI}\left(\frac{-wL^2x^2}{4} + \frac{wLx^3}{6} - \frac{wx^4}{24}\right)$$

(2.11)

From eq. 2.11 deflection obtained by substituting respective parameters. Values of displacement and load at different locations along the beam spam were calculated by using the assumed cubic displacement function.

$$\hat{v}(x) = \frac{1}{L^3}\left(-2x^2 + 3x^2L\right)d_{2y} + \frac{1}{L^3}(x^3L - x^2L^2)\phi_2$$

(2.12)

The deflection of beams is necessary to make several real world decisions. Many times, maximum deflection on a beam under a set of loads is required and where it occurs as well. By using the boundary conditions $\hat{d}_{1y} = \hat{\varnothing}_1 = 0$ in above equation, displacement at certain length of the beam was calculated.

Bending moments and shear forces in the present problem were evaluated based on FEM simulation and beam theory. The bending moment is given by

$$M = EIv'' = EI\,\frac{d^2(Nd)}{dx^2} = EI\,\frac{(d^2N)}{dx^2}\,d \tag{2.17}$$

As d is not a function of *x*. Or in terms of the gradient matrix B we have

$$M = EIBd \tag{2.18}$$

$$B = \frac{d^2N}{dx^2} = \left[\left(\frac{-6}{L^2}+\frac{12x}{L^3}\right)\left(\frac{-4}{L}+\frac{6x}{L^2}\right)\left(\frac{6}{L^2}-\frac{12x}{L^3}\right)\left(\frac{-2}{L}+\frac{6x}{L^2}\right)\right] \tag{2.19}$$

The shape functions are used to obtain the recent above equation for the B matrix. For the single element solution, the bending moment is then evaluated by substituting this above equation into M = EIB d and by multiplying B by d to obtain

$$M = EI\left[\left(\frac{-6}{L^2}+\frac{12x}{L^3}\right)\hat{d}_{1x}+\left(\frac{-4}{L}+\frac{6x}{L^2}\right)\hat{\phi}_1+\left(\frac{6}{L^2}-\frac{12x}{L^3}\right)\hat{d}_{2x}+\left(\frac{-2}{L}+\frac{6x}{L^2}\right)\hat{\phi}_2\right] \tag{2.20}$$

By evaluating the moment at the fixed end, x = 0, with $d_{1x} = \hat{\varnothing}_1 = 0$ and d_{2x} and $\hat{\varnothing}_2$ are given by eq. 2.6 and eq. 2.7 we have,

$$M(x) = -\frac{10wL^2 + 12wLx}{24} \tag{2.21}$$

Shear forces was calculated by using formulas of beam theory and FEM simulation, by using eq. 2.4; F = *w**L.

Beam normal stresses are computed from the flexure formula, which relates the internal moment and the beam cross-sectional properties to the normal stress. Transverse shear stresses were computed from the shear formula for beams, which relates the internal shear force and the beam cross-sectional properties to the shear stress. The internal forces and moments were used to compute the shear stress and normal stress, which were found from the shear force and bending moment diagrams.

Fig. 3 showed BMD and SFD of cantilever beam in MD Solid software. The bending moment was derived by taking two derivatives on the displacement function. It took more elements to model the second derivative of the displacement function. Therefore, the finite element solution function does not predict the bending moment as well as it does the displacement.

2.2 Cracked Cantilever Beam

Regardless of the method used for calculating deflections, a careful calculation is required to measure the moment of inertia for cracked cantilever beam. The trouble lies in the amount of cracking that has not occurred. If the bending moment is less than the cracking moment (that is, if the flexural stress is less than the modulus of rupture of about $7.5\sqrt{f_c'}$ for normal-weight concrete), the full un-cracked section provides rigidity, and the moment of inertia for the gross section I_g is available. When larger moments are present, variable sized tension cracks occurred and the position of natural axis varies. Effective moment of inertia was calculated by using Brandon equation. (Pytel and Singer 1987)

$$I_e = \left(\frac{Mcr}{Ma}\right)^m * I_{gt} + \left[1 - \left(\frac{Mcr}{Ma}\right)^m\right] * I_{cr} \qquad (2.32)$$

Where:

m = experimentally determined exponent

I_e = effective moment of inertia

Icr = cracked moment of inertia

Igt = Moment of inertia of transformed cross-section

Ma = applied moment
Mcr = cracking moment

Now values of above mentioned variables were calculated as mentioned below:

$$M_a = \frac{wL^2}{2} = \frac{1*200^2}{2} = 20{,}000\ N-mm \quad (2.33)$$

$$M_{cr} = \frac{f_r*I_g}{y_t} = \frac{9.0369*6666.667}{9} = 6694.065\ N-mm \quad (2.33)$$

I_g = gross moment of Inertia = 6666.667 N-mm

$$f_r = 0.7\lambda\sqrt{f_c'} = 0.7*1*\sqrt{166.67} = 9.0369\text{N-mm-}^2 \quad (2.34)$$

Where f_r is modulus of rupture and f_c' is load factor having specific value for each material.

$$I_{cr} = \frac{bd^3}{12} = \frac{10*19^3}{12} = 5715.83\ \text{mm}^4 \quad (2.35)$$

$$\mathrm{I_{gt}} = \frac{\mathrm{bd}^3}{12} + \mathrm{Ad}^2 = \frac{10*20^3}{12} + (10*20)*9^2 = 22866.667\ \ \text{mm}^4 \quad (2.36)$$

While practical results for rectangular cross-section give m= 3 and putting the values in equation 2.32

$$I_e = \left(\frac{6694.065}{20{,}000}\right)^3 *22866.667 + \left[1-\left(\frac{6694.065}{20{,}000}\right)^3\right]5715.83 = 6358.104\ \text{mm}^4 \quad (2.37)$$

Now all calculations will be done using the effect moment of inertia

$$d = \left[\frac{-wL^4}{8EI_g}\right] = \left[\frac{-1*200^4}{8*200{,}000*6358.104}\right] = -0.1572\ \text{mm} \quad (2.38)$$

The slope can be calculated by using the formula

$$\emptyset = \frac{-wL^3}{6EI} = \frac{-1*200^3}{6*200{,}000*6358.104} = -0.001048\ \text{rad} \quad (2.39)$$

The shear force and bending moment will be same as calculated for simple cantilever beam using because shear force and bending moment are not function of moment of inertia. By using the boundary conditions $\hat{d}_{1y} = \hat{\phi}_1 = 0$ in eq. 2.11 and eq 2.12, deflections at specified spam length were calculated.

2.3 Analysis using ANSYS

ANSYS (2006, 1) academic version is used for finite element analysis. This software is being used for solving structural, dynamic, electromagnetic and fluid flow problems. The Results obtained by ANSYS simulations are shown in fig 4 (a-b) for specific node.

2.4 Computer Programming

A specific programming code was made in C++ computer language, which can generate results for deflection, bending moment and shear force at any points by entering its location along beam length and moment of inertia. Fig. 5 showed result windows for considered node.

3 Conclusion

The theoretical and software based simulation work for simple bending problem were performed by considering example of cantilever beam. Numerical simulations for simple and cracked cantilever beams using finite element stiffness method, analytical beam theory and finite element package (ANSYS) were evaluated and then results were verified with code generated in C++ language. The results gathered by mathematical modeling are very beneficial. They enable a better understanding and ability to predict deformation process; however, when using the mathematical modeling assumptions may have to be made. In order to compensate, a factor of safety is always added into the equations when the results are going to be used in applications.

The author concluded that the beam theory solution for the mid length displacement y = -0.053mm is greater than the finite element solution for displacement v = -0.050mm. In general, the displacements evaluated using the cubic function for *v* are lower as predicted by the beam theory at particular nodes. The variations in deflection, shear force and bending moment results for simple and cracked cantilever beam obtained from finite element method and beam theory are presented in Fig. 6, Fig. 7, Fig. 8 and Fig. 9. The comparison of all results obtained from finite element method, beam theory, ANSYS and C++ code are summarized in Table 1. This is always true for the beam subjected to some form of distributed load that were modeled using the cubic function displacement. The exception to this result at the nodes where the beam theory and the finite element results are identical because of the work equivalence concept use which was used to replace the distributed load by work equivalent discrete loads at the nodes.

The analytical or exact solution gives results at any point or at any location in the structures. While the numerical solution is capable of generating results only at nodes, it is not capable of finding results between the two consecutive nodes. The FEM simulation shows less deflection at any specific point as compared with beam theory. The shear force curve shown in Fig. 8 represents the comparison between FEM simulation and beam theory, the curve of FEM simulation is a straight line representing constant value for shear force, which is due to the fact that the FEM does not have the capacity of evaluating results in between the nodes, while the slope of beam theory predicts results for all desired locations. This work hopes to be helpful in macro to micro designs to consider bending problems.

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Research Highlights

- Simple bending problem was analyses using simple and cracked cantilever beam example.
- Theoretical and Software based simulations were performed using finite element method, beam theory, ANSYS and code generated in C++.
- A competitive approach was developed to understand the results and there reliability obtained by theoretical and software methods.
- The constraints of finite element method and benefits of beam theory were highlighted.
- Special C++ code was developed to obtain instant results of deflection, shear force and bending moments by putting moment of inertia and any location along spam length

Figure Captions

Figure 1: Nodal solution for simple cantilever beam with uniform distributed load

Figure 2: (2a) Beam subjected to uniformly distributed load.
(2b)The equivalent nodal forces to be determined

Figure 3: BMD and SFD of cantilever beam in MD Solid

Figure 4: (4a) Nodal results of deflection in SCB,
(4b) Graphical representation of deflection in CCB

Figure 5: Result window of C++ at x=150mmfor CCB

Figure 6: Comparison of displacement obtained by FEM simulation and beam theory for SCB

Figure 7: Comparison of moment obtained by FEM simulation and beam theory for SCB

Figure 8: Comparison of shear force obtained by FEM simulation and beam theory for SCB

Figure 9: Comparison of displacement obtained by FEM simulation and beam theory for CCB

Table 1: Detailed results obtained by comparative study of simple and cracked cantilever beam.

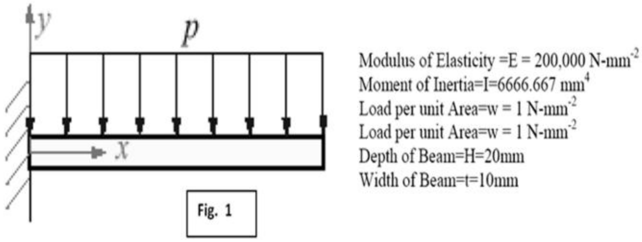


Figure 1: Nodal solution for simple cantilever beam with uniform distributed load

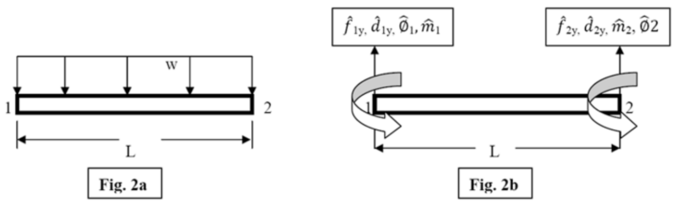


Figure 2: (2a) Beam subjected to uniformly distributed load.
(2b)The equivalent nodal forces to be determined

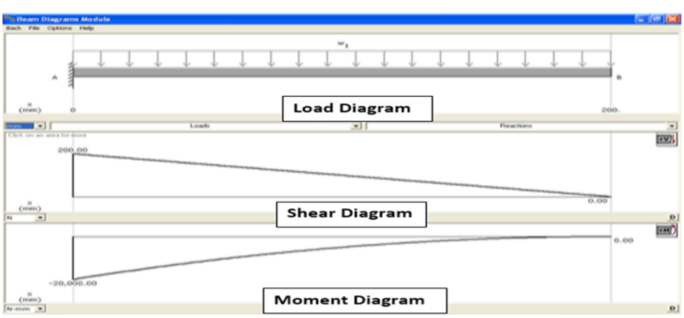
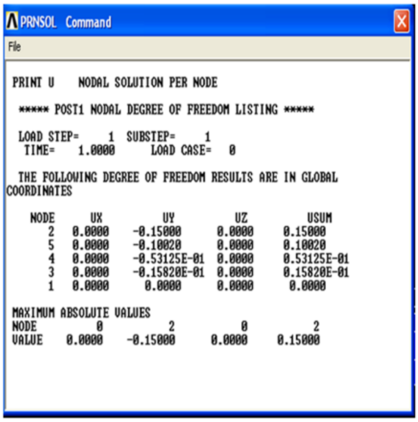
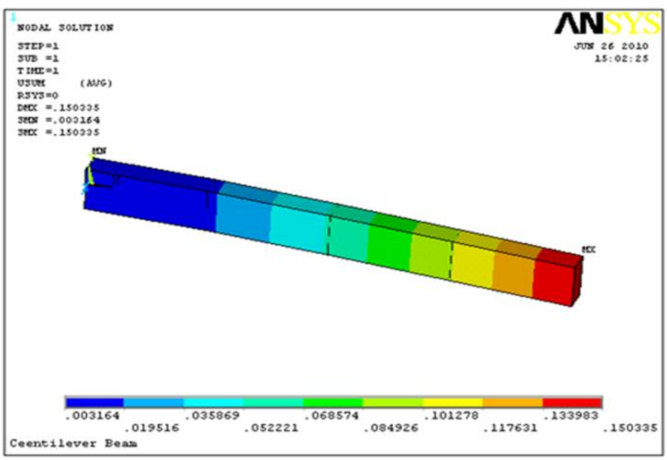


Figure 3: BMD and SFD of cantilever beam in MD Solid



```
PRNSOL Command
File

PRINT U    NODAL SOLUTION PER NODE

 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN GLOBAL
COORDINATES

    NODE      UX          UY          UZ          USUM
       2   0.0000    -0.15000     0.0000     0.15000
       5   0.0000    -0.10020     0.0000     0.10020
       4   0.0000    -0.53125E-01 0.0000     0.53125E-01
       3   0.0000    -0.15820E-01 0.0000     0.15820E-01
       1   0.0000     0.0000      0.0000      0.0000

 MAXIMUM ABSOLUTE VALUES
 NODE          0           2           0           2
 VALUE    0.0000    -0.15000     0.0000     0.15000
```

Figure 4: (4a) Nodal results of deflection in SCB



(4b) Graphical representation of deflection in CCB

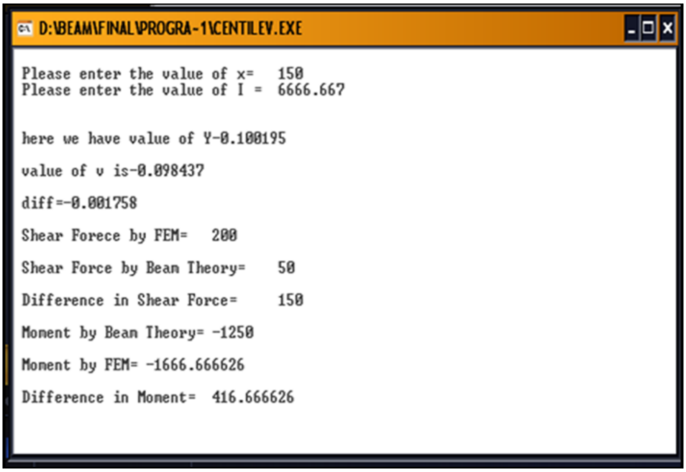


Figure 5: Result window of C++ at x=150mmfor CCB

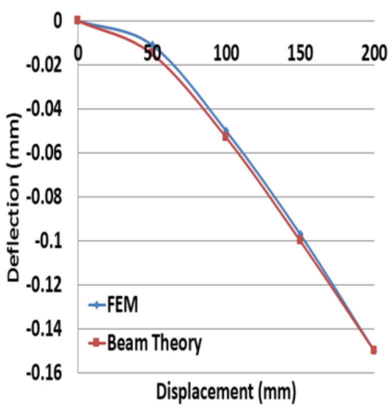


Figure 6: Comparison of displacement obtained by FEM simulation and beam theory for SCB

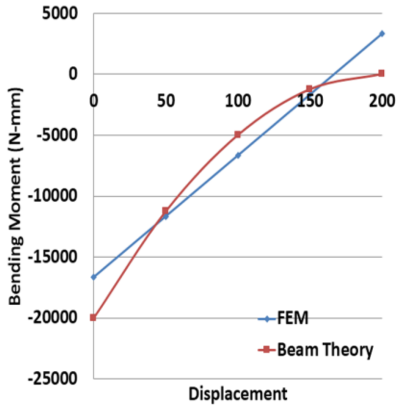


Figure 7: Comparison of moment obtained by FEM simulation and beam theory for SCB

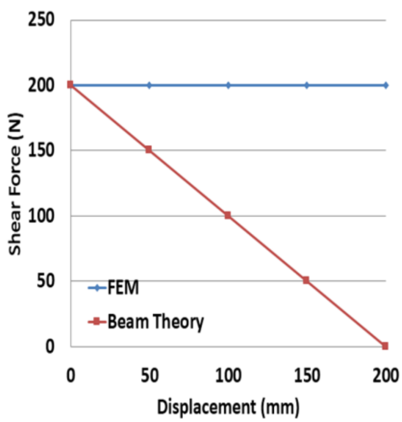


Figure 8: Comparison of shear force obtained by FEM simulation and beam theory for SCB

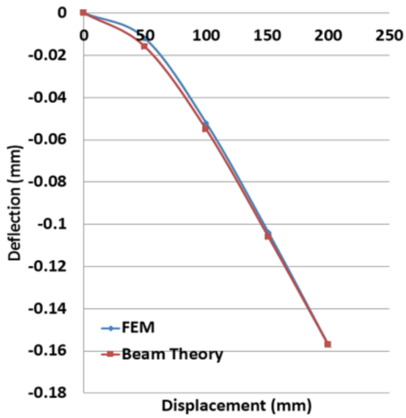


Figure 9: Comparison of displacement obtained by FEM simulation and beam theory for CCB

Results for Simple Cantilever Beam

Node No	x	FEM Analysis			Beam Theory			ANSYS 9 Analysis			C++					
											FEM Results			Beam Theory Results		
		Def.	B. M.	S F.	Def.	B. M.	S. F.	Def.	B.M.	S.F.	Def.	B.M	S.F.	Def.	B.M	S.F
	mm	mm	N-mm	N	mm	N-mm	N	mm	N-mm	N	mm	N-mm	N	mm	N-mm	N
1	0	0	-16666.67	200	0	-20000	200	0	-20000	200	0	-16666.67	200	0	-20000	200
2	50	-0.011	-11666.67	200	-0.015	-11250	150	-0.0158	-11250	150	-0.014	-11666.67	200	-0.015	-11250	150
3	100	-0.050	-6666.67	200	-0.053	-5000	100	-0.0531	-5000	100	-0.050	-6666.67	200	-0.053	-5000	100
4	150	-0.097	-1666.67	200	-0.100	-1250	50	-0.1002	-1250	50	-0.098	-1666.67	200	-0.100	-1250	50
5	200	-0.149	+3333.33	200	-0.150	0	0	-0.1500	$-0.9e^{-11}$	$0.5e^{-12}$	-0.150	+3333.33	200	-0.150	0	0

Results for Cracked Cantilever Beam

Node No	x	FEM Analysis			Beam Theory			ANSYS 9 Analysis			C++					
											FEM Results			Beam Theory Results		
		Def.	B. M.	S F.	Def.	B. M.	S. F.	Def.	B.M.	S.F.	Def.	B.M	S.F.	Def.	B.M	S.F
	mm	mm	N-mm	N	mm	N-mm	N	mm	N-mm	N	mm	N-mm	N	mm	N-mm	N
1	0	0	-16666.67	200	0	-20000	200	0	-20000	200	0	-16666.67	200	0	-20000	200
2	50	-0.012	-11666.67	200	-0.016	-11250	150	-0.0158	-11250	150	-0.015	-11666.67	200	-0.016	-11250	150
3	100	-0.052	-6666.67	200	-0.055	-5000	100	-0.0531	-5000	100	-0.052	-6666.67	200	-0.055	-5000	100
4	150	-0.104	-1666.67	200	-0.106	-1250	50	-0.1007	-1250	50	-0.103	-1666.67	200	-0.105	-1250	50
5	200	-0.157	+3333.33	200	-0.157	0	0	-0.1506	0	0	-0.157	+3333.33	200	-0.157	0	0

Table 1: Detailed results obtained by comparative study of simple and cracked cantilever beam.

ACKNOWLEDGMENTS

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